**On SD-Prime Cordial Labeling of Graphs**

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**Abstract.** Out of the vast branches of mathematics, graph theory plays a vital role in the fields of applied mathematics and scientific computing. Graph labelling is a prominent area in the field of graph theory. There exist various types of graph labeling in the available literature, such as graceful labeling, even graceful labeling, odd graceful labeling, odd-even graceful labeling, and even-even graceful labeling. The present work deals with a unique kind of graph labeling known as SD-prime cordial labeling. This paper presents an SD-prime cordial labeling of certain classes of graphs like fan graphs of particular type, tristar, quadrilateral snake, -snake, double -snake, alternate cycle pendant graphs, and a type of Coconut tree.

**Keywords:** Graph labeling; SD-prime cordial labeling; Coconut tree; Cyclic snake;

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**INTRODUCTION**

Among various domains within graph theory, graph labelling is a significant area of study. Most graph labeling problems can be traced back to Rosa’s initial formulation. This concept involves assigning integers to the vertices, edges, or both, provided certain conditions are met. Idea of graph labeling emerged in 1960s as an effort to address challenges arising from the decomposition of larger graphs into smaller components. Since then, over 2600 research articles on this topic have been published, leading to the development of several innovative graph labeling methods. Various types of graph labeling have been proposed, including graceful labeling, even graceful labeling, odd graceful labeling, even-even graceful labeling, odd-even graceful labeling, super edge graceful labeling, and prime labeling, among others. In our work, we have concentrated on a specific type of graph labeling known as SD-prime cordial labeling, which introduced by Lau et al. [7]. Consider one-to-one and onto mapping  from set of vertices, which produces labeling  for edges  into. For each edge in the graph , we define  if , where represents the sum of  and , and denotes absolute value of difference between  and . Conversely, in all other cases. Let  and denote count of edges labelled with 0 and 1, respectively. The mapping has SD-prime cordial labeling when absolute value of the difference between  and is less than or equal to 1. For further information on the results concerning SD-prime cordial labeling of graphs, please refer to the works of the term SD-prime cordial labeling of graphs does not appear to be widely recognised or documented as of my last knowledge update in September 2021. It is possible that this concept has emerged or gained prominence since that time, or it may represent a relatively novel or niche area within the field of graph theory. In graph theory, cordial labeling refers to a specific form of vertex labeling in which the counts of vertices labelled with each label, along with the absolute differences in the numbers of vertices labelled with adjacent labels, must either be precisely the same or differ by no more than one. In a similar vein, prime cordial labeling involves a vertex labeling wherein the counts of vertices assigned each label and the absolute differences in the counts of vertices labelled with adjacent labels are prime numbers. The SD component in SD-prime cordial labeling requires further clarification, as its meaning remains ambiguous without additional context or information. It may represent a particular variation or constraint related to cordial labeling; however, a definitive explanation cannot be provided without more details. If SD-prime cordial labeling is indeed a recent development or a specific variant introduced after my last update, I recommend consulting current graph theory literature, research publications, or academic resources for comprehensive information on this topic.

J.A. Gallian [1] shows in his paper about all graph labelings, including graceful labeling, odd-even graceful labeling, even-even graceful labeling, Sd-prime cordial labeling, etc., in detail. Wency et.al [2] show that certain classes of zero-divisor graphs of commutative rings are SD-prime cordial graphs. Lau et. al [3] investigate SD-prime cordiality of some standard graphs. A. Lourdusamy et al. [4] showed that several types of graphs-like the union of star and path, subdivisions of certain graphs, and a path with an attached star, are all SD-prime cordial. In the same paper, they proved that if you join two SD-prime cordial graphs together, the new graph is not necessarily also SD-prime cordial. A. Lourdusamy et. al [5] prove some results on SD-Prime cordial labeling. U.M. Prajapati et. al [6] show some graphs that satisfy the properties of SD-prime cordial labeling. U.M. Prajapati et. al [7] prove SD-prime cordial labeling of alternate k-polygonal snake of various types.

**Definition 1.1.** m-cyclic snake is generated from path  by substituting each edge of  with cyclic graph  (where ). This structure is denoted as  and referred to as a -snake.

**Definition 1.2.** A double -cyclic graph consists of two copies of cyclic graph  (where ) that share one edge.

**Definition 1.3.** Double -cyclic snake formed by transforming path  such that each edge of  is replaced with double -cyclic graph in manner that edge corresponds to shared edge of the double -cyclic graph. This construction is denoted as  and known as a double - snake.

**MAIN RESULTS**

The SD component remains somewhat ambiguous in my prior understanding; however, it may signify a further refinement or constraint associated with prime cordial labeling. In the absence of specific details, it is difficult to accurately delineate the concept of SD-prime cordial labeling. One interpretation could suggest that it involves the integration of certain characteristics pertaining to the distribution of prime numbers or particular differences in labeling.

**THEOREM 1.**

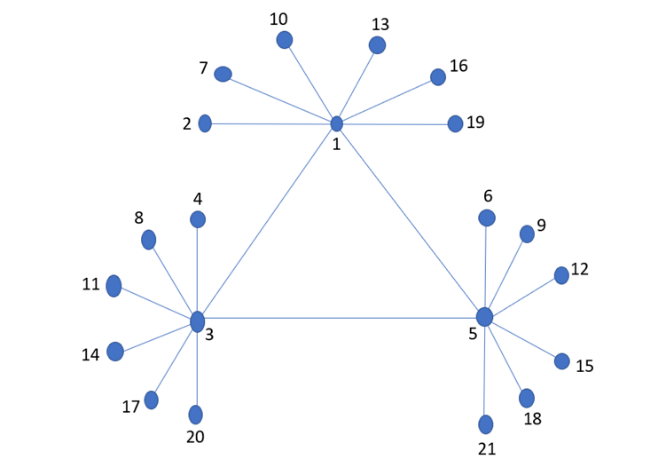
 is characterised as SD-prime cordial for .

Proof. Consider the graph , which is constructed with attaching  pendant edges to each vertex of cycle graph . Let  denote vertices of , and graph  has a total of edges. Additionally, let represent the vertices connected to , where .

The vertex labeling of the graph is defined as 

Consequently, it holds that . Therefore, we conclude  satisfies SD-prime cordial.

Illustration.



**FIGURE 1.** The graph  with SD-prime cordial labeling.

**THEOREM 2.**

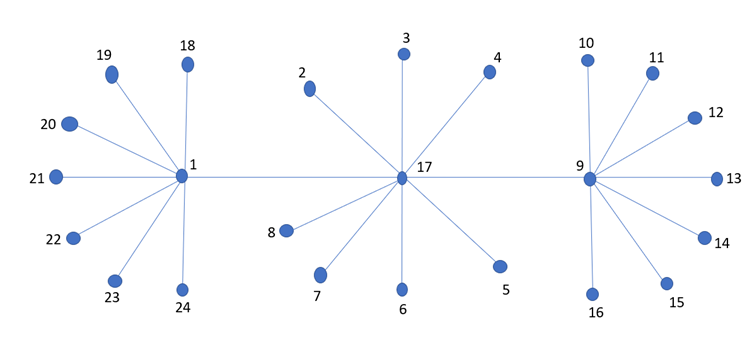
Tristar  is demonstrated to be SD-prime cordial for .

Proof. Consider the tristar graph , which comprises  edges and features a central path . This path is characterised by three vertices , with serving as the centre of the tristar graph .  for  represent vertices connected with the vertices and  respectively. Vertex labeling of tristar graph  is prescribed as

.

Consequently, it is established that . This leads to the conclusion that . Hence, the tristar  can be recognised as SD-prime cordial.

Illustration.



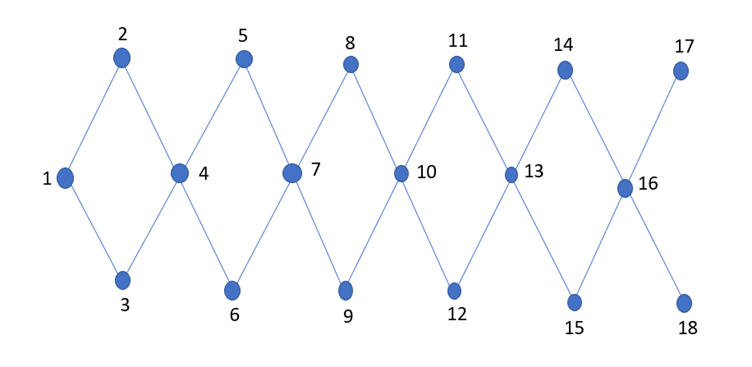
**FIGURE 2.** The tristar  is presented with an SD-prime cordial labeling.

**THEOREM 3.**

Quadrilateral snake  is classified as SD-prime cordial for .

Proof. Consider graph denoted as , which represents a quadrilateral snake graph that has its tail end connected to , resulting in a total of  edges. Let us denote the vertices located at the top of the quadrilateral snake by for , those in the middle by  for , and those at the bottom by  for . Vertex labeling for  is defined as . Consequently, we observe that , leading to the conclusion that $|. This confirms quadrilateral snake  classified as SD-prime cordial.

Illustration.

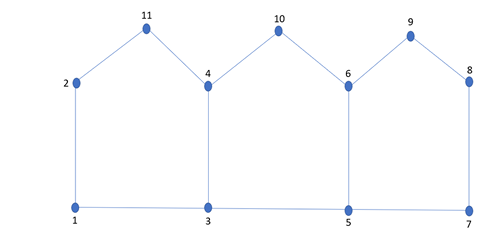


**FIGURE 3.** The quadrilateral snake  with SD-prime cordial labeling.

**THEOREM 4.**

The pentagonal snake is characterised as SD-prime cordial for .

Proof. Let  denote a pentagonal snake characterised by its central path . The vertices along this central path are labeled as , while the vertices attached to each  (for ) are designated as and the vertices  (for ) are also included. The vertex labeling for the aforementioned pentagonal snake is defined as and. It follows that the inequality  holds. Consequently, we conclude pentagonal snake $C\_5S\_n$ is classified as SD-prime cordial.



**FIGURE 4.** The pentagonal snake with an SD-prime cordial labeling for is presented.

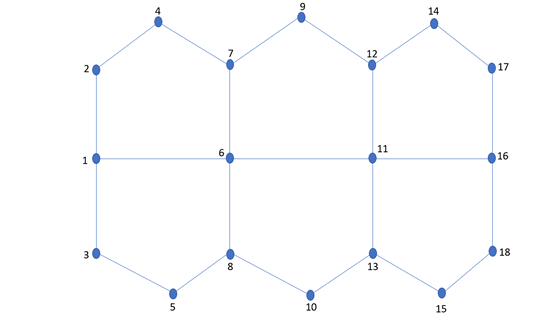
**THEOREM 5.**

Double pentagonal snake  is classified as SD-prime cordial for .

Proof. Assume that  represents a double pentagonal snake connected to a central path , where each successive pentagon overlaps with adjacent ones through a common edge in both the upper and lower sections of the snake. Let denote vertices along central path . Vertices  refer to alternate vertices situated on the upper section of the pentagonal snake, while denote alternate vertices on the lower section of the snake. Let represent the vertices connecting and , and refer to the vertices connecting and . The vertex labeling for the aforementioned double pentagonal snake is defined as



Consequently, it follows . Therefore, this can be concluded double pentagonal snake  exhibits the properties of being SD-prime cordial.

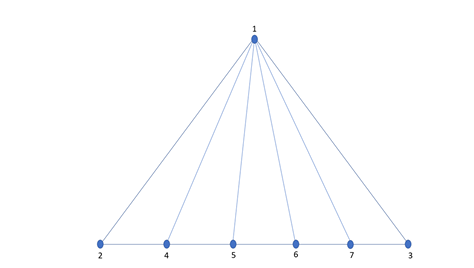


**FIGURE 5.** The double pentagonal snake with an SD-prime cordial labeling for .

**THEOREM 6.**

 is classified as SD-prime cordial.

Proof. The graph  is constructed by overlapping  with . In this configuration, vertices of  are aligned along the path . The head vertex of  is connected to the remaining  vertices that are situated on the path. Denote the head vertex as  and the vertices along the path of as . Assignments are made as . For , we have . Consequently, it follows . Therefore, graph  is classified as SD-prime cordial.

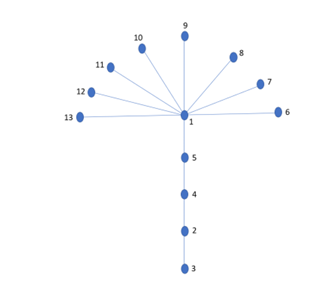


**FIGURE 6.** The graph  features an SD-prime cordial labeling.

**THEOREM 7.**

Coconut tree is classified as SD-prime cordial.

Proof. Consider the class of coconut trees denoted as , which is formed by connecting pendant vertices to the central structure of the coconut tree. In this context, represents the number of vertices on the central path . Let the pendant vertices be labeled as , while the vertices along the central path are designated as . The vertex labeling for the coconut tree  established as . Consequently, we find that the absolute difference in the edge count satisfies condition . Thus, we classify the coconut tree  as being SD-prime cordial.

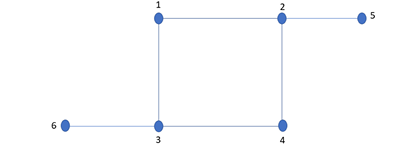


**FIGURE 7.** The Coconut tree with an SD-prime cordial labeling.

**THEOREM 8.**

Alternative is classified as SD-prime cordial.

Proof. Consider the alternative graph , which is constructed by attaching pendant edges to the alternating vertices of . Let  represent the vertices of . We define  as the vertex connected to vertex  of , and  as the vertex connected to vertex  of . The vertex labeling for the alternative graph is specified as follows: . Consequently, it follows . Thus, we conclude that alternative is classified as SD-prime cordial.

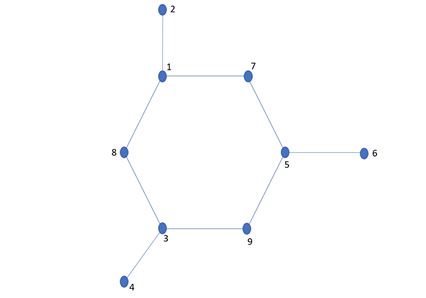


**FIGURE 8.** An Alternative to  featuring an SD-prime cordial labeling.

**THEOREM 9.**

Alternative is classified as SD-prime cordial.

Proof. Consider the graph designated as alternative , where the alternative vertices of  are connected to pendant edges. Let  represent vertices of , with  being attached to the pendant edges and the vertices having a degree of two. The vertex labeling for the alternative is defined as . Consequently, it follows that . Hence, alternative classified as SD-prime cordial.

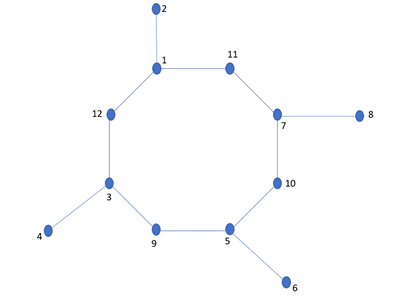


**FIGURE 9.** Alternative  with an SD-prime cordial labeling.

**THEOREM 10.**

Alternative is SD-prime cordial.

Proof. Consider the alternative graph denoted as , where alternate vertices of are connected to pendant edges. Let  represent vertices of , with connecting to the pendant edges, while vertices exhibit a degree of two. Vertex labeling for the alternative is defined as and . It can be shown that . Thus, alternative  classified as SD-prime cordial.

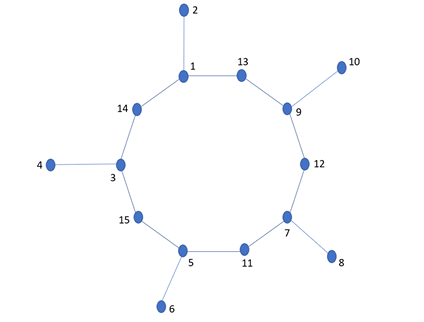


**FIGURE 10.** Alternative with an SD-prime cordial labeling.

**THEOREM 11.**

Alternative is characterized as SD-prime cordial.

Proof. Consider the alternative graph , which is formed by attaching pendant edges to the alternating vertices of . In this graph, let  represent the vertices of that are connected to the pendant edges, while vertices maintain a degree of two. Vertex labeling for alternative graph  is defined as . Consequently, it can be derived  Thus, the alternative graph classified as SD-prime cordial.



**FIGURE 10.** Alternative with an SD-prime cordial labeling.

**CONCLUSION**

The present paper focused on SD-prime cordial labeling of graphs like class of fan graphs, tristar, quadrilateral snake, -snake, double -snake, alternate cycle pendant graphs, and the Coconut tree. The results are found to be more interesting and inspiring for strengthening more research on SD-prime cordial graphs. The problems are still open for *n*- star graphs (). One can also try for *n*-gonal snakes with (). Furthermore, hairy cycle graphs with cycle of length  may be proved SD-prime cordial labeling.

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