**Method for calculating the main and differential leakageinductancesof the stator winding using the energy method in machines with a uniform air gap**

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**Abstract.** This paper examines the problem of accurately determining the main and differential leakage inductances of stator winding in AC electric machines. Related parameters have a significant impact on the dynamic properties, stability, and energy efficiency of the machines, especially in modern designs with a fractional number of slots, magnetic circuit saturation, and complex winding structures. It is shown that traditional calculation methods do not always provide the required accuracy in the presence of pronounced higher harmonics of the magnetic field. The proposed energy method takes into account both the fundamental and higher spatial harmonic components of the magnetic field in the air gap, which enables more realistic modeling of operating conditions. Testing the method on synchronous generators with a wide range of design parameters confirms its relevance and practical applicability in the design and optimization of electric machines.

**INTRODUCTION**

Accurate modeling and analysis of electromagnetic processes in AC electric machines remain key challenges in modern electrical engineering. Among the most important parameters affecting machine operation, the main and differential leakage inductances of the stator winding are particularly significant. These inductances, determined primarily by the magnetic field in the air gap, significantly affect the dynamics, energy efficiency, and operational stability of electric machines. In modern high-efficiency machines, especially those with a fractional number of slots, pronounced poles, or complex winding structures, the influence of geometric parameters and magnetic steel saturation becomes particularly significant. Accurately determining leakage inductances under such conditions requires consideration of numerous design and operating factors: air gap shape and dimensions, slot width, tooth pitch, number of slots per pole and phase, winding turn distribution, magnetic properties of the stator and rotor cores, and others. One of the universal and reliable methods for determining these inductances is the energy method, based on the calculation of the electromagnetic energy stored in the air gap. This approach allows for a more complete description of the magnetic field distribution, taking into account both the fundamental wave and the higher spatial harmonics generated by the stator winding currents. This work is devoted to the development and application of the energy method for calculating the main and differential leakage inductances of single-phase and three-phase stator windings in AC machines with a uniform air gap. The technique takes into account the field distribution in the active zone of the machine and allows for analysis at various levels of magnetic saturation. Particular attention is paid to the influence of higher spatial harmonics and the design parameters of the winding. The proposed method is implemented in computational form and tested on the example of a four-pole synchronous generator of the MSA 72/4 type (15 kVA, 230 V, 1500 rot/min). The obtained results confirm the practical applicability and high accuracy of the method in the analysis of leakage inductances under conditions close to actual operation [1-15].

The main and differential leakage inductances of the stator winding are the main parameters determined by the field of the main air gap of modern highly utilized AC electric machines. The fields created by the stator windings are complex functions of the radial value of the air gap between the stator and rotor cores, the number of slots per pole and phase, the pitch, the number of phases and winding zones, the slot spline width, the tooth pitch, the magnetic state of the steel sections of the stator and rotor magnetic circuits and the damping effect of currents in the secondary circuits of the machine. We present a method for calculating the main and differential leakage inductances [1-2], which allows for taking into account all the above factors influencing these inductances, except for the effect of the unevenness of the air gap between the stator and rotor cores and the damping effect of currents induced in the secondary circuits by the fields of higher spatial harmonics created by the stator winding of the AC machine. Accounting for the influence of the latter is the subject of separate research. The calculation method proposed in this paper is based on a model of the air gap field of an AC electric machine. The model assumes that the total stator slot current is concentrated in a thin layer located along the circular arc of the smooth surface of the stator bore and equal in width to the slot spline, which corresponds to an internal spatial angle of 2α. The radial air gap δ between the ferromagnetic cores of the stator and rotor of the machine is assumed to be uniform [16-24].

**METHODS AND MATERIALS**

As it is known, the differential leakage inductance is the difference between the winding inductance due to the actual field in the air gap and its fundamental wave. The fundamental wave of the field determines the main inductance of the winding. Inductances can be determined for the case of power supply of each phase of a multiphase winding separately, for the case of multiphase power supply, and also for zero-sequence currents. This section considers the issue of calculating the main and differential leakage inductances for a single-phase winding and for positive-sequence currents of three-phase windings. Usually, when calculating the reactance due to the magnetic field of the main air gap of AC machines, the energy and harmonic analysis methods are used [13-15]. We will apply the first of these methods, based on the energy method for determining inductances. When applying the method for one phase of multiphase stator windings, expressions (1) can be used (two-layer windings with integer q),

.





 (1)

where



(2) (single-layer windings with integer q),

 (2)

where 

(3) (two-layer windings with fractional q with the fractional base equal to two)

,

,

from (2) we obtain

 (3)

When =, expression (3) turns into (1) for a two-layer winding with integer q and expressions similar to (4) (for any fractions of q).







. (4)

It can be seen from (4), the axes of adjacent coil groups are shifted relative to each other by angles that differ from the pole division of the machine by  or  .

The expression for the magnetic field strength in the air gap of a machine created by a three-phase stator winding is compiled for one of the instantaneous values of its current [6-8]. In particular, for the moment in time when the current in one of the phases of a three-phase two-layer winding is zero, and in the other two phases the currents are equal in magnitude and opposite in sign, the expression for the field strength is determined by expression (5)



, (5)

where ω1=2πf1 is the angular frequency; t is the time.

Expression has the form

 (6)

where -is the number of parallel branches of the stator winding;

- the number of effective turns of one phase of the winding: ;

After some transformations from (6) we obtain

 (7)

Differential leakage inductance

, (8)

where  - is the inductance of the winding due to the actual field in the gap,  - main inductance.

As it is known, the inductance of the winding is related to the electromagnetic energy of the actual field by the expression

, (9)

where - is the electromagnetic energy of the actual field in the machine gap.

Similar to (3-4), the main inductance of the winding

 (10)

where  - is the electromagnetic energy of the main working field in the machine gap.

Taking into account (9) and (10) from (8) we obtain an expression for the differential leakage inductance of a single-phase winding:

 (11)

where  - is the electromagnetic energy of the actual field of a single-phase winding in the machine gap.

 - electromagnetic energy of the main working field of a single-phase winding.

Similarly to (11) the differential leakage inductance of one phase of a three-phase winding when supplied with three-phase current

, (12)

where - is the electromagnetic energy of the actual field of the three-phase winding;

- electromagnetic energy of the main working field of the three-phase winding.

In (11) and (12), the ‘o’ and ‘t’ designations in the indices correspond to the single-phase and three-phase windings. On the other hand, the electromagnetic energy of the field in the air gap

, (13)

where -is the effective value of the magnetic field strength,

*V* -is the volume of air space between the stator and rotor cores of the machine.

, (14)

where - is the calculated length of the air gap.

As applied to the single-phase and three-phase windings, expression (13), taking into account (14), the expression can be written as:

, (15)

, (16)

, (17)

, (18)

where *H0P, HTP* - are the fundamental harmonic intensities of the magnetic fields of the single-phase and three-phase windings.

The values *H0P* and *HTP* are obtained by substituting n=p into (6) and (7), respectively, i.e.

, (19)

, (20)



Substituting (19) and (20), respectively, into (16) and (18) and integrating, we obtain

, .

Integration in (15) and (17) with a large number of harmonics in the composition of *H0* and *HT* is difficult, and in some cases practically impossible. Therefore, the quantities *H0* and *HT* can be replaced by equivalent quantities *H0E* and *HTE*, which are found graphically by dividing the corresponding field curve into *s* equally spaced ordinates *h*1, *h*2, *h*3, etc. according to the expressions [25-31].

, (21)

. (22)

Substituting *H0E* and *HTE* instead of *H0* and *HT*, respectively, (15) and (17) and performing integration, we have

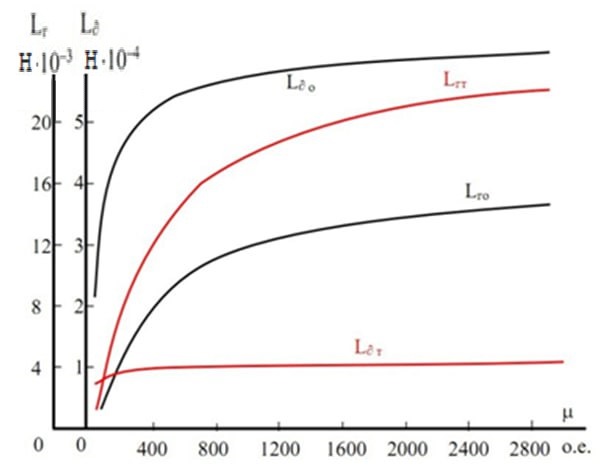
, .

**RESULTSANDDISCUSSION**

Noted that for finding out *H0E* and *HTE* when dividing the field curve into equally spaced ordinates, for armature windings with integer *q*, it is sufficient to limit the analysis to one pole division. While for windings with fractional *q*, it is necessary to use the field curve within the repeating part of the stator winding structure [9-12]. With an even fractional base, the repeating part of the winding occupies a number of pole divisions equal to the fractional base, and with an odd base, it occupies twice the fractional base.

As noted earlier, the above expressions for calculating the main and differential inductance leakage are valid for two-layer windings with integer *q*. Using these expressions, similar expressions can be obtained for machines with single-layer windings with integer *q*, as well as for single-layer and two-layer fractional windings, and for any type of special stator windings [13-18].

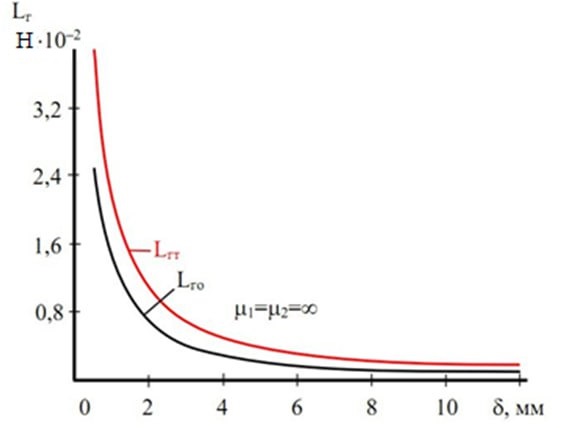
Calculations of the main and differential inductance scattering were performed on a computer for a four-pole AC machine with a two-layer armature winding at a=0; b=0.1241 m; ρ=c=0.125; d=0.184 m; α=0.028 rad; αz z=0.1496 rad; β=1.3464 rad; q=0; W1=56. These values generally correspond to a three-phase salient-pole synchronous generator of the MSA 72/4 type, 15 kVA (12 kW), 230 V, stator winding connection diagram “star”, 1500 rpm [19-24]. Fig.1 shows the dependencies .



**FIGURE 1.**Calculation results of leakage inductances of a four-pole alternating current machine

Here, it is assumed that the value *μ=μ1=μ2*. Calculations were performed on a computer. Summation of individual harmonic components to determine the dependencies *H0, HT=f(φ)* was performed up to *n*=150. As can be seen, the differential leakage inductance of one phase with a three-phase power supply remains virtually unchanged in the range of variation of the relative value of the magnetic permeability of the steel parts of the stator and rotor magnetic circuits from 200 to 4000 and decreases only at those values of μ that correspond to deep saturation of the magnetic circuit. At the same time, the main inductances of the windings, both with single-phase and three-phase power supply, vary over a fairly wide range depending on the value of *μ*. The differential leakage inductance with a single-phase power supply varies over a wider range than with a three-phase power supply [32-48].

Figure 2 shows the dependences of the main inductances of the three-phase and single-phase stator windings for the same MSA-72/4 machine as a function of the air gap between the stator and rotor cores. As can be seen, these inductances vary over a fairly wide range depending on the machine’s air gap size.



**FIGURE 2.** Dependencies of the main inductances of three-phase and single-phase stator windings

**CONCLUSION**

The developed methods for calculating air-gap field-induced reactance, and in particular differential leakage inductance and its components for positive-sequence stator winding currents, allow calculations to be made taking into account various design and operating factors that influence these reactance’ values. The differential leakage inductance of a single-phase winding, due to the influence of higher spatial harmonic multiples of three, is greater than that of a three-phase winding.

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