**Conductivity, absorption, and reflection of a wave at the end of a T-shaped graph**

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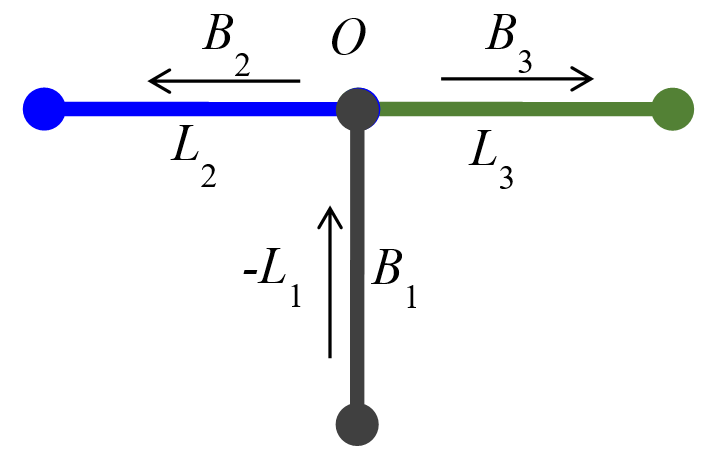
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**Abstract.** The branched structures can be observed in most of physical and natural systems. The study of dynamics of the wave and particle’s flow on the base of evolution equations which are determined on the metric graphs with the boundary conditions is the actual problems.

**INTRODUCTION**

Branched structures appear widely in physical and natural systems, such as optical fibers, nanowires, and biological networks. Understanding the wave and particle dynamics in such systems requires studying the boundary conditions defined on metric graphs. Despite recent developments, the mathematical modeling of wave propagation on branched graphs remains insufficiently explored. This study focuses on analyzing the conductivity, absorbency, and reflectivity of waves at the end of a T-shaped graph. [1].



**FIGURE 1.** Simple metric graph obtained by combining three segments at one point O.

In the given article on the simple metric graph in the T - form is constructed the solutions of Klein-Gordon’s equation for study of physical process. The structure which we studies constructed the following [2, 3]:

The region which we regarded consist of three segments crossing by the one point, moreover the point O is the graph vertex (figure 1).

**METHODOLOGY**

The connections (segments) of this graph we indicates via *В*1, *В*2 and *В*3. The coordinates of connection *В*1 is (–*L*1, 0). The coordinates of *В*2 and *В*3 connections are determined as (0, *L*). Here each connection of the point O has coordinate is equal zero.

We consider the simple graph founded from three segments uninfected in one point O. The first segment *B*1 of this graph is [-*L*1, 0], the second segment *B*2 is [0, *L*2] and third segment *B*3  is [0, *L*3]. Moreover, on each segment for the point O corresponds of the coordinate 0.

On each connection of the graph the process is determined the equation



   (1)

here *а* = 1 *k* = 1, 2, 3.

On the borderline points of the graph:

 (2)

We input the boundary conditions. On the graph vertex the continuous and Kirchhoff’s conditions:

 (3)

 (4)

For the correct formation of the problem of determination of the wave with mathematical point of view is to input two initial conditions:

  (5)

This solutions of the intended problem with the initial conditions (5), from its the initial functions φ(*x*) and η(*x*) can be satisfies to (5) only by cases. For this purpose, we consider the wave packet with arbitrary initial form (*t* = 0), the solution corresponds to wave packet.

As the equation (1) is the linear and homogeneous that sum of its eigen solutions also is the solution of given equation



 (6)

and satisfies to boundary conditions (2). The initial conditions allows to determine of coefficients  and . We will demand in such a way the relation (6) satisfies to conditions (3), (4), that is





 (7)

It is known from the theory of Fourier series, determined on the interval –*L* ≤ *x* ≤ *L* continuous and differentiate arbitrary function *f*(*x*) can be expanded into Fourier series. The functions φ(*x*) and η(*x*) which are determined on the graph can be expanded into Fourier series on the system  functions. In this case, with prevision for the function system  is the orthogonal and total one we obtains

 (8)

Similar obtains





 (9)

Thus, the function (6) which is the solution of studying problem was total determined.

**RESULTS**

Analytical solutions were derived for each segment using eigenfunction expansion techniques. The continuity and Kirchhoff conditions determined the relationships among amplitude coefficients, which describe the transmission and reflection of waves between branches. The results show that:

Reflection increases with asymmetry in branch lengths (*L*1, *L*2, *L*3);

Conductivity improves when the graph segments are symmetric;

Wave absorption depends on the relative impedance and potential boundary damping.

These findings confirm that wave behavior in branched structures can be controlled by adjusting boundary geometry and physical parameters.

**DISCUSSION**

The obtained results are consistent with known models of wave propagation in quantum graphs and photonic lattices. The T-shaped structure provides a simplified representation of complex branched systems in nanoelectronics. The study highlights the importance of vertex boundary conditions in determining the stability and transmission efficiency of wave motion.

**CONCLUSION**

This research demonstrates that the Klein–Gordon equation on a T-shaped metric graph successfully models the interplay between wave reflection, absorption, and conductivity. The analytical approach provides useful insights for designing branched nanostructures and waveguides with desired transmission characteristics. Future work may include numerical simulations and experimental validation for nonlinear and dissipative cases.

**REFERENCES**

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