Mathematical Model of a Compensating Device with a Movable Screen

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**Abstract.** This article explores the influence of the displacement of a movable screen within the magnetic system of a compensating device-developed by the authors and equipped with a movable electromagnetic screen—on the distribution of magnetic flux and magnetic stresses along the magnetic circuit. The study presents theoretical expressions for magnetic flux and magnetic potential distributed along the magnetic circuit, using a calculation method that divides the circuit into discrete segments. These expressions apply to cases where the active resistance of the movable screen is either infinite or known. The analysis of these expressions reveals that magnetic flux varies nonlinearly along the magnetic circuit and reverses direction at the magnetic neutral point. Furthermore, a decrease in the distribution coefficient enhances linearity, while magnetic potential also exhibits nonlinear variation along the circuit.

**Keywords:** compensating device, magnetic capacitance, magnetic circuit, magnetic flux, magnetic potential, mathematical model, movable screen.

# INTRODUCTION

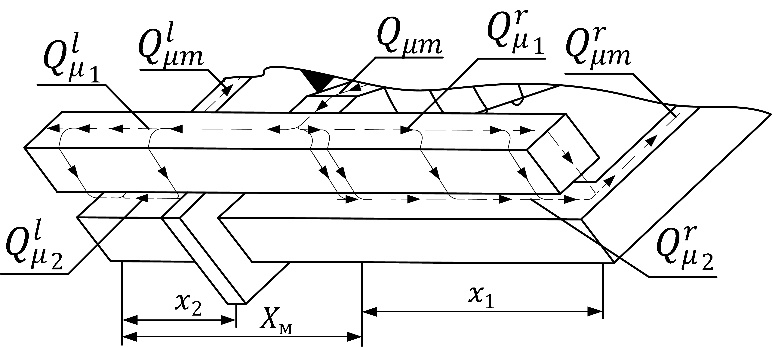
Moving electromagnetic screen (MES) compensating devices (transducers) are a type of measuring equipment used for non-contact measurement of direct and alternating currents. Analysis of several devices of this type has revealed that they typically exhibit relatively low sensitivity and a narrow linear range in their static characteristics. Significant contributions to the development of moving screen and distributed parameter transducers—as well as to the improvement of their design, theoretical foundations, and implementation in the production of magnetic cores—have been made by numerous researchers, including D.D. Israel, Ch.J. Cyril, D.W. Shen, J.R. Andresen, N. Burnell, C. Farrand, L.F. Kulikovsky, B.K. Bul, P.A. Butyrin, K.S. Demirchyan, M.F. Zaripov, M.I. Bely, N.E. Konyukhov, A.A. Koltsov, M.A. Urakseev, Y.R. Abdullaev, B.N. Lichtzinder, L.A. Brovkin, O.V. Tarkhanov, I.A. Limanov, R.K. Azimov, S.F. Amirov, and N.A. Akhrarov, among others. [13, 14, 15, 16, 17]. At the same time, there has been insufficient research on the development of moving screen and distributed parameter transducers with high sensitivity, accuracy, reliability, and linearity of static characteristics—specifically those intended for monitoring and control systems in railway power supply facilities. Additionally, the theoretical foundations of magnetic circuits within this category remain underdeveloped. A more in-depth study of the magnetic systems of these transducers is essential, including the selection of methods that yield high accuracy in calculating electrical and magnetic parameters, as well as identifying existing limitations and developing measures to address them. In response to these challenges, the authors have developed a new MES compensating device that overcomes the aforementioned shortcomings [1, 2, 3, 4].

New designs of MES compensating devices with enhanced measurement accuracy, sensitivity, and linearity of the conversion function have been proposed by optimizing the structure of their magnetic circuits and the placement of magnetically sensitive elements.

**METHOD OF RESEARCH**

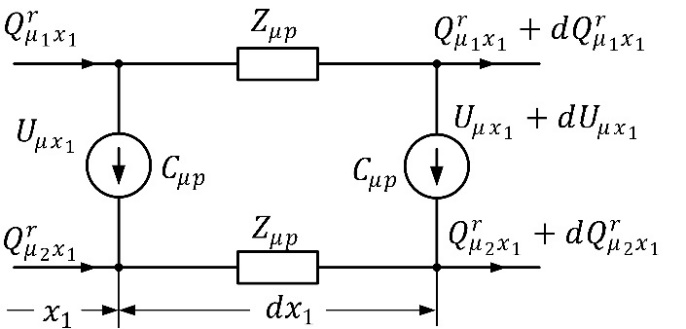
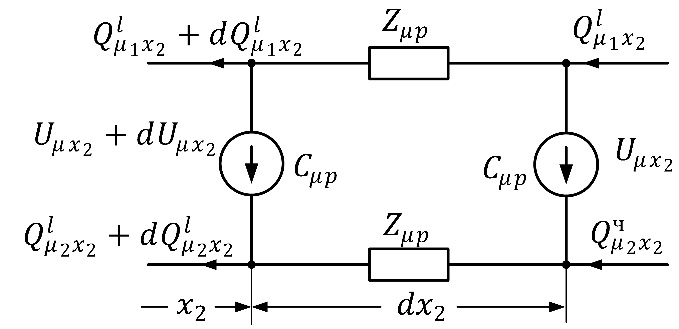
To calculate the magnetic circuits of the newly developed device, the most convenient and accurate method - used specifically for magnetic circuits with a movable electromagnetic screen (MES) - was applied. This method involves dividing the magnetic circuit into discrete sections. Based on this approach, expressions were derived for the distribution of magnetic flux and magnetic field strength within the device's magnetic system, considering cases in which the MES has varying resistance values[5, 6, 7].

In the case where the MES has infinitely large resistance, the elementary sections of the device’s magnetic system are constructed separately for the left and right sides relative to the MES. To perform this analysis, it is necessary to determine the distribution of magnetic flux along the magnetic circuit under consideration (see Fig. 1).

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**FIGURE 1.** Distribution of magnetic flux in the magnetic system of the compensating device

Based on Kirchhoff’s laws, a differential equation is constructed for the magnetic field and magnetic flux generated by the magnetizing force of the excitation loop (QEL), along the coordinates and of the magnetic circuit corresponding to the elementary sections and , Through a series of relatively straightforward transformations, the following equations are obtained (see Figure 2) [8, 9, 10, 11, 12].



**FIGURE 2.** Schematic representation of alternating sections of the magnetic circuit with elementary lengths and

, (1)

, (2)

, (3)

, (4)

(5)

(6)

Where is ; – The magnetic fluxes on the left and right sides of the MES in the parallel sections of the “T” and “O” shaped magnetic circuit, respectively, and the magnetic force between them; , – the magnetic resistances of the "T" and "O"-shaped rods and the pagon values of the magnetic capacity between them, respectively; - The magnetic permeability of a magnetic conductor material; , - “О” The distance along which the MES can move along the rod-shaped structure and its maximum value .

By differentiating equations (5) and (6) and substituting the resulting expressions into equations (1) through (4), the following second-order differential equations are obtained:

(7)

(8)

It is known that the general solution of differential equations (7) and (8) has the following form:

(9)

(10)

Where is – constants of integration, ; – the speed of propagation of the magnetic field in the magnetic circuit.

**RESEARCH RESULT**

Using expressions (5), (6), (9) and (10) and the maximum magnetic flux expressions, we obtain the following expressions for the magnetic fluxes , we form the following expressions for magnetic fluxes [7]:

(11)

(12)

(13)

(14)

– we determine the integration constants based on the following boundary conditions:

(15)

(16)

(17)

(18)

By substituting the boundary conditions (15)-(18) into the expressions (11)-(14), we solve the resulting system of equations to determine the integration constants [6].

(19)

(20)

(20)

(21)

– we put the defined expressions of integration constants into equations (9)-(14) and form the following expressions:

(22)

(23)

(24)

(25)

(26)

(27)

To determine the maximum magnetic flux in the previously derived expressions, we apply Kirchhoff’s second law to both the left and right sections of the magnetic circuit. Based on these equations, we derive expressions for the maximum magnetic flux on each side of the circuit:

*,*  (28)

*,* (29)

Where is – Magnetic driving force of EC, ; - Magnetic resistance of the base of the T-shaped magnetic conductor.

*,*  (30)

*,*  (31)

*,*

*.*

Taking into account expressions (30) and (31), expressions (22)-(27) become as follows:

(32)

(33)

(34)

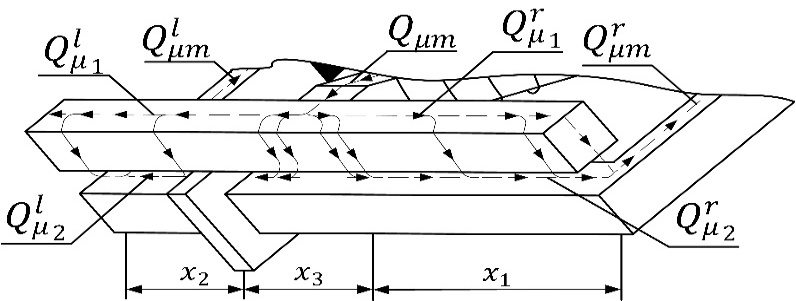
(35)

(36)

(37)

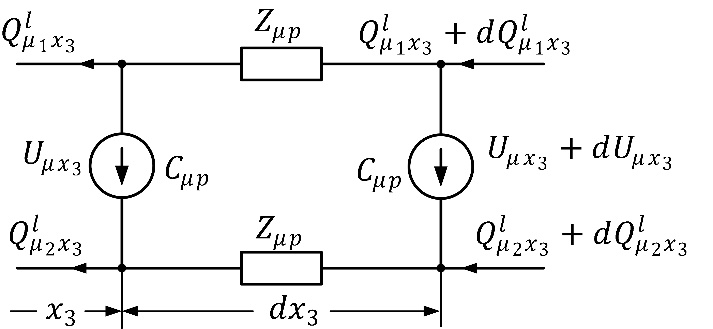
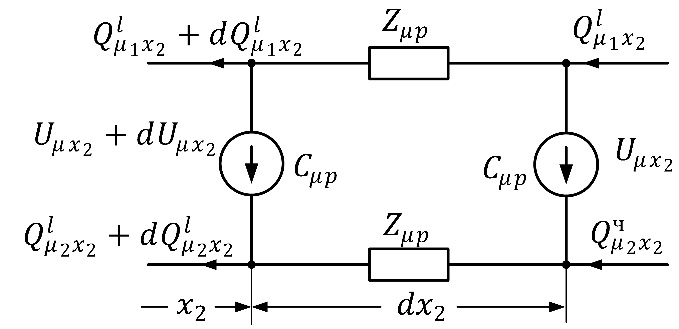
**ANALYZES AND DISCUSSION**

Now, for the case in which the MES has a known resistance, we construct the elementary section exchange schemes for the left and right sides of the device’s magnetic system relative to the MES. To accomplish this, the distribution of magnetic flux along the magnetic circuit under consideration is determined (see Figure 3).



**FIGURE 3.** Distribution of magnetic flux in the magnetic system of the compensating device

The sizes of the elementary plots are identical to the differential equations (35)-(37). Therefore, in this section we will consider the sequence of determining differential equations related to the elementary parts and .



**FIGURE 4.** Schemes of alternating sections of the magnetic circuit of elementary lengths and

According to the given exchange schemes, for the corresponding elementary sections, we construct differential equations for the magnetic field and magnetic flux generated by the magnetizing force of the EL based on Kirchhoff's laws for the elementary sections and of the magnetic circuit along the coordinates and , and by making some not very complicated changes, we obtain the following equations:

. (38)

(39)

It is known that the general solution of differential equations (38) and (39) has the following form:

, (40)

, (41)

Using the expressions (38), (39), (40) and (41) and the expression of the maximum magnetic flux, we create the following expressions for magnetic fluxes:

(42)

(43)

(44)

(45)

and we determine the integration constants based on the following boundary conditions:

(46)

(47)

(48)

(49)

By substituting the boundary conditions (46)-(49) into expressions (40)-(45), we solve the resulting system of equations and determine the integration constants and .

, (50)

, (51)

, (52)

(53)

Where is , .

ва putting the defined expressions of integration constants into equations (40)-(45), we form the following expressions:

(54)

(55)

(56)

(57)

(58)

(59)

To determine the maximum magnetic flux in the last expressions, we write an equation for the left side of the magnetic circuit in question according to Kirchhoff's second law, and based on this equation, we determine the expression for the maximum magnetic flux on the left side:

*,* (60)

By putting the appropriate quantities based on the boundary conditions in this equation, the expression of the maximum magnetic flux becomes the following:

(61)

Where is .

Taking equality (60) into account, expressions (54)-(59) become as follows:

(62)

(63)

(64)

(65)

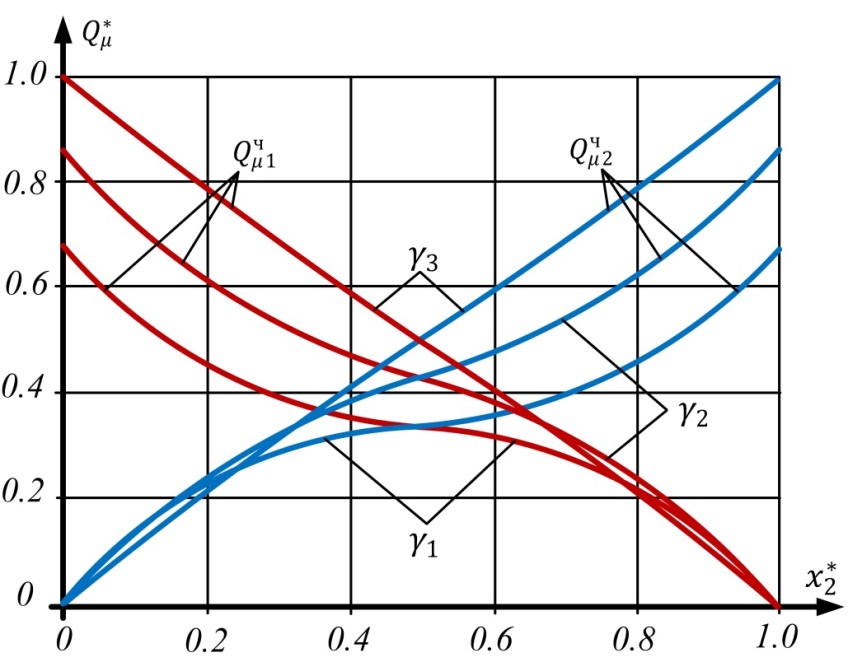
(66)

(67)

All calculations of moving screen magnetic circuits were performed manually using theoretical expressions. It is also possible to perform the calculations in Matlab to obtain fast and high-precision results.

**CONCLUSION**

Based on the expressions for magnetic flux distribution along the magnetic circuit—derived for cases where the MES has infinite active resistance and a known resistance—it is possible to construct curves illustrating the dependence of magnetic flux on the displacement of the moving screen (see Figure 5).



**FIGURE 5.** Curves showing the dependence of magnetic flux on MES displacement at different values of the diffusion coefficient continuous curves-theoretical; continuous curve-experimental

Analysis of the magnetic flux and magnetic potential curves in the magnetic circuit of the developed device showed that the magnetic flux varies nonlinearly along the circuit and reverses direction at the magnetic neutral point. As the value of the dispersion coefficient decreases, the linearity of the flux distribution improves. Additionally, the magnetic potential also exhibits nonlinear variation along the circuit. Based on the conducted studies, it was determined that the difference between the theoretical and experimental results does not exceed 6–9%.

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