**Equivalent Electromagnetic Capacitance of a Nonlinear Inductance**

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**Abstract.** The article addresses the problem of determining the equivalent electromagnetic capacitance of a nonlinear inductance. The dependence of the equivalent capacitance on geometric, dielectric, and magnetic parameters, as well as on the saturation frequency, is analyzed both theoretically and experimentally.

**INTRODUCTION**

At present, nonlinear inductances are widely used in high-frequency secondary power sources, pulse converters, and highly sensitive electronic devices. The electrical properties of such components, particularly their equivalent capacitance, have a significant impact on the dynamic and frequency characteristics of the device. Toroidal multilayer inductors are widely used in practice, and accurately assessing the resulting internal capacitances is of significant scientific and practical importance [1-4].

The equivalent capacitance of a multi-layer toroidal winding is determined by the mutual capacitances between the turns and layers. Since the toroid has different outer and inner diameters, the distance between turns on the outer surface increases, and a phenomenon occurs in which subsequent layers of windings settle into the gaps of the previous layers. As a result, capacitances are formed not only between adjacent layers but also between more distant layers [5-8]. It is known that as the number of layers increases, the equivalent capacitance initially increases, but then decreases due to the series connection of the capacitors. This maximum value mainly depends on the ratio of the toroid's outer and inner diameters. Based on the theoretical model, the equivalent capacitance is expressed through the number of layers, insulation parameters, and geometric dimensions [9-10].

*Methods Used.* The following methods were employed during the research process [11-15]:

- Circuit for determining the equivalent capacitance based on the resonance method;

- Analytical modeling and approximation methods;

- Computational experiment;

- Methods for comparing experimental results with theoretical calculations were applied.

**METHODS AND MATERIALS**

The problem of calculating the capacitance of multilayer toroidal windings of transformers with a ferromagnetic core, as well as other configurations of nonlinear inductances, is of practical interest. The works of M.A.Rosenblat, I.B.Negnevitsky, S.I.Eliseev, and others are dedicated to this topic. The use of nonlinear inductances in high-frequency secondary power supply devices is receiving increased attention. In well-known articles, it has been shown which parameters can be affected by the equivalent capacitance of a nonlinear inductance, particularly in highly sensitive operational devices. The cited articles experimentally investigated the capacitances of toroidal windings mounted on metallic enclosures for laminated cores. In this case, the capacitances were measured using the resonance method according to the circuit shown in Fig.1 [16-22].

In Fig.1, *G* is a sinusoidal oscillation generator (50-20 kHz); *C*std is a standard variable capacitor (100-10⁶ pF); *R*add is an additional resistance, *Ω* [23-25].

*G*

*V*

*C*std

*R*add

*w*

**FIGURE 1.** The scheme for the experimental determination of the equivalent capacitance of a nonlinear inductance

As is known, the main distinction and specificity of a multilayer toroidal winding is determined by the different lengths of the layers on the inner and outer lateral surfaces of the toroid. At the same time, there will be a gap on the outer surface between the turns of the layer, which becomes larger as the ratio of the core's outer diameter to its inner diameter increases. Therefore, some turns of the subsequent layers on the outer surface of the winding settle between the turns of the previous layer. The capacitance exists not only between layers 1 and 2, 2 and 3, and so on, but also between layers 1 and 3, 1 and 4, 2 and 4, and so forth [26-29].

As the authors of the studies note, with an increase in the number of layers, the equivalent capacitance of the winding initially increases due to the sinking of turns in subsequent layers, and then decreases, as a kind of series connection of interlayer capacitances gradually arises [30-31].

The number of layers at which the maximum capacitance is achieved mainly depends on the ratio of the outer diameter of the container, *Dk*, to the inner diameter, *dk*. If we consider 𝐶1 as the capacitance between two adjacent layers, each of which has a capacitance 𝐶0, then, using the equivalent circuit shown in Fig.2, the following expression is obtained for the case 𝐾 ≥ 2 [32-34]:

 (1)

where *K* is the number of winding layers.



**FIGURE 2.** Equivalent capacitance scheme of a multilayer winding

It follows from formula (1) that the equivalent capacitance depends on *C*0*, C*1and *K.* In real cases, as the authors claim, the equivalent capacitance depends on the type of insulation and the thickness of the wire, as well as on the ambient temperature [35-37].

Finally, the following expressions for the equivalent capacitance have been obtained:

 (2)

Where 𝐷𝐾, 𝑑𝐾, and ℎ𝐾 are the outer and inner diameters and the height of the container, respectively; 𝜎ins is the insulation thickness of the wire; 𝑑ins is the diameter of the wire including insulation; 𝜀0 is the dielectric permittivity of the wire material; and 𝜀𝑒 is the equivalent dielectric permittivity of the wire material [38-39].

Under the appropriate assumptions, the relationship between the number of layers and the number of turns is as follows:

 (3)

Where *γ* is the coefficient that accounts for the loose packing of turns in a layer.

From expressions (1), (2), and (3), it follows that, with the constancy of the corresponding parameters and geometric dimensions, the equivalent capacitance of a nonlinear inductance does not depend on the electromagnetic parameters of the nonlinear inductance [40-42].

However, our research shows that the equivalent capacitance depends not only on the geometric parameters and dielectric constants but also on the magnetization frequency of the nonlinear inductance, as well as on the values of the magnetic and electric quantities. Therefore, the equivalent capacitance of a nonlinear inductance will be referred to as the equivalent electromagnetic capacitance [43-46].

As is known from our research, under the condition , it follows from the expression that:

 (4)

Where *iL=aψ+bψ*3 is the well-known approximation of the Weber-Ampere characteristic of a nonlinear inductance, obtained based on the magnetization curve 𝐵=(𝐻); 𝐶𝑒 is the equivalent electromagnetic capacitance of the nonlinear inductance; and ** is the equivalent active conductance of the nonlinear inductance [47-48].

 (5)

Considering that: 



then, after some transformations, from

 (6)

From (6), we obtain the following expression for the equivalent electromagnetic capacitance:

 (7)

The analysis shows that the coefficient “α” varies inversely with a certain linear inductance, which is determined by the following formula [49]:

 (8)

 (9)

Where *μ* - Absolute magnetic permeability of a ferromagnetic material



Taking into account (8) and (9), from expression (6) we obtain the equivalent electromagnetic capacitance:

 (10)

Where .

After some transformations, we obtain the electromagnetic equivalent capacitance:

 (11)

Expression (10) makes it possible to determine the electromagnetic capacitance of a nonlinear inductance over a wide range of frequency variation and to take into account the influence of individual parameters of the nonlinear inductance on the value of this capacitance. In particular, the influence of the number of turns of a nonlinear inductance on the value of 𝐶𝑒 is shown in Fig.3 [50-51].

On the generalized model of nonlinear inductance

*C*e

*w*

0

200

300

100

500

1000

1500

2000

2500

Experimental

According to the known model

**FIGURE 3.** The experimental dependencies of the equivalent capacitance of a nonlinear inductor on the number of turns

The comparative analysis shows that expression (11) practically coincides with the experimental data [52-54].

*Analytical part.* In classical approaches, the equivalent capacitance is considered to depend solely on geometric and dielectric parameters. The conducted studies, however, have shown that the equivalent capacitance of a nonlinear inductance depends on the magnetic parameters, the values of current and magnetic flux, as well as the high magnetization frequency.

For this reason, the concept of “equivalent electromagnetic capacitance” is introduced in this article. Analytical expressions obtained based on the approximation of the Weber-Ampere characteristic of a nonlinear inductance are compared with experimental results. It has been demonstrated that the obtained theoretical expressions allow determining the equivalent electromagnetic capacitance over a wide frequency range. Comparison with experimental graphs showed that the computational model is sufficiently accurate.

**CONCLUSION**

In the course of the article on “Equivalent Electromagnetic Capacitance of a Nonlinear Inductance” the following scientific results were obtained:

1. It has been proven that the active resistance and the electromagnetic equivalent capacitance depend not only on the electrical and geometrical parameters of a nonlinear inductor, but also on its magnetic parameters.

2. The computational experiment and comparative analysis with existing studies show that the equivalent electromagnetic capacitance determined using the proposed methodology practically coincides with the experimental curves.

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