**Mathematical modeling and analysis of the stability of an autonomous station based on an asynchronous generator with a short-circuited rotor**

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**Abstract.** The article presents a mathematical model of an autonomous station based on an asynchronous generator (AG) with a short-circuited rotor. The model is built on the Park-Gorev system of equations, which describes electromagnetic and electromechanical processes when the load and parameters of the external circuit change. A linear approximation of the system of equations was carried out, the characteristic equation of the autonomous station was obtained, and its stability conditions were determined based on the Hurwitz criterion. It has been shown that the change in output voltage and frequency of the asynchronous generator during load fluctuations can be stabilized by automatically adjusting the capacitance of the capacitor cell, which maintains the constancy of the main magnetic flux. It has been established that the choice of optimal capacitance is a key factor in ensuring the stable operating mode of the autonomous generator.

**INTRODUCTION**

Short-circuited rotor asynchronous generators are widely used in autonomous power sources due to their simple design, high reliability, and affordability. In autonomous mode, the asynchronous generator provides stable output parameters (voltage and frequency) with varying load. However, due to the nonlinear nature of electromagnetic processes, as well as the dependence of generation on reactive power, precise mathematical description is required for stability analysis and regulation.

For asynchronous generators operating on self-excitation from a capacitor battery, it is especially important to determine the conditions under which the system remains stable and is able to maintain the required voltage level. This work presents a mathematical model, its linearization, and an analysis of the stability of the considered autonomous station.

The work serves as a theoretical basis for developing control systems for autonomous generators used in wind power plants, mini-hydroelectric power plants, and mobile power plants [1, 2].

**EXPERIMENTAL RESEARCH**

The Park-Gorev system of equations, which represents the equation of nonlinear motion of electromagnetic and electromechanical moments, reflecting the dependence of the state parameters of the asynchronous generator on external disturbances, has the following form:

|  |  |
| --- | --- |
| = – ωK –  = – ωK –  = +  = – (SK – SA) +  = – (SK – SA) +  = Mт – МА  = –  = –  =  = +(–id)  = + (– ) | (1) |

The following notations are introduced in the above equations:

Ud, Uq, Ѱd, Ѱq – components of the asynchronous stator voltage generator and the flow clutches on the longitudinal and transverse axes; rs, rr, rrd – active resistances of the stator, rotor, and generator damper windings;

x1d, x1q, xm – components and mutual inductive resistances on the longitudinal and transverse axes of the damper winding's inductive resistance; id, iq, idr, iqr, ir1d, ir1q - longitudinal and transverse components of the generator stator, rotor, and damper windings currents;

Electromagnetic moment of AG: MА = Xm [iqidr–idiqr] (1+Sк);

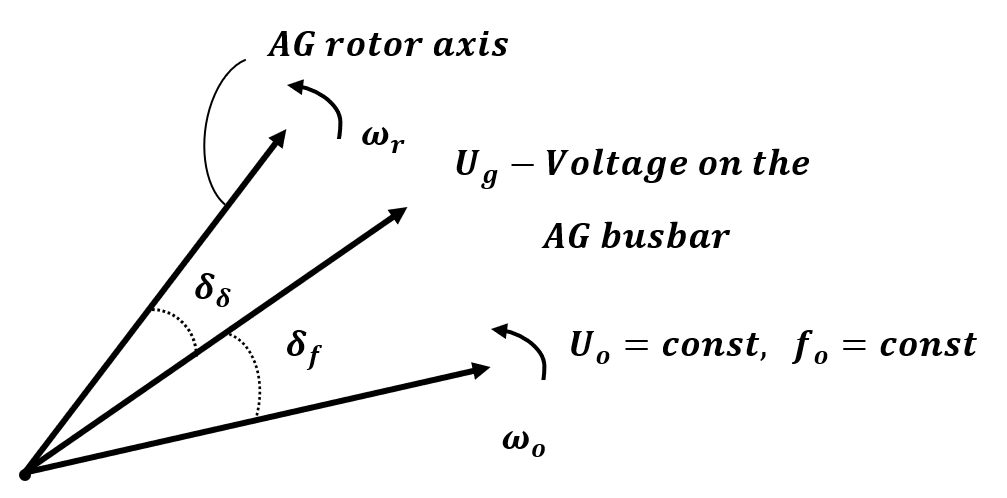
MТ – rotational torque of the primary motor (pipe);

– AG sliding; ωr=ω0(1+SA) – rotor rotation speed;

ω0 – synchronous speed; SK – sliding relative to the coordinate axes.

Let's assume that due to a slight push (turtle) in the system, there is a slight change in the power and sliding of the blood pressure: РА=РА0±ΔР, SА=SА0±ΔS.

The AG slip consists of two components: the slip associated with the change in voltage in the bus and the slip associated with the rotational speed of the primary motor. SA=SAf + SAT; expression in another form: δδA= δf + δr.



**FIGURE 1.** Slippage in AG

To linearize the given system of nonlinear equations for small oscillations with Taylor series decomposition, the following equation of characteristics of an autonomous operating system consisting of one generator was obtained [3, 4].

λ3 + (–С1 – Р1 – D2)λ2 + (P1C1 + С1D2 + Р1D2 – С2D1)λ +(С2P1D1 – С1P1D1) = 0 (2)

where:

j – constant of inertia, reduced to the angular frequency ω;

E, U – EMF and generator voltage; х1 – inductive resistance of the generator;

δ – internal angle of the machine (angle between the voltage and EMF vectors); хЮ, хҚ.С, хС – respectively, the inductive resistance of the load, the excitation resistance, and the variable capacitance of the capacitor.

According to the Hurwitz criterion, the stability of such a system is ensured under the following conditions:

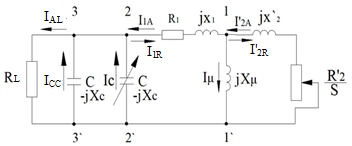
а1 = (–С1 –Р1 – D2) > 0;

а1а2 – а0а3 = С1P1D1 + (С1 + Р1)(D1D2 + С1P1 + С2D1) – С2D12+ D2(С1 + Р1)2 + +Р1(С2D1 – С1D2) > 0. (3)

C2P1D1- C1P1D2 = 0 stability reserve:

(4)

where: and it becomes clear that to increase stability, it is necessary to reduce the capacitive resistance. The value of the capacitive resistance is determined based on the parameters of the AG and the load values. The measures for ensuring stability, determined by solving the equations, correspond to the analysis based on the "T" form of the equivalent substitution scheme for AG, presented in Figure 2 [5, 6].



**FIGURE .2.** Scheme of equivalent replacement of AH in the form of "T"

The replacement scheme consists of the following branches:

1. The reactive conductivity of the magnetization branch consisting of Xμ, is 1/Xμ=b0;

2. R'2/S - the main branch of reactive conductivity, consisting of the active resistance 1/X'2=b'2;

3. Reactive conductivity of a branch consisting of a capacitor with variable capacitance 1/Xc=bc;

4. The reactive conductivity of the excitation branch, consisting of a capacitor with constant capacitance, is equal to 1/ХC.С=bC.С;

5. Reactive conductance of a branch consisting of a mixed load, 1/ХЮ=bЮ;

Here: I1A - is the active component of the AG current.

I1R – is the reactive component of the AG current.

I'2А – active component of the AG rotor current.

I'2R – is the reactive component of the AG rotor current.

IА.L – active current load.

ICС – Capacitor excitation current.

Iµ – is the reactive current of the magnetizing branch AG.

When the AC load changes, the stability of its initial nominal parameters can be ensured by regulating the rotational speed of the primary motor or by controlling the AC magnetic flux using capacitor banks.

The output voltage of the AG depends on the reactive current Iμ of the magnetization branch in the replacement circuit, where;

(5)

*IК =U1ω*1*С* – capacitor current.

Output frequency of AG:

(6)

In this case; *E*- EMF arising in the stator phase

Ф- The main magnetic flux creating EMF *E*.

**RESEARCH RESULTS**

From the above, it follows that the preservation of the output voltage and frequency of the AG during load changes occurs due to ensuring the constancy of the main flow Ф by automatically regulating the capacitance of the capacitor connected to it.

In the classical type of excitation, the source of reactive energy of which is a battery of capacitors, the stored energy of the excitation unit is characterized by the following expression [7, 8]:

(7)

where U - operating voltage, V;

q - charge between the capacitor plates, C;

C - capacitance of the capacitor, F.

The most well-known and frequently encountered formula for calculating reactive power is:

(8)

The quantities that determine the power are the voltage across the capacitor and its capacitance. By expressing the voltage through the value of the charge on the plates, it is possible to obtain other types of connections. However, the most common and convenient form is the function, which includes a quadratic dependence on the voltage and a linear dependence on the capacitance value [9, 10].

Depending on the rotor speed of the electric machine, it is possible to estimate the required capacitance value of the capacitor. On the one hand, the frequency of generated electrical energy is determined by the phase capacitance and the inductance of the magnetizing circuit of the asynchronous machine.

On the other hand, based on the dependence of the rotor rotation frequency of the asynchronous machine, the frequency of the supply or generated voltage is determined as follows [11,12]:

(9)

On the other hand, based on the dependence of the rotor rotation frequency of the asynchronous machine, the frequency of the supply or generated voltage is determined as follows:

(10)

From the proposed formula for calculating the value of the capacitor's phase capacitance, it follows that this value depends not only on the inductive reactance of the magnetizing circuit but also on the number of pole pairs (p) and the rotor rotation speed (n), however, the verification was carried out under the condition that the rotor rotation frequency is close to and slightly higher than the synchronous frequency. The resulting expression gives the same results as the empirically obtained values. Choosing a value exceeding the calculated value allows the AG to operate under an active load not exceeding 10% of the nominal value. The final choice of the condenser battery's phase capacity should be made based on the maximum value and nature of the AG load [13, 14].

The form of capacitance dependence on the value of the power factor at the nominal load has been empirically determined:

(11)

where .

Maintaining a constant rotational speed at its nominal value allows us to consider the *p2n2*product constant with a value of *p2n2=9·106* which leads to the following expression (1.15):

(12)

The magnetization inductance *Lm* is higher the lower the load power. This value changes depending on the saturation of the electric machine's steel in working condition [15].

**CONCLUSIONS**

1. It is established that a change in load causes deviations in output voltage and frequency, but the system can be stabilized by automatically regulating the capacitance of the capacitor bank.

2. The constant magnetic flux Φ is a key condition for the stability of an autonomous asynchronous generator.

3. Based on the Park-Gorev equations, a model was obtained that allows for the analysis of generator stability and the development of automatic control systems.

4. The Hurwitz criterion allows us to establish a range of parameters under which an autonomous station remains stable.

5. Optimal capacitance selection of capacitors should take into account both the parameters of the AG and the type and level of load; excess capacitance leads to instability.

6. The research confirms the need for dynamic control of capacitor capacitances for high-precision autonomous power systems.

**REFERENCES**

1. Allayev, K.R., Fedorenko, G.M.,Postnikov, V.I.,Ostapchuk, L.B. Asynchronous generators as power system’s natural dampers. 43rd International Conference on Large High Voltage Electric Systems 2010, CIGRE 20102010, 9p43rd International Conference on Large High Voltage Electric Systems 2010, CIGRE 2010; Paris; France; 22 August 2010.https://doi.org/10.1051/e3sconf/202021601177
2. Fazylov, Kh.F., Allaev, K.R. Analysis of the operation of an electrical system during simultaneous operation of synchronous and asynchronous generators. Power engineering New York Volume 18, Issue 3, 1980, Pages81–88. https://doi.org/10.1088/1757-899X/163/1/012033
3. Fazylov, Kh.F., Allaev, K.R. Asynchronous turbogenerators with stator excitation and the prospects for their utilization. Power engineering New York Volume 23, Issue 2, 1985, Pages7–13. <https://doi.org/10.1051/e3sconf/202021601177>
4. Ibadullaev, M., Esenbekov, A., Muratov, A., Kurbaniyazov, T., & Berdanov, T. A. (2025, November). On damping mechanical vibrations in an electromagnetic vibration exciter. In American Institute of Physics Conference Series (Vol. 3331, No. 1, p. 040003). <https://doi.org/10.1063/5.0306124>
5. Kurbaniyazov, T.U., Nietbaev, A.D., Najmatdinov, K.M., Kadirov, Y.B. (2025). Mathematical Modeling the Processes of Converting Asymmetric Three-Phase Reactive Power Currents into Secondary Voltage. In: Stanimirović, P.S., Mourtas, S.D., Sahoo, J.K. (eds) Hybrid Methods for Modeling and Optimizing Complex Systems. HMMOCS 2024. Lecture Notes in Networks and Systems, vol 1481. Springer, Cham. <https://doi.org/10.1007/978-3-031-95649-2_5>
6. Siddikov, I. K., Abubakirov, A. B., Djalilov, A. U., Kurbaniyazov, T. U., & Abdumalikov, A. A. (2023). Statistical descriptions of multiphase current sensers of reactive power control systems in renewable power supply power systems. Problems in the textile and light industry in the context of integration of science and industry and ways to solve them:(PTLICISIWS-2022), 2789(1), 060002. <https://doi.org/10.1063/5.0145430>
7. Fazylov, Kh.F., Allaev, K.R. Calculation and experimental analysis of conditions of electrical power systems containing induction generators Power Engineering New York Volume 27, Issue 6, 1989, Pages27–34.
8. Yu.Bobozhonov, B. Seytmuratov, B. Fayzullaev, A Sultonov. Study of the influence of different designs of massive rotor of asynchronous generator on their maximum power // E3S Web of Conferences 216, 01168 (2020) RSES 2020 (Scopus). https://doi.org/10.1051/e3sconf/202021601168.
9. Yu.M. Bobozhonov, K. M. Reymov, B.T. Seytmuratov, Khakimov T.Kh. Research of the dependence of the resistance of asynchronous generators with massive rotors on their design // E3S Web of Conferences 384. 2023. РР, 01042, 1-4. <https://doi.org/10.1051/e3sconf/202338401042>.
10. Yu.Bobozhonov, B. Seytmuratov, B. Fayzullaev, A Sultonov, Sh. H. Husanov. Calculated studies of the vibrational properties of the mode parameter of the electric power system containing asynchronous turbogenerators by their frequency characteristics // E3S Web of Conferences 289 EDP Sciences 2021 (Scopus) <https://doi.org/10.1051/e3sconf/202128907025>.
11. Bobojonov Y.M., Reymov K.M., Seitmuratov B.T. Development of two-speed asynchronous electric motors for the undercarriage of mine self-propelled cars // E3S Web of Conferences 384. 2023. РР, 01044, 1-5. <https://doi.org/10.1051/e3sconf/202338401044>.
12. Abubakirov A., Kurbaniyazov T., Bekimbetov M. Analysis of three-phase asymmetrical currents in the secondary voltage of signal change sensors in the power supply system using graph models //E3S Web of Conferences. – EDP Sciences,2024.–Т.525.–С.03013. <https://doi.org/10.1051/e3sconf/202452503013>
13. Siddikov, I., Kurbaniyazov, T., Esenbekov, A., Najimatdinov, K., & Kushmonov, E. (2025, November). Three-phase electromagnetic current transducers for control of reactive power of electricity consumption. In AIP Conference Proceedings (Vol. 3331, No. 1, p. 030035). AIP Publishing LLC. <https://doi.org/10.1063/5.0306170>
14. Bobojonov Y.M., Saidkhodjaev A.G. Reactive power compensation for sustainable development of power grids in cities of Uzbekistan // E3S Web of Conferences 384, 01040 (2023). doi.org/10.1051/e3sconf/202338401040.
15. F.Mamarasulova, Y.Bobojonov, Sh.Djurayev, N.Karimova, Stimulating environmental protection activities in the energy sector // 202346101099E3S Web of Conferences 461, 01099 (2023), 1-4. https://doi.org/10.1051/e3sconf/202346101099.
16. Melikuziev M.V. Determination of the service area and location of transformer substations in the city power supply system // E3S Web of Conferences 384. 2023. РР, 01033, 1-5. <https://doi.org/10.1051/e3sconf/202338401033>.
17. Bobojonov Y.M., Saidkhodjaev A.G. Critical evaluation of energy use in industrial enterprises // E3S Web of Conferences 384. 2023. РР, 01048, 1-5. <https://doi.org/10.1051/e3sconf/202338401048>.
18. I U Rakhmonov, K M Reymov, A M Najimova, B Uzakov and B T Seytmuratov. Analysis and calculation of optimum parameters of electric arc furnace // Journal of Physics: Conference Series 1399 (2019) 055048, 1-5. doi:10.1088/1742-6596/1399/5/055048.
19. M Erejepov, Novikov A N, B M Khusanov, Bayram Seytmuratov, Z Sayimbetov. Algorithm for estimating the mode and electricity losses in distribution electric networks 6-110 kV conditions of incomplete information // E3S Web of Conferences 289, 07018 (2021), 1-5. https://doi.org/10.1051/e3sconf/202128907018.
20. Bobojanov M.K., Rismukhamedov D.A., Tuychiev F.N., Shamsutdinov Kh.F. Development of new pole-changing winding for lifting and transport mechanisms // E3S Web of Conferences 365. 2023. РР, 04024, 1-10. <https://doi.org/10.1051/e3sconf/202336504024>.