Mathematical Modeling of the Dynamic Loads of Steel Cables of the Boom of a Single-Bucket Quarry Excavator

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**Abstract.** This article proposes a mathematical approach and a computational algorithm aimed at accurately modeling the dynamic load of steel cables used in the boom of a single-bucket quarry excavator. In the study, based on the consideration of a steel cable as a linear medium with elastic springs, its dynamic equations of motion were formulated, local and global stiffness and mass matrices were calculated by the discretization method. Integration over time was carried out by the central difference method, and the dynamics of the temporal change in displacement, acceleration, and internal stresses were determined. The obtained graphical results showed the nature of vibrational vibrations arising under real operating conditions of the rope, a significant increase in the amplitude of dynamic stresses compared to the static state, and the presence of critical points leading to fatigue damage. The proposed model has important theoretical and practical significance in determining the reliability of the cable, the service life, and the frequency of maintenance, and can be used as an effective tool for determining the optimal operating modes of the working mechanisms of mining equipment.

**INTRODUCTION**

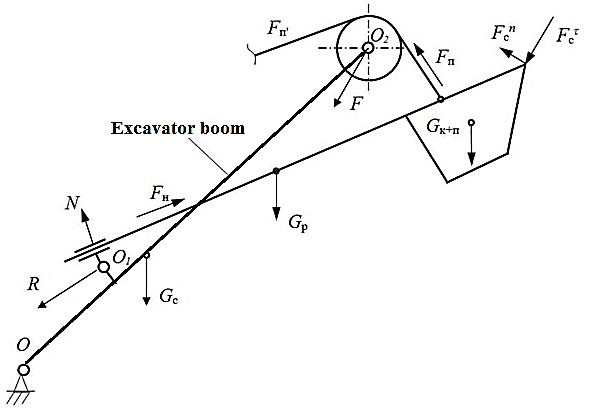
The reliable operation of single-bucket excavators is of great importance in ensuring the efficiency and safety of quarry excavation work. One of the main working equipment of such machines is a steel cable that provides the movement of the boom, which is subjected to complex dynamic loads during operation. Variable inertia forces acting on the steel cable, acceleration and deceleration processes, disproportionate load distribution, and external influences significantly reduce its service life [1-4].

Dynamic stresses and vibrations arising in steel cables affect the overall reliability of the unit, the frequency of maintenance, and energy efficiency. Therefore, the problem of correctly assessing the load on the steel cable under real operating conditions, modeling its mechanical state, and determining optimal operating modes is relevant from a scientific and practical point of view [8-9].

In recent years, numerical modeling methods - in particular, the finite element method, multifactorial dynamic analysis, and algorithmic calculations - have been widely used in the study of dynamic processes in quarry equipment. These methods allow taking into account the geometric and physical nonlinearity of the steel cable, the characteristics of the load changing over time, and the interaction of machine mechanisms in real operating conditions. However, to achieve an accurate and reliable result, it is necessary to correctly construct the mathematical model, form global matrices, apply initial conditions, as well as develop an effective computational algorithm [2-3].

**MATERIALS AND METHODS**

To accurately model the dynamic loads arising in the steel cable of the boom of a single-bucket quarry excavator, a mathematical model of the mechanical system, its discretization principles, and a calculation algorithm were developed. The model is based on the division of the steel cable into unit elements along its length, the formation of local stiffness, inertia, and load matrices for each element. This approach allows one to accurately describe the dynamic state of the steel cable over time [10-12].



**FIGURE 1.** Scheme for determining the forces arising in the working parts of the excavator

Stresses along steel cables [7]:

 (1)

 (2)

where: *EA* and *EB* - wool modulus, MPa.

*SA* and *SB* - cross-sectional areas of steel cables, mm2.

*s* - coordinates of the steel cables, s ϵ [0, LA] and s ϵ [0, LB].

 and  - displacements of steel cables during longitudinal tension.

Integrated model for steel cables When we take each steel cable as an elastic spring line, the dynamic equilibrium looks like this [5-6]:

 (3)

 (4)

The integral representation of moments in a steel wire rope shows that the force of each steel wire rope is generated at the point where it connects to the boom. Simplified, for right and left steel cables:

 (5)

 (6)

Based on the Lever mechanism, taking into account the semi-block and its geometry, the moment generated by the boom is determined using the following expression:

 (7)

where: *lA* and *lB* - right and left shoulders of force, m.

*αA* and *αB* are the angles of direction of the steel cables.

The general integral equation for two steel cables and a boom, combined with the above PDE and ODE, gives the general system as follows [11]:

(8)

The boundary conditions for the obtained expression (12) are as follows:

, (9)

This is a complex model with the resulting integral, connected by variables. Now this is converted to matrix form.

**RESULTS AND DISCUSSION**

Discretization by nodes and matrix equation of the numerical model By dividing each wire rope into n elements, we obtain the following nodal displacements in vector form:

 (10)

 (11)

Then the general expression of the matrix dynamic model is:

(12)

where:

 (13)

 (14)

 (15)

where: *BA* and *BB* - kinematic matrices expressing the relationship between the change in the angle of the boom and the nodal deformations of the steel cable (taken from geometric relationships).

*CA* and *CB* - matrices of internal friction on steel cables.

The force vector expressions are as follows:

(16)

where: *fA* and *fB* are the elementary integrals of the forces *qA* and *qB* distributed along the steel cable.

*Mext(t)* - additional external moments.

The difference in the forces of the steel cable is indicated separately, i.e., since the main problem is *TA≠TB*, the forces at the end of the steel cable in the matrix model are:

 (17)

 (18)

where: *eA* and *eB* - vectors separating the corresponding node at the end of the steel cable.

The equality condition for a half-block mechanism is defined by the following expression:

 (19)

This can be added as a constraint equation and incorporated into the global matrix system through the Lagrange coefficient or constraint matrix (mathematically expresses the automatic tension balance). A brief general view of the resulting complex model will be in the form of the following expression.

(20)

Additional integral with defining expressions:

,  (21)

,  (22)

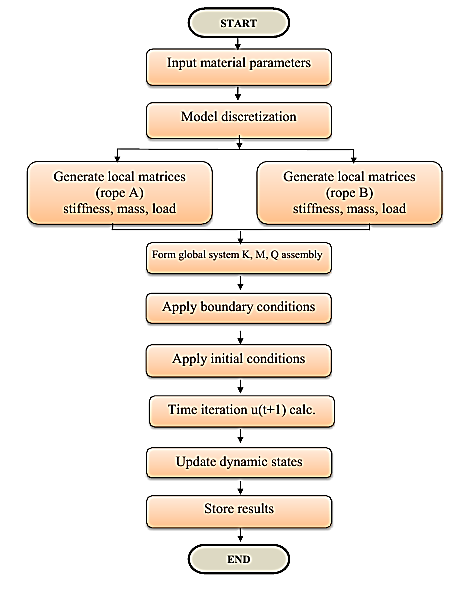
This created mathematical model takes into account the mass and stiffness distributed in integral form along the steel cables, the matrix relationship between the boom and the steel cables, forces and moments that can be nonlinear in time.

Initially, the change in the displacement amplitude of the steel cable at designated points over time was analyzed. According to the calculation results, the displacement function has a rapidly changing harmonic character, and a decrease in amplitude over time was observed. This is due to the internal vibrational properties of the steel cable and the dynamics of external load attenuation. The amplitude of the displacement on the graph (Fig. 2a) was represented in the form of a sinusoidal oscillation. This circumstance indicates that the oscillation mode of the rope is stable.

|  |  |
| --- | --- |
|  |  |
| a) | b) |

**FIGURE 2**. Graph of the amplitude of displacement of internal oscillations of the excavator cable: a) amplitude of oscillations in the initial position; b) amplitude of oscillations after modeling.

As the second important data, the change in dynamic stress in the composition of the steel cable over time was calculated. Based on the obtained graph (Fig. 2b), it was determined that it has a periodic component that varies around its average value. Such a change in the load function is the result of the uncontrolled (disproportionate) dynamic action of the needle movement, the interaction of inertial forces and the movement of the mechanism under real conditions.



**FIGURE 3**. Algorithm for determining the dynamic load on the steel cable of the excavator boom in real conditions

The obtained results indicate that the steel cable in real conditions is subjected to complex dynamic influences that cannot be described by a static model. The presented mathematical model and computational algorithm made it possible to accurately model and evaluate these processes.

The conducted calculations and the obtained results clearly showed that the dynamic loads of the steel cable are not fully represented by traditional static or quasi-static models. The proposed mathematical model takes into account a number of factors necessary for the accurate description of the speech and stress state of the steel cable during its real operation. In particular, the time-dependent change in the load propagating along the rope, the correct description of inertial forces and the nature of oscillations made it possible to determine the hot points of dynamic processes.

**CONCLUSION**

In this study, a mathematical model and a calculation algorithm were developed, aimed at accurately assessing the dynamic loads arising on the steel cable of the boom of a single-bucket quarry excavator. The model was formed taking into account the elastic properties of the rope, the variability of external influences over time, inertial forces, and the dynamic reaction of the mechanical system. The proposed model and computational algorithm are of great practical importance for accurate prediction of the mechanical state of the cable, optimization of maintenance intervals, risk assessment, and modernization of leading mining machines.

Also, due to the versatility of the model, it can be used in the analysis of the dynamics of various types of excavators, cranes, and steel cable mechanisms. The results serve to ensure safety and reliability in the operation of mining equipment.

**REFERENCES**

1. Toshov J.B., Rabatuly M., Khaydarov Sh., Kenetayeva A.A., Khamzayev A., Usmonov M., Zheldikbayeva A.T. Methods for Analysis and Improvement of Dynamic Loads on the Steel Wire Rope Holding the Boom of Steel Wire Rope Excavators. Kompleksnoe Ispolzovanie Mineralnogo Syra = Complex Use of Mineral Resources 2026; 339(4):87-96 <https://doi.org/10.31643/2026/6445.43>
2. Rabatuly M., Myrzathan S.A., Toshov J.B., Nasimov J., Khamzaev A. Views on drilling effectiveness and sampling estimation for solid ore minerals. Kompleksnoe Ispolzovanie Mineralnogo Syra = Complex Use of Mineral Resources. №1(336), 2026. <https://doi.org/10.31643/2026/6445.01>
3. Zokhidov O.U., Khoshimov O.O., Khalilov Sh.Sh. Experimental analysis of microges installation for existing water flows in industrial plants. III International Conference on Improving Energy Efficiency, Environmental Safety and Sustainable Development in Agriculture (EESTE2023), E3S Web of Conferences. Volume 463. Pages 02023. 2023. <https://doi.org/10.1051/e3sconf/202346302023>
4. Zokhidov O.U., Khoshimov O.O., Sunnatov S.Z. Selection of the type and design of special water turbines based on the nominal parameters of Navoi mine metallurgical combine engineering structures. AIP Conf. Proc. 3331, 050022 (2025). <https://doi.org/10.1063/5.0306554>
5. Khamzaev A.A., Mambetsheripova A., Arislanbek N. Thyristor-based control for high-power and high-voltage synchronous electric drives in ball mill operations/ E3S Web Conf. Volume 498, 2024/ III International Conference on Actual Problems of the Energy Complex: Mining, Production, Transmission, Processing and Environmental Protection (ICAPE2024) DOI: <https://doi.org/10.1051/e3sconf/202449801011>
6. A. S. Zhuraev, S. A. Turdiyev, S. T. Jurayev, and S.S. Q. Salimova, "Characteristics of packing gland seals in hydraulic systems of quarry excavators and results of comparative analysis of experimental tests," Vibroengineering Procedia, Vol. 54, pp. 252–257, Apr. 2024, <https://doi.org/10.21595/vp.2024.24051>
7. Akbar Zhuraev, Sardorjon Turdiyev; Analyses and studies of working fluid flow in the hydraulic system of hydraulic excavators at the Auminzo-Amantaytau open pit mine. AIP Conf. Proc. 4 November 2025; 3331 (1): 030067. <https://doi.org/10.1063/5.0305703>
8. Mislibaev I.T., Makhmudov A.M., Makhmudov Sh.A. Theoretical generalisation of functioning modes and modelling of operational indicators of excavators. // Mining information-analytical bulletin. - 2021. №1. p. 102-110. DOI: 10.25018/0236-1493-2021-1-0-102-110
9. Makhmudov Sh, Makhmudov A, Khudojberdiev L, Izzat Rakhmonov, “Criteria for assessing the performance of mining and transport equipment of mining enterprises,” Proc. SPIE 12986, Third International Scientific and Practical Symposium on Materials Science and Technology (MST-III 2023), 129860P (19 January 2024); doi: 10.1117/12.3017722
10. Ataqulov L.N., Haydarov Sh.B., Polvonov N.O. Impact forces on side and middle rollers. SPIE 12986, Third International Scientific and Practical Symposium on Materials Science and Technology (MST-III 2023), 129860Q (19 January 2024); doi: 10.1117/12.3017724
11. Atakulov L.N., Kakharov S.K., Khaidarov S.B. Selection of optimal jointing method for rubber conveyor belts. Gornyl Zhurnal, 2018. (9), 97-100. DOI:10.17580/gzh.2018.09.16
12. Mahmudov A, Musurmanov E, Chorikulov A, Tukhtaev Sh. Justification of the development of the ventilation network and increasing the efficiency of ventilation equipment by controlling themovement of air flow. Third International Scientific and Practical Symposium on Materials Science and Technology (MST-III 2023), Proc. of SPIE Vol. 12986, 1298610. doi: 10.1117/12.3017914