**Experimental And Numerical Study Of A Counter-flow Double Pipe Heat Exchanger With Ball Turbulator**

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**Abstract**. This study investigates the complex thermo-hydrodynamic behavior of water within a counter-flow double pipe heat exchanger (DPHX), emphasizing the influence of internal flow disruption using ball turbulators. In the present study, the inner copper pipe transports heated water, whereas the annular passage carries cooling water, forming a typical counter-flow configuration under steady-state operating conditions. To enhance heat transfer, small Mild Steel balls—alternating in diameter at a 2:3 ratio (12 mm and 18 mm)—are brazed onto a central rod using Copper and mounted equidistantly along the inner pipe. This arrangement disturbs the internal flow, promoting thermal mixing and turbulence. A combined experimental and numerical analysis is performed to assess both the pressure drop and the overall thermal performance of the system. The study compares heat exchanger performance with and without ball turbulators, revealing significant enhancements in the overall heat transfer coefficient, particularly as a function of the outer pipe diameter (0.04 m *< Do <* 0.18 m) and the annular inner diameter (0.015 m *< Di <* 0.045 m). Meanwhile, variations in inner diameter of the pipe and the outer diameter of the annular show minimal influence on 'U' fluctuation. The impact of geometrical and flow parameters such as pipe length (0.5 m *< L <* 3 m), mass flow rate (0.2 kgs-1 ≤ *ṁ* ≤ 0.8 kgs-1), and flow velocity (1 ms-1 *< V <* 4 ms-1) on pressure drop is systematically examined. Thermal and hydrodynamic behaviors are further characterized by variations in Nusselt number and friction factor across a wide Reynolds number range (5000 ≤ *Re* ≤ 30000). In the final phase of the work, a machine learning algorithm—incorporating decision tree and neural network models—is developed to optimize the heat exchanger’s performance. This intelligent system enables real-time adjustment of process variables via sensor feedback, providing adaptive control for maximizing thermal efficiency and output under dynamic operating conditions.

**Keywords:** Ball turbulator, Double-pipe heat exchanger, Overall heat transfer coefficient, machine learning.

1. **INTRODUCTION**

Double-pipe heat exchangers (DPHXs) are widely employed owing to their simple construction, ease of maintenance, and adaptability under diverse operating conditions. They consist of two concentric pipes in which hot and cold fluids flow, typically in parallel-flow or counter-flow arrangements. Because of their compactness and reliability, DPHXs find extensive application in process industries, HVAC systems, refineries, and laboratory-scale thermal systems [1].

Improving the thermal efficiency of DPHXs has long been a focus of research. Passive enhancement methods, which operate without external energy input, are particularly attractive due to their simplicity and robustness. Numerous turbulator concepts were investigated, such as twisted tapes, perforated inserts, baffles, and vortex generators [2–3]. Studies showed that twisted and perforated tape inserts improved heat transfer while also reducing pressure losses, with perforated versions typically performing better than continuous ones [4–5]. Specifically, Vaisi et al. reported that discontinuous and perforated tapes achieved a favorable trade-off between heat transfer and pressure drop [4], while Sheikholeslami and Ganji demonstrated that perforated turbulators in the annular region yielded significant thermal enhancement when their geometry and flow parameters were optimized [5]. More recently, hybrid approaches, including the integration of turbulators with bubble injection or magnetic fields, demonstrated significant heat transfer improvements, though at the expense of higher pumping power [6]. Longitudinal vortex generators (LVGs) also gained attention for creating secondary swirling flows that thin the thermal boundary layer boosting heat transfer [7]. Beyond these, innovative turbulator geometries continued to emerge. Şener and Demir tested two new shapes, designated TY and TZ, inserted into the inner tube of a DPHX. Their experimental and CFD evaluations showed that TY improved temperature difference by 28% at 2.5 m/s, while TZ achieved up to 118% at 3 m/s compared to a plain tube [8]. Special attention has also been given to ball-type turbulators (BTs). Yuan et al. [9] showed that BT efficiency depends on ball size and spacing, with larger balls enhancing heat transfer with intensified turbulence but increasing resistance, while smaller ones provide a better balance between efficiency and pressure drop.

Parallel to physical improvements, machine-learning (ML) approaches are increasingly used for modeling, prediction, and control of heat exchangers. A recent comprehensive review documented successful use of Artificial Neural Networks (ANN), Support Vector Machines (SVM), Random Forest, Gradient Boosting ML methods for predicting heat transfer coefficients, friction factors, and exchanger performance under varying conditions, showing strong promise in real-time simulations and control applications [10]. Another recent study trained a neural network on numerical data to optimize and predict counter-flow DPHX performance, demonstrating effective design and operating point selection using hybrid Artificial ANN genetic algorithm (GA) methods [11].

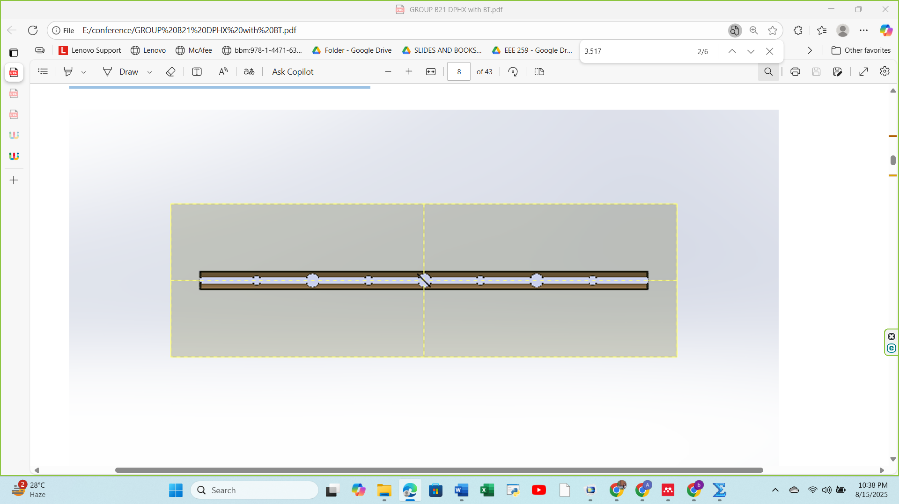
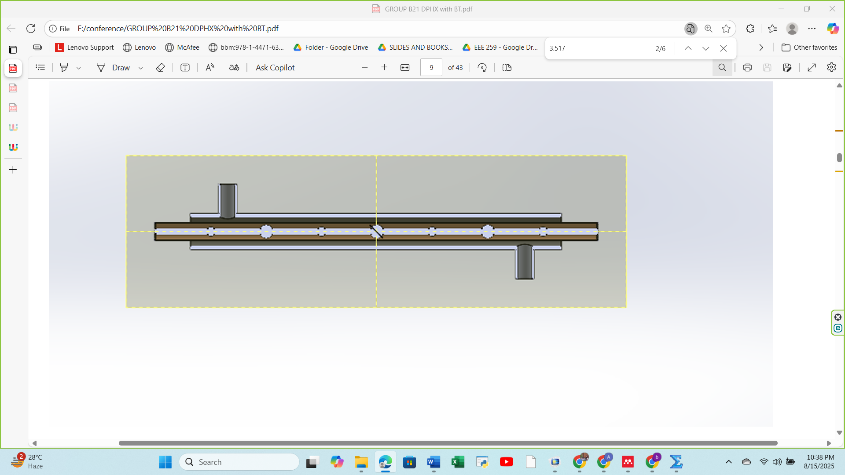
While many studies explore twisted tapes, perforated inserts, or active methods, fewer have systematically examined mixed BT configurations, particularly those integrating different ball sizes or combining BTs with other passive elements. Moreover, experimental validation remains limited, and there is a compelling opportunity to develop optimized BT-based DPHX designs that maximize enhancement while controlling pressure drop. Therefore, the present study aims to design and fabricate a counter-flow DPHX equipped with ball-type turbulators of two alternating diameters, arranged at fixed spacings, to maximize boundary layer disruption, conduct experimental testing and analytical evaluation to quantify heat transfer improvement and pressure loss, comparing configurations with and without BTs and analyze the influence of ball diameter ratio and spacing on performance metrics Nusselt number, overall heat transfer coefficient and friction factor. To further advance design intelligence, the study integrates machine learning (ML), specifically decision tree and neural network models trained on experimental and numerical datasets. This ML framework enables real-time adaptive control of operating parameters using sensor feedback, offering an innovative pathway for maximizing thermal efficiency and output under dynamic conditions. The combination of mixed-diameter BTs, comprehensive parametric assessment, and ML-based control provides a novel contribution to DPHX performance optimization, bridging a clear gap in both experimental validation and intelligent thermal system design.

1. **PHYSICAL OUTLINE AND MATHEMATICAL STRATEGY**

**2.1 Experimental Configuration**

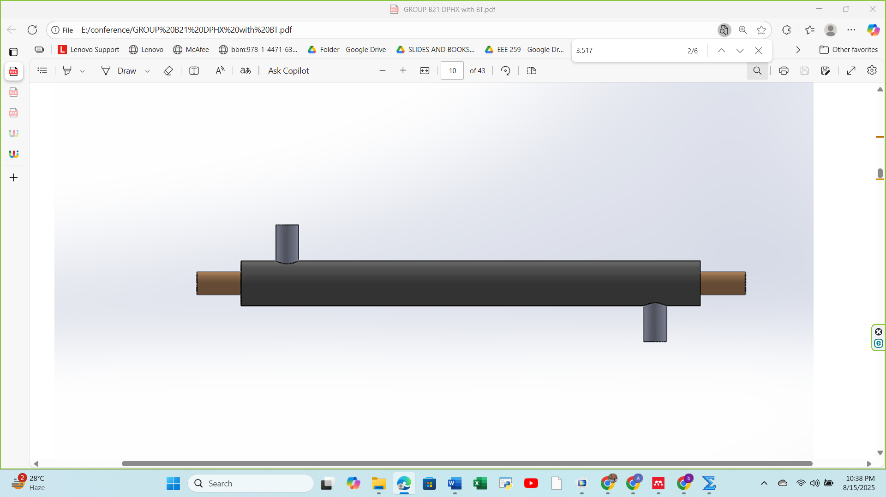
The present study investigates a counter-flow double-pipe heat exchanger (DPHX) equipped with ball-type turbulators (BTs) mounted inside the inner tube. The inner pipe, fabricated from copper, is concentrically positioned within a mild steel outer pipe. Hot water flows through the inner tube, while cold water is supplied through the annular space between the two pipes in a counter-flow arrangement. The working fluid in both streams is water, chosen for ease of laboratory testing, although the initial design considered ethylene glycol in the inner tube.

The BTs consist of mild steel spheres of two diameters, 12 mm and 18 mm, mounted alternately on a central copper rod by brazing. This rod is connected to the inner pipe by welding to maintain structural stability and spacing. The ratio of ball diameters (2:3) was selected to balance heat transfer enhancement with acceptable pressure drop, based on geometric constraints and prior studies. The spacing between adjacent balls was fixed at 76 mm to induce repeated flow disturbances and boundary layer disruption. The schematic representation of the physical domain is modeled using “SolidWorks 2018” shown in Fig. 1 and Fig. 2.

1. (b)

**FIGURE 1.** *Cross-section of the (a) inner pipe with turbulator, (b) heat exchanger.*



**FIGURE 2.** *3D model of the heat exchanger*

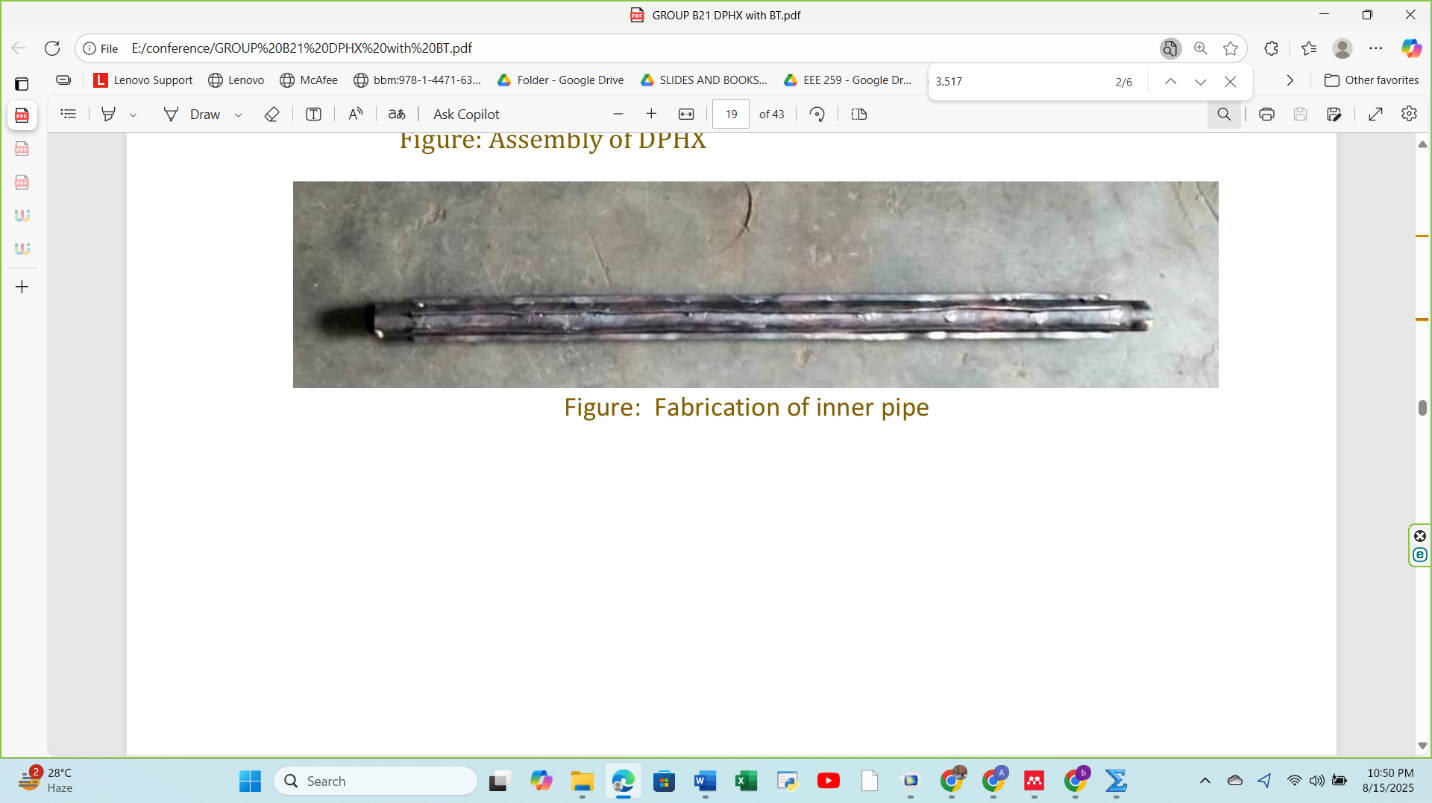
* 1. **Design Specification**

The principal geometric parameters of the exchanger are summarized in Table 1. The inner copper tube has an outer diameter of 25 mm, inner diameter of 23 mm, and a wall thickness of 1 mm. The outer mild steel pipe has an inner diameter of 40 mm, outer diameter of 50 mm, and a wall thickness of 5 mm. The total active heat transfer length, *L* is 0.6096 m (24 in).

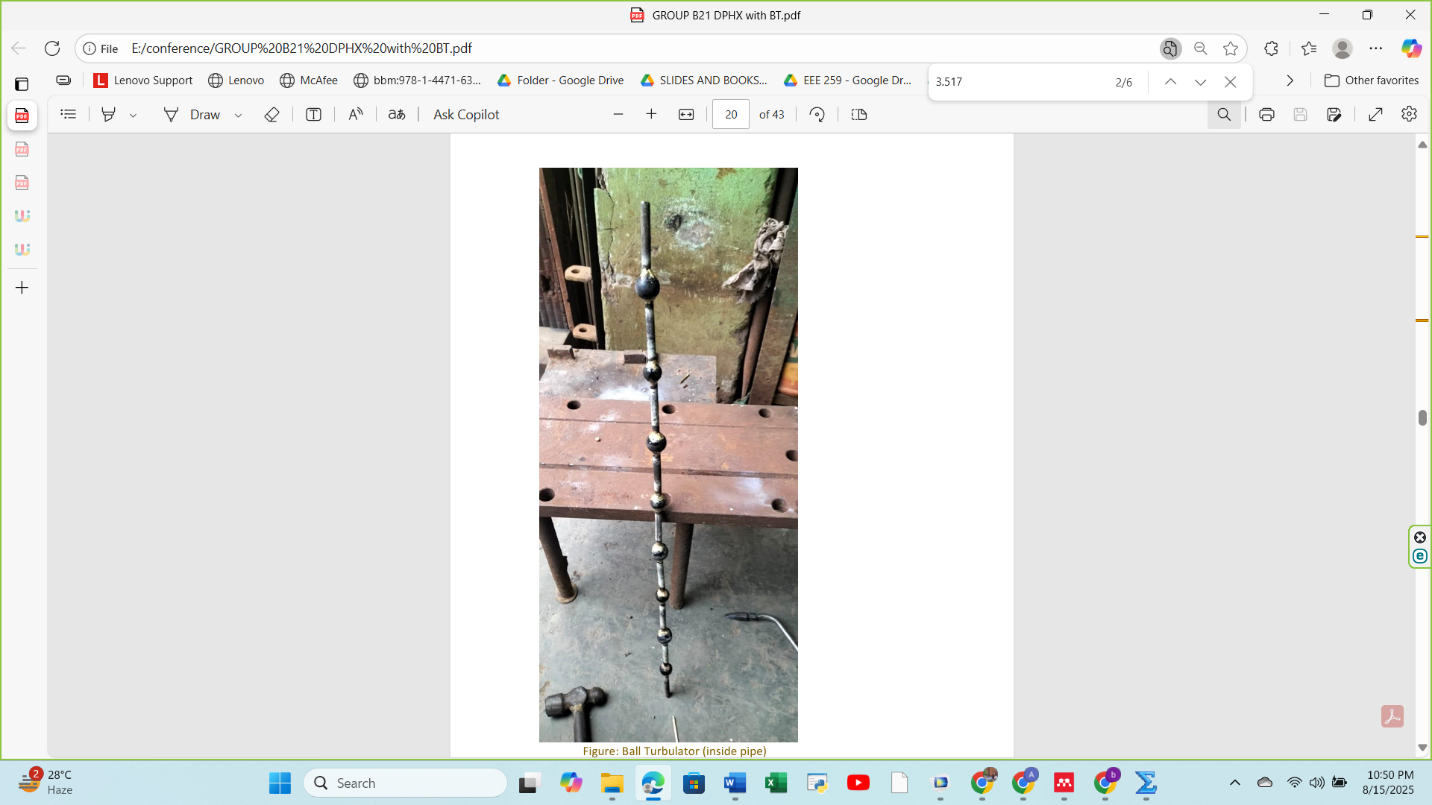
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| --- | --- | --- | --- |
| **TABLE 1.** *Geometric parameters of the DPHX*. | | | |
| **Component** | **Material** | **Outer Diameter (mm)** | **Inner Diameter (mm)** |
| Inner Tube | Copper | 25 | 23 |
| Outer Tube | Mild Steel | 50 | 40 |
| BT – Large | Mild Steel | 18 | — |
| BT – Small | Mild Steel | 12 | — |

* 1. **Manufacturing Procedure**

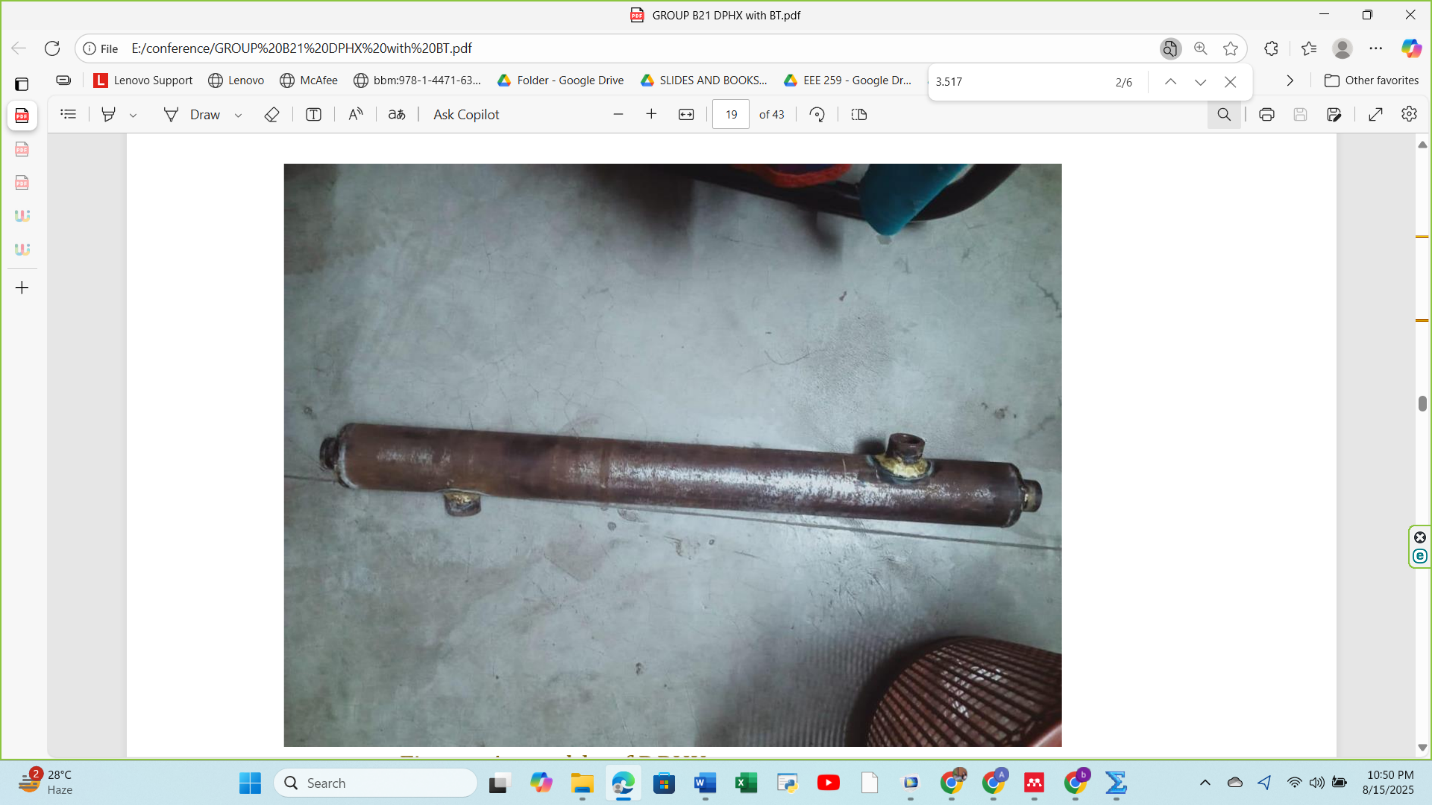
The fabrication process began with material selection, using copper for the inner pipe due to its high thermal conductivity, mild steel for the outer pipe for structural strength, and mild steel balls for the BTs. The BTs, with alternating diameters and fixed spacing, were brazed onto a central copper rod to form the assembly. The inner tube was prepared with a slotted fin arrangement on its outer surface to ensure coaxial alignment within the outer pipe. Metal Inert Gas (MIG) welding was employed to join the rod to the inner pipe and to seal the inlet and outlet connections. The completed inner tube–BT assembly was then inserted into the outer pipe, and inlet/outlet fittings were welded into place. Finally, hydrostatic pressure testing was performed to verify leak-free operation under laboratory conditions.



**FIGURE 3.** *Fabrication of inner pipe*



**FIGURE 4.** *Ball Turbulator (inside pipe).*



**FIGURE 5.** *Assembly of DPHX*

**2.4 Assumption**

The analysis and design of the DPHX with BTs are conducted under the assumption of steady-state operation, with both hot and cold streams comprising incompressible, single-phase water. Thermophysical properties are considered constant and evaluated at the mean fluid temperature for each stream, while changes in kinetic and potential energy between the inlet and outlet are assumed negligible. The system is regarded as perfectly insulated, with no heat loss to the surroundings, and a fouling factor of 0.00035 m²·K/W is applied for city water in both flow paths.

* 1. **Governing Equations**

The heat transfer analysis is based on the energy balance equation for steady-flow systems:

 (1)

The convective heat transfer coefficients for the inner tube (ℎ*p*) and annulus (ℎ*a*) are calculated from the Nusselt number correlation.

For smooth tubes (without BTs), the Dittus–Boelter correlation is applied:

 (2)

where 𝑛 = 0.3 for cooling (hot stream) and 𝑛 = 0.4 for heating (cold stream).

For the BT-equipped tube, an empirical correlation for twisted-tape-type swirl flow is adopted:

 (3)

where 𝐻 = 0.025 m is experimental constant obtained from

The overall heat transfer coefficient is then obtained from:

 (4)

* 1. **Hydraulic Performance**

Pressure drops in both the inner tube and annulus are determined using the Darcy–Weisbach equation:

 (5)

For smooth tubes, the Blasius-type friction factor is used:

 (6)

For BT-equipped tubes, the modified friction factor correlation is applied:

 (7)

where *ρ* is the fluid density, 𝑉 is the mean velocity, and *Dh* is the hydraulic diameter.

1. **NUMERICAL SCHEME**

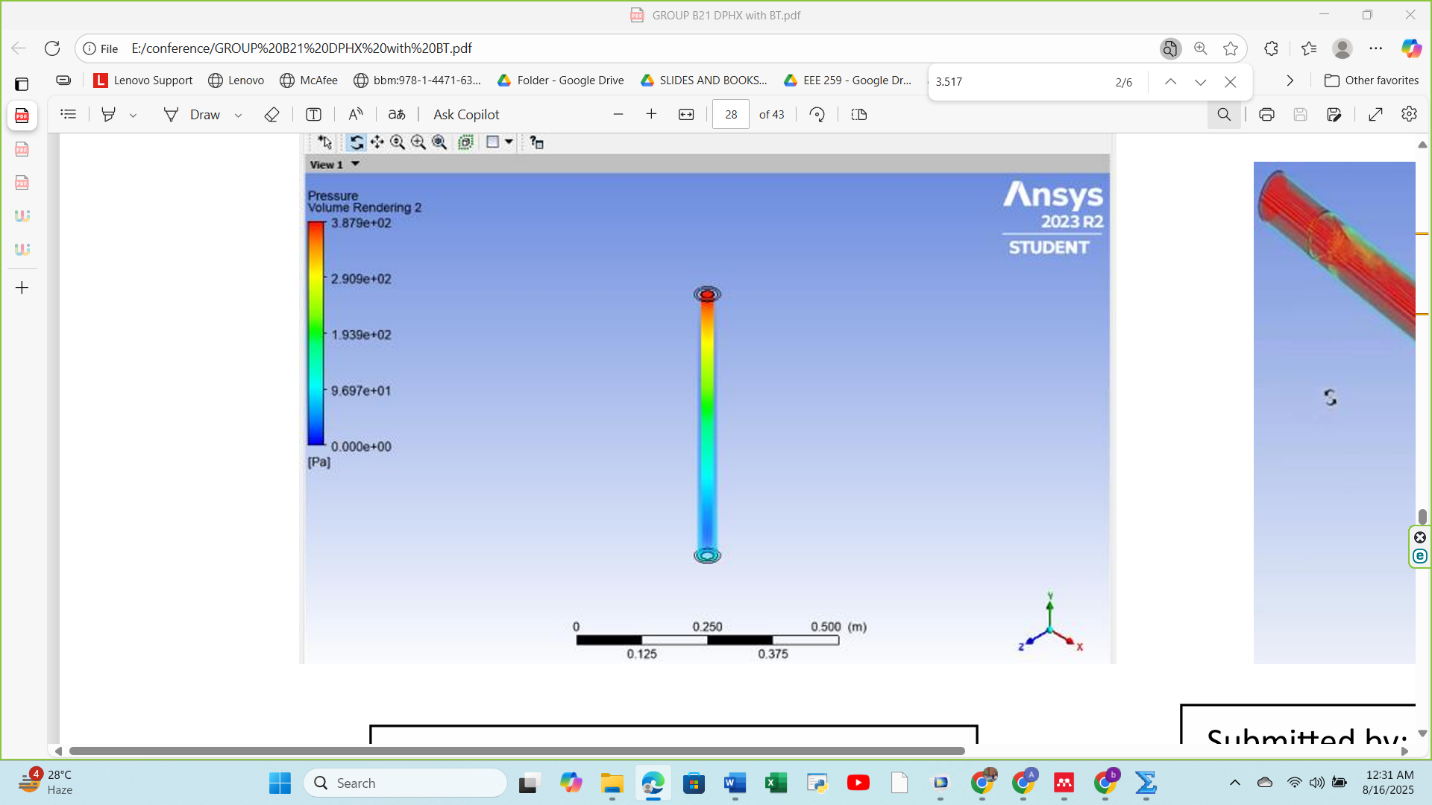
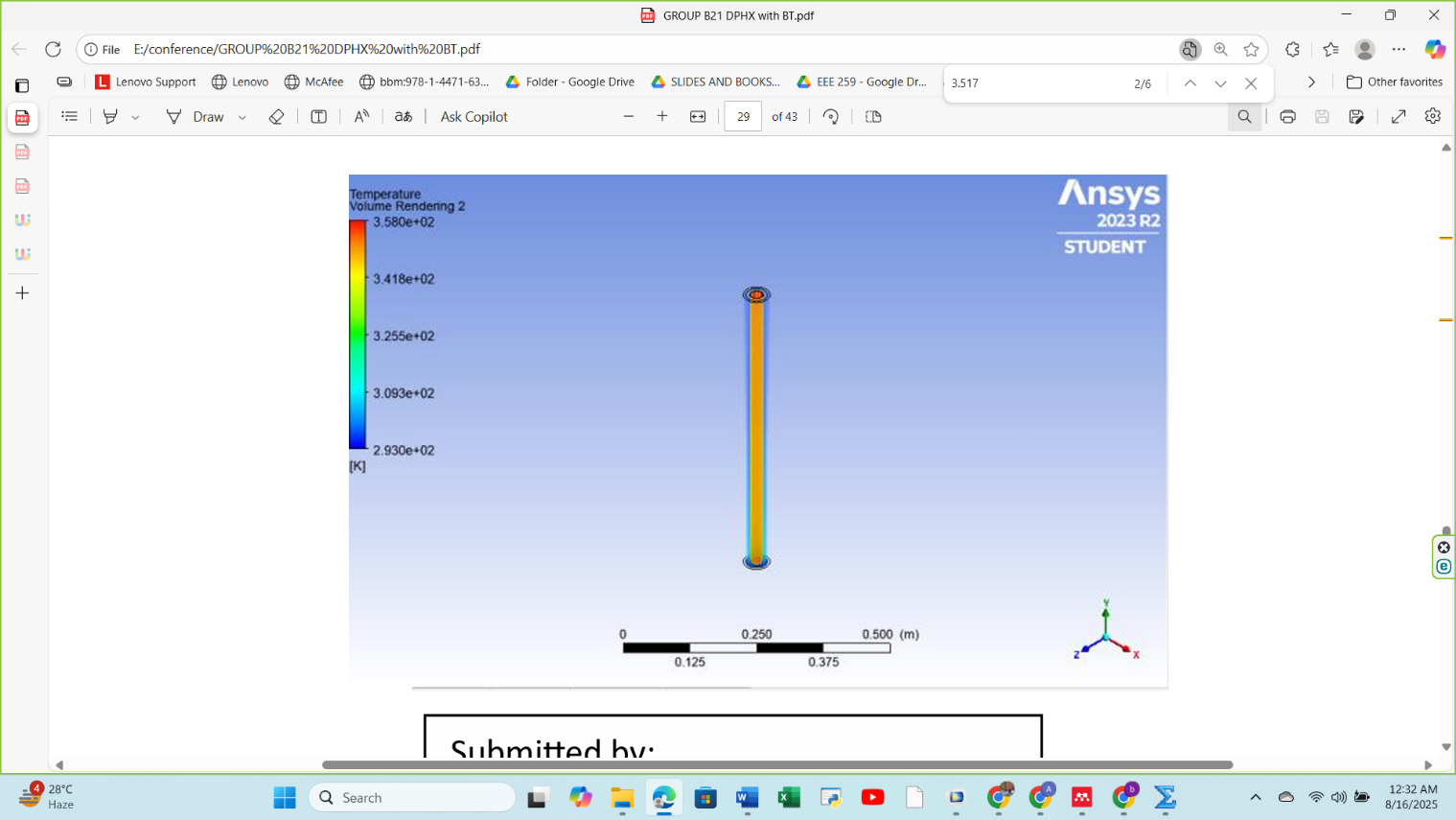
Complementary computational analysis was performed in “Ansys 2023 R2: Ansys Fluent”, adopting the same geometry and flow conditions without ball turbulators as in the experiments. The turbulence was modeled using the k-𝜀 RANS model with enhanced wall treatment. A structured hexahedral mesh was generated. Inlet temperatures and flow rates were matched to experimental values, and simulations were run for both configurations (smooth tube with no BT-equipped). In addition, Python programming was employed to implement a machine-learning model for predicting heat transfer rates based on operating parameters, enabling rapid performance estimation without re-running CFD or experiments.

1. **RESULTS AND DISCUSSION**

This section presents the thermal–hydraulic performance of the double pipe heat exchanger (DPHX) with and without the ball turbulator (BT), based on “Ansys 2023 R2: Ansys Fluent” simulations, and machine-learning-based predictive modelling. The aim is to evaluate the BT’s effect on heat transfer enhancement, hydraulic resistance, and the potential of data-driven approaches for operational optimization.

**4.1 CFD Simulation Results (without Ball Turbulators)**

The baseline smooth-tube case was simulated in “Ansys 2023 R2: Ansys Fluent” using the same inlet conditions, fluid properties, and geometry as the experimental DPHX but without BTs in the inner tube. The velocity contours indicated predominantly axial flow with minimal cross-sectional mixing, while the temperature fields showed clear thermal stratification along the length and a thicker thermal boundary layer near the tube wall, limiting heat transfer. The calculated overall heat transfer coefficient for this configuration was *U0* = 606.62 W·m⁻²·K⁻¹, with corresponding pressure drops of 253.70 Pa in the inner pipe and 88.75 Pa in the annulus, providing the reference for quantifying the BT-induced enhancement.

1. (b)

**FIGURE 6.** *Contour plots of (a) pressure, (b) temperature.*

* 1. **Quantitative Comparison of Performance**

**TABLE 2.** *The Computed Performance Parameters for both Configurations.*

|  |  |  |
| --- | --- | --- |
| **Parameters** | **Without BT** | **With BT** |
| U0 (W. m-2. K-1) | 606.62 | 966.18 |
| Pipe Pressure Drop (Pa) | 253.70 | 912.63 |
| Annulus Pressure Drop (Pa) | 88.75 | 278.69 |

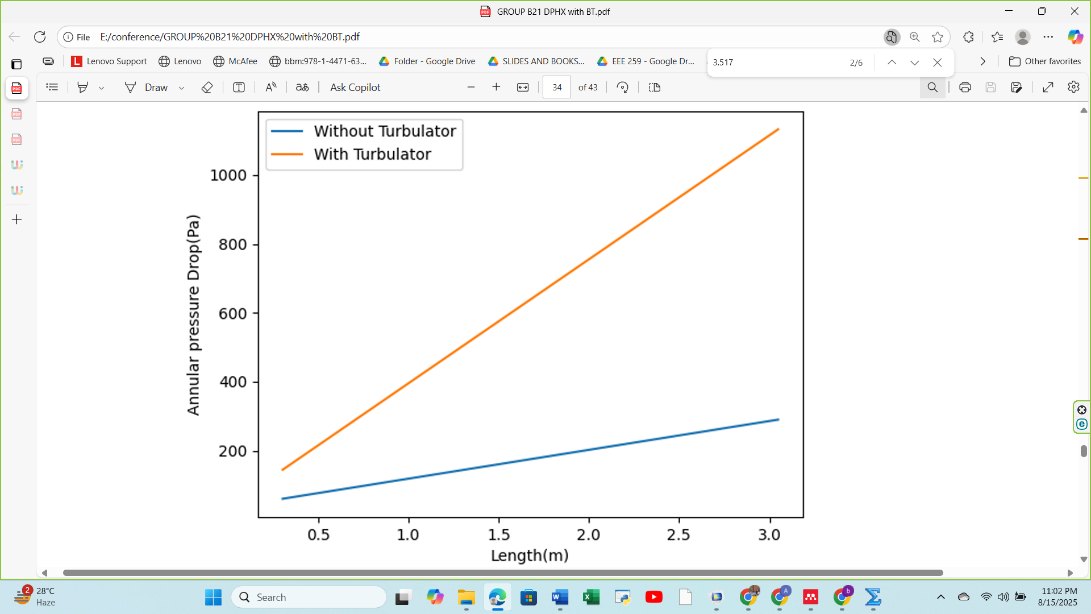
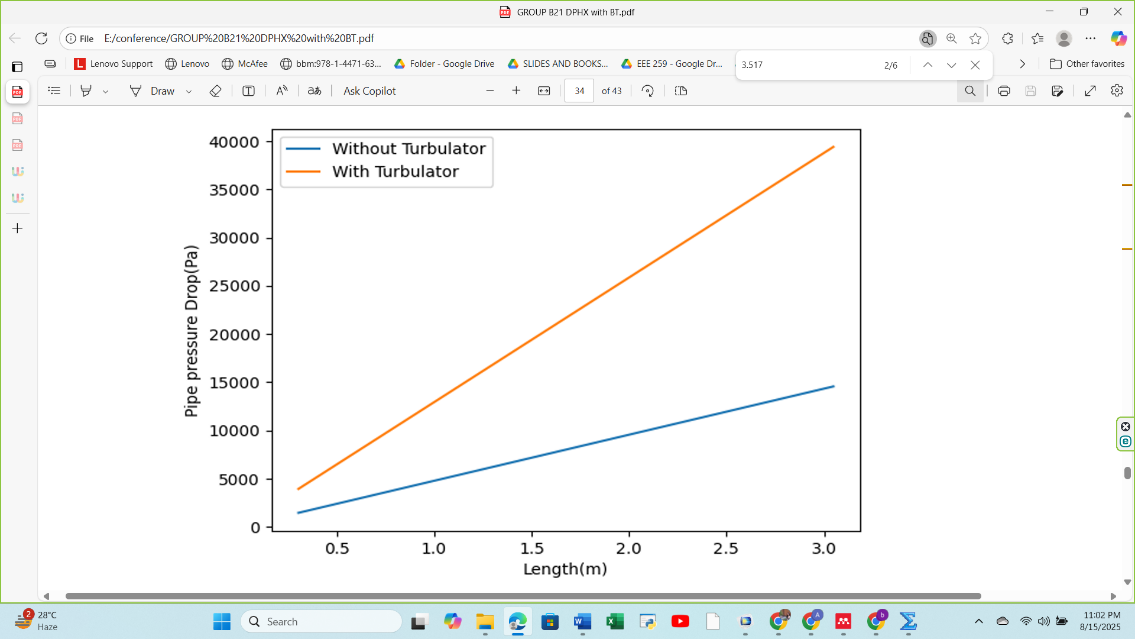
Introducing ball turbulators yielded nearly a 60% gain in overall heat transfer coefficient, which aligns with the turbulence patterns observed in CFD. However, this gain came at the expense of hydraulic performance: pipe and annulus pressure drop increased by factors of 3.60 and 3.14, respectively. This trade-off reflects the balance between turbulence-induced enhancement and additional pumping power requirements.

* 1. **Fluctuation of Heat Transfer Characteristics over Different Parameters**

The variation of heat transfer characteristics with different operating parameters was analysed to understand the sensitivity of the DPHX performance to flow and thermal conditions. The plotted trends reveal distinct response patterns for the configurations with and without BT.

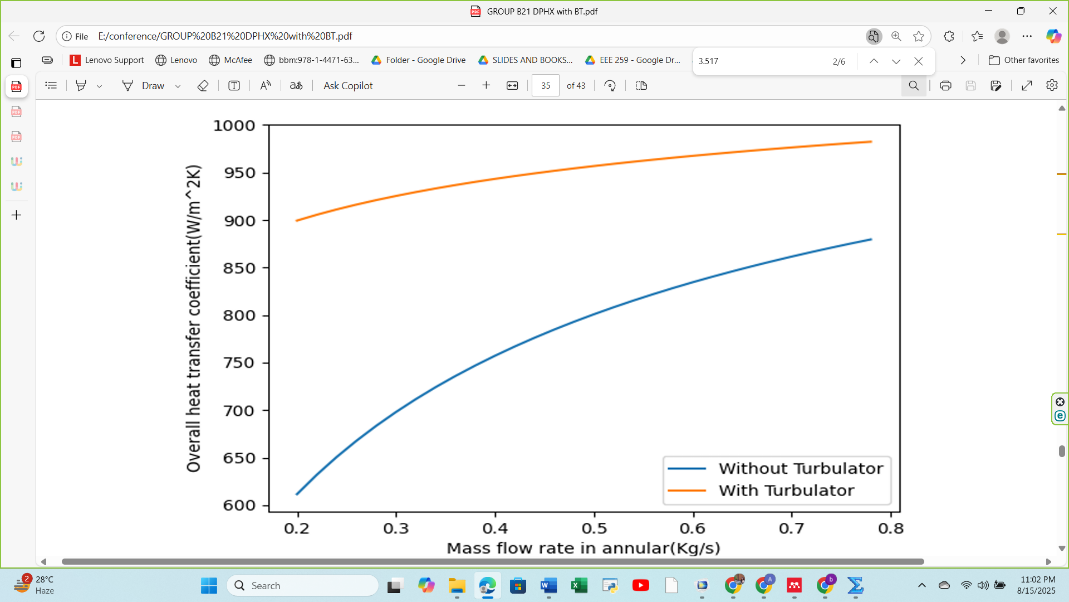
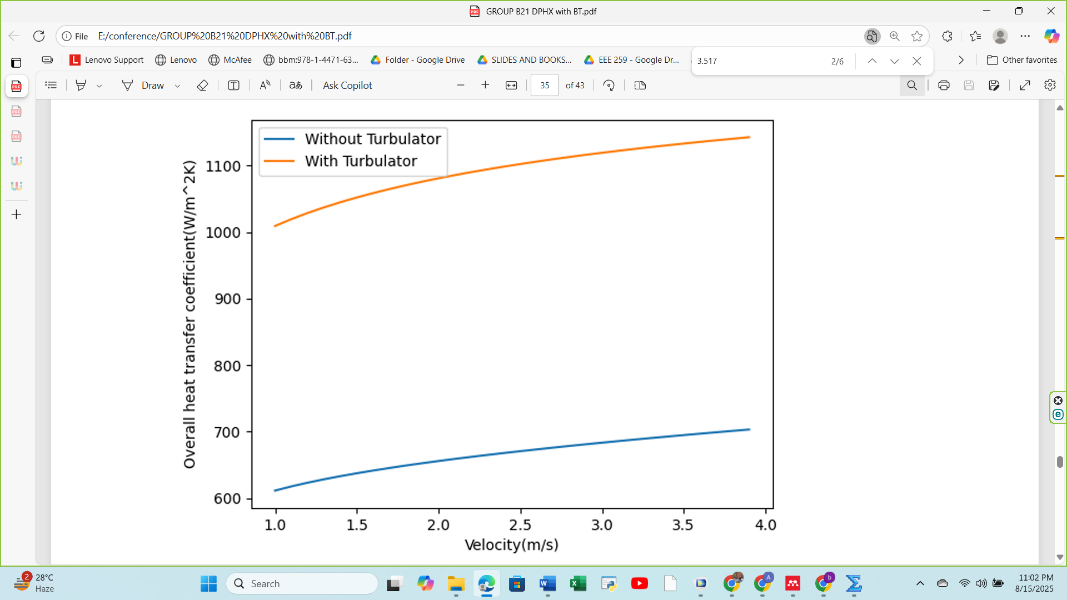
Fig. 7 illustrates the variation of annular and pipe pressure drops with length *(L*) for the DPHX with and without the ball turbulator. In both cases, pressure drop increases linearly with length due to cumulative frictional and form losses. The BT-equipped configuration consistently exhibits higher values in both annulus and pipe, reflecting increased flow obstruction and turbulence generation. The difference between the two configurations widens with length, indicating that the penalty from BT insertion becomes more pronounced over longer heat exchanger sections.

In Fig. 8(a), the overall heat transfer coefficient *U0* increases with annular mass flow rate for both configurations, but the BT-equipped DPHX shows a consistently higher value, confirming the turbulence-enhancement effect of the turbulator. Fig. 8(b) presents *U0* variation with pipe velocity, showing a similar upward trend. At higher velocities, the rate of increase slows as the thermal boundary layer is already thin, leading to diminishing returns, though the BT case still maintains a clear advantage.

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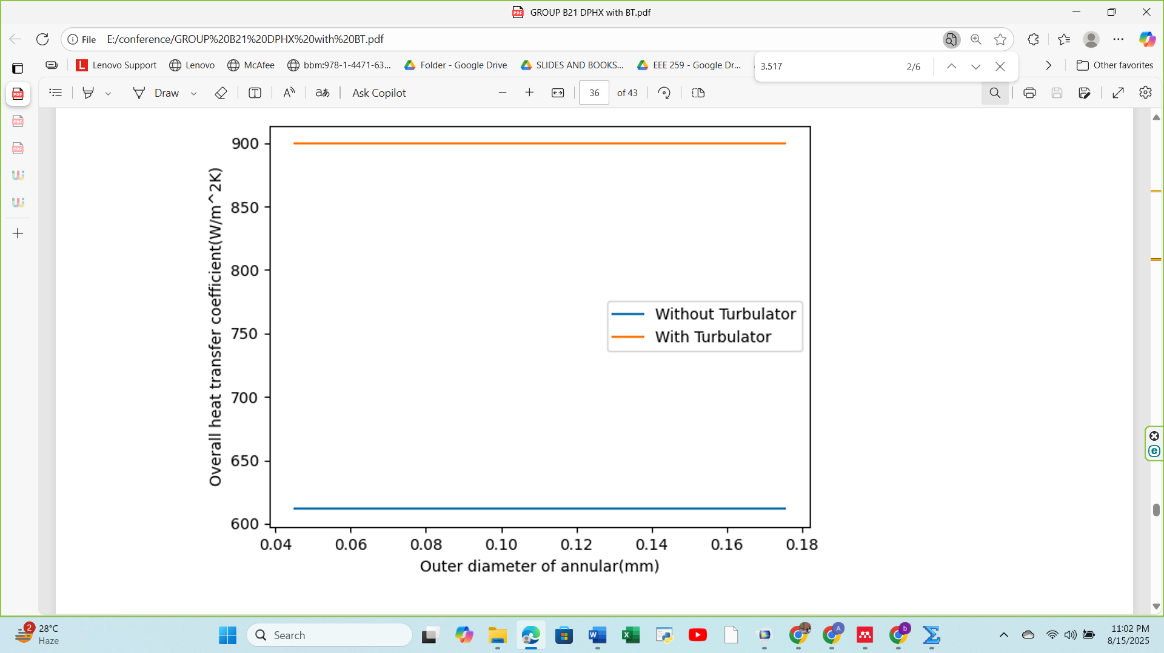
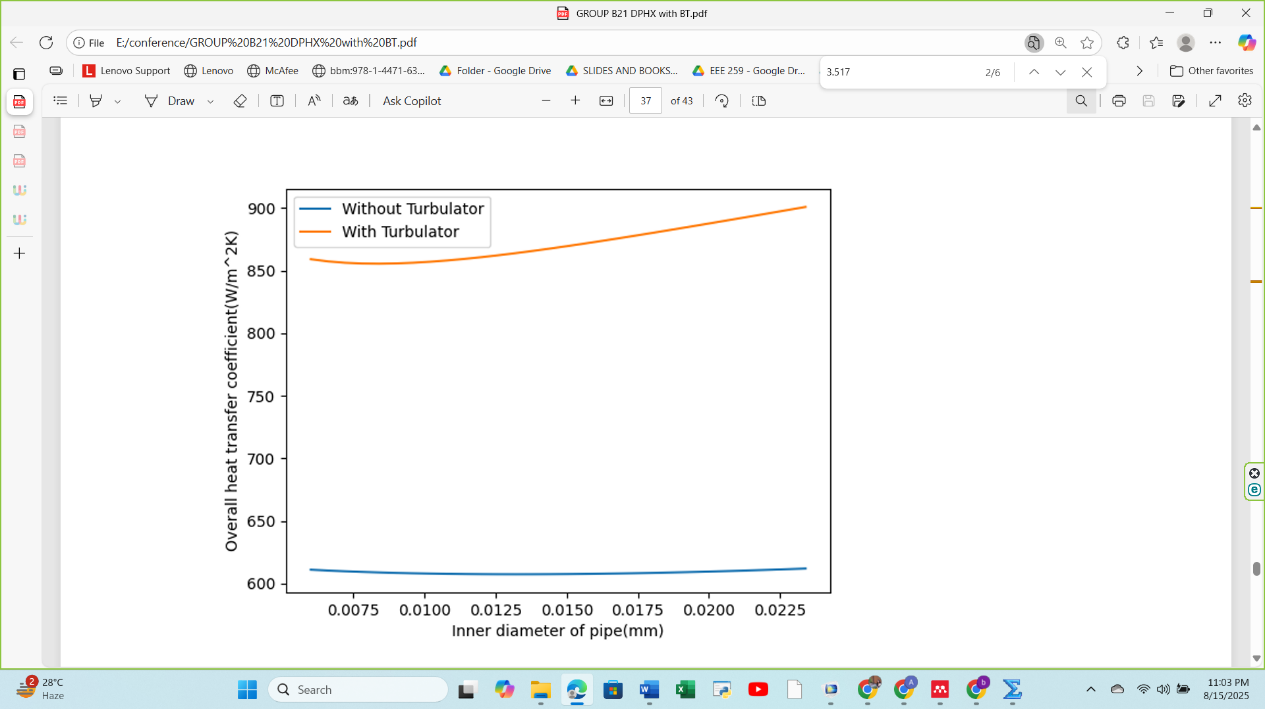
1. (b)

**FIGURE 7.** *Comparison of the variation of the (a) annular pressure drop, (b) pipe pressure drop with length (L) for DPHX with and without ball turbulator.*

** **

1. (b)

**FIGURE 8.** *Comparison of the variation of the overall heat transfer coefficient (U0) with (a) mass flow rate (ṁ ) and (b) velocity (V) for DPHX with and without ball turbulator.*

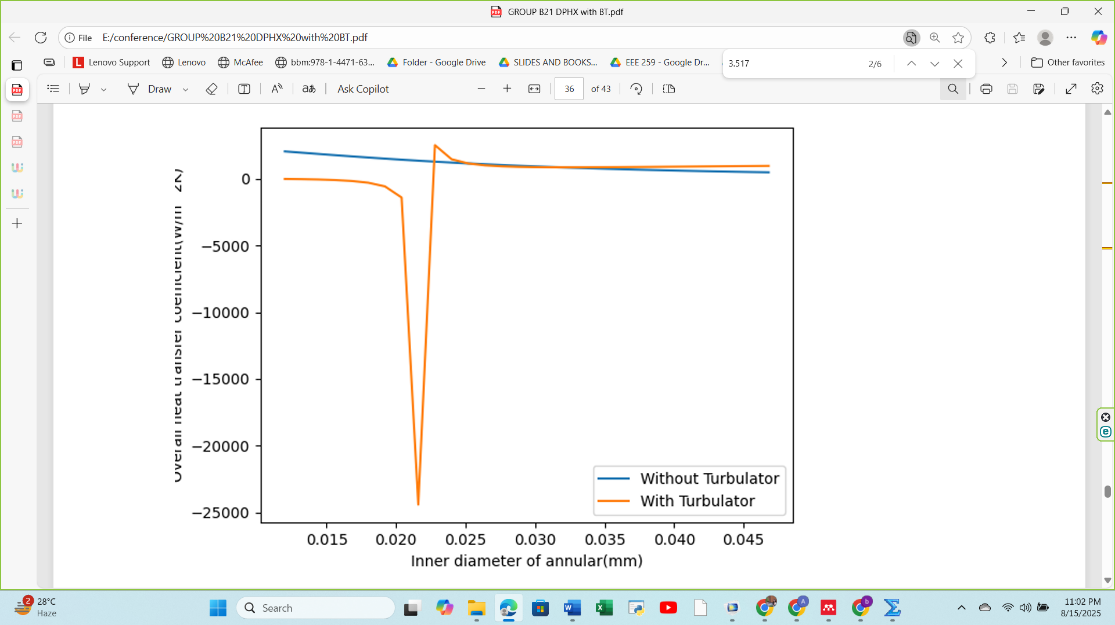
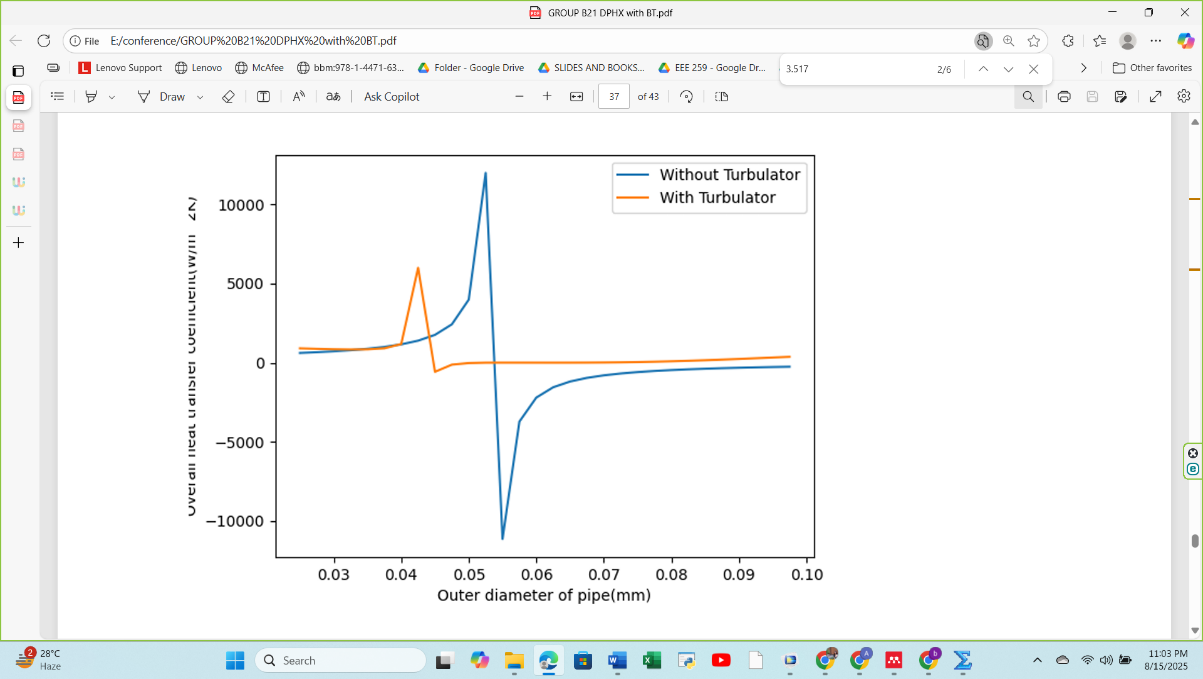
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1. (b)

**FIGURE 9.** *Comparison of the variation of the overall heat transfer coefficient (U0) with (a) outer diameter of the annular and (b) inner diameter of the pipe for DPHX with and without ball turbulator.*

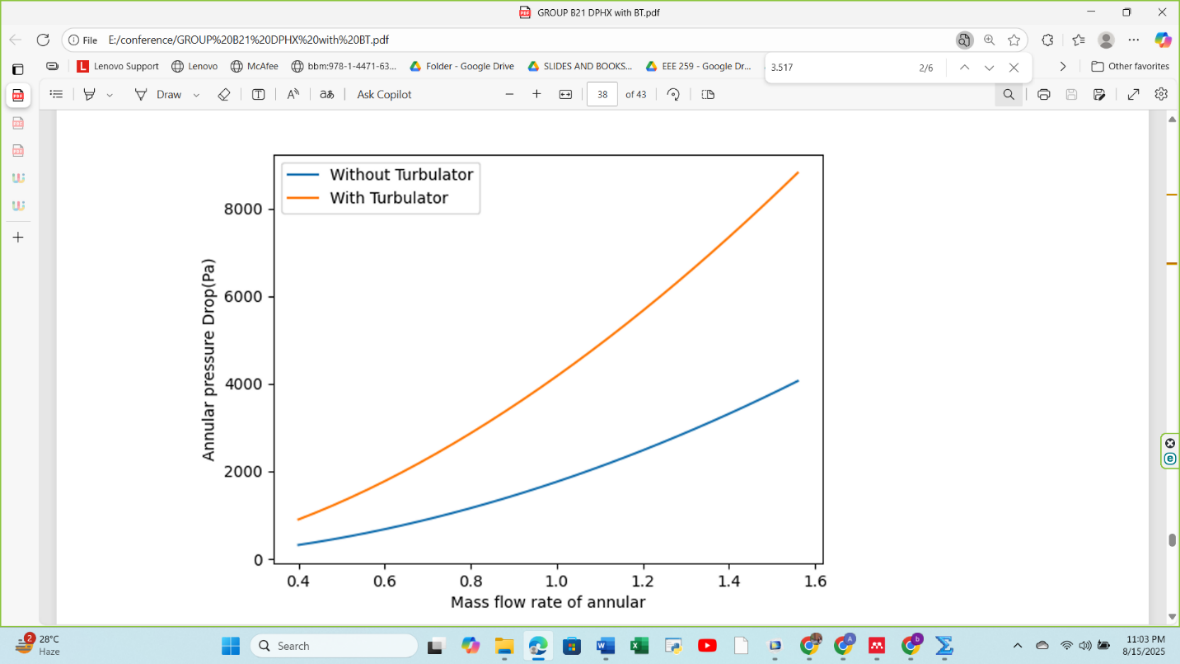
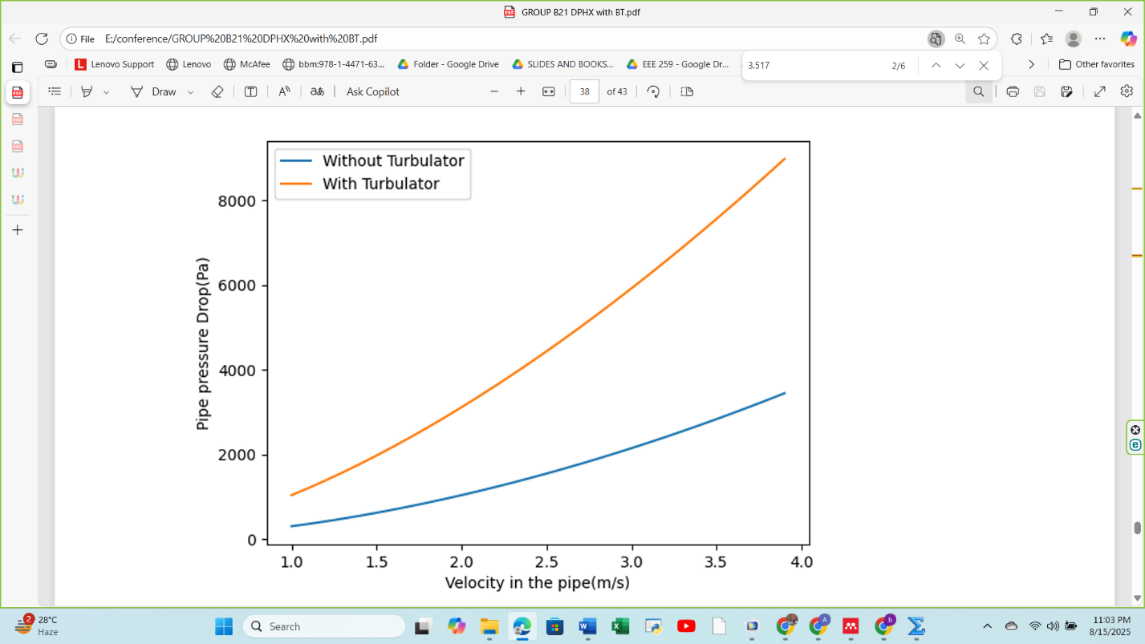
Fig. 9(a) shows that *U0*​ varies slightly with the outer diameter of the annulus, with the BT configuration maintaining a marginally higher coefficient. Fig. 9(b) indicates a modest increase in *U0*​ with inner pipe diameter for both cases, with the BT-equipped exchanger again outperforming the smooth tube.

Fig. 10(a) and Fig.10(b) present *U0​* variations for inner annulus diameter, *Di* and outer pipe diameter, *D0* respectively. The irregularities and sudden drops in some curves are attributed to numerical instabilities in the simulation at extreme geometric ratios. Despite these anomalies, the BT-equipped exchanger generally shows better performance in most geometric configurations, confirming that turbulence generation outweighs the slight change in flow cross-section.

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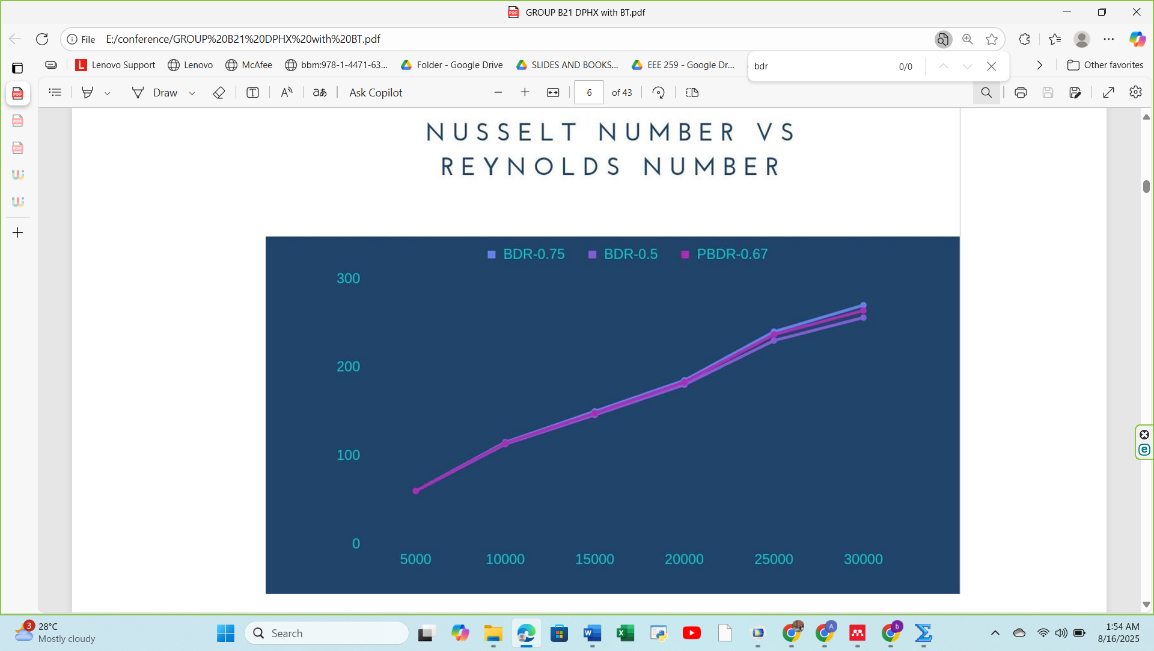
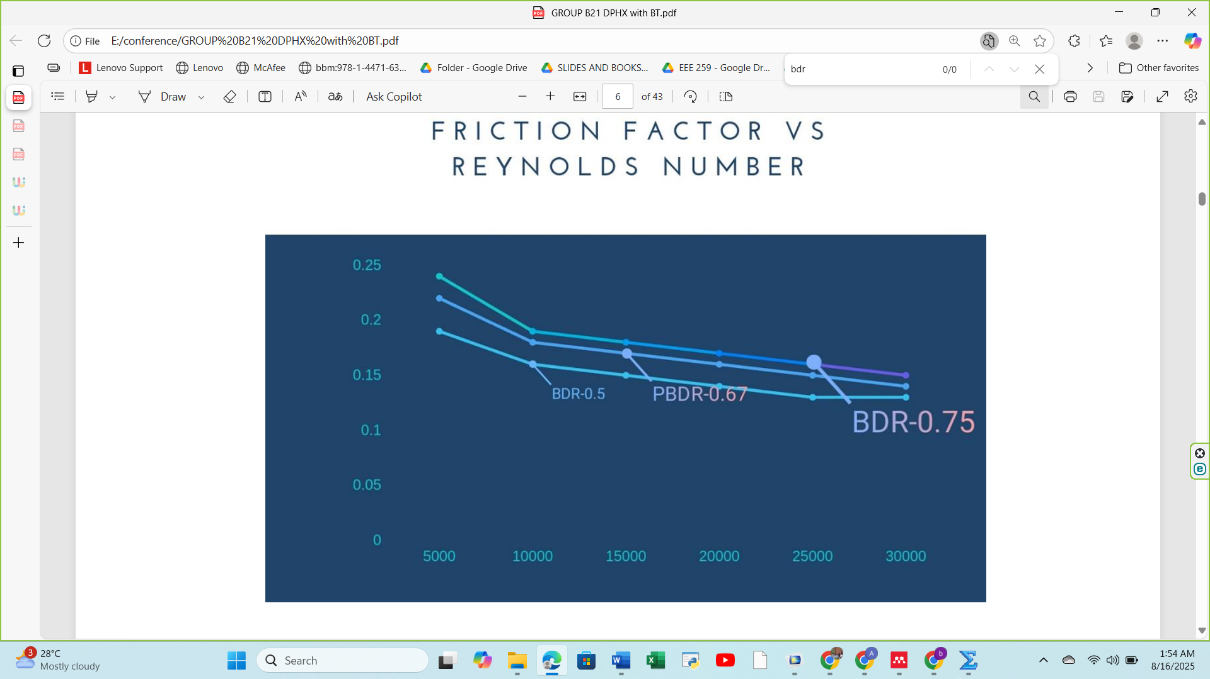
1. (b)

**FIGURE 10*.*** *Comparison of the variation of the overall heat transfer coefficient (U0) with (a) inner diameter of the annular, D­i and (b) outer diameter of the pipe, D0 for DPHX with and without ball turbulator.*

** **

1. (b)

**FIGURE 11.** *Comparison of the variation of the (a) annular pressure drop with annular mass flow rate and (b) pipe pressure drop with velocity in the pipe for DPHX with and without ball turbulator.*

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1. (b)

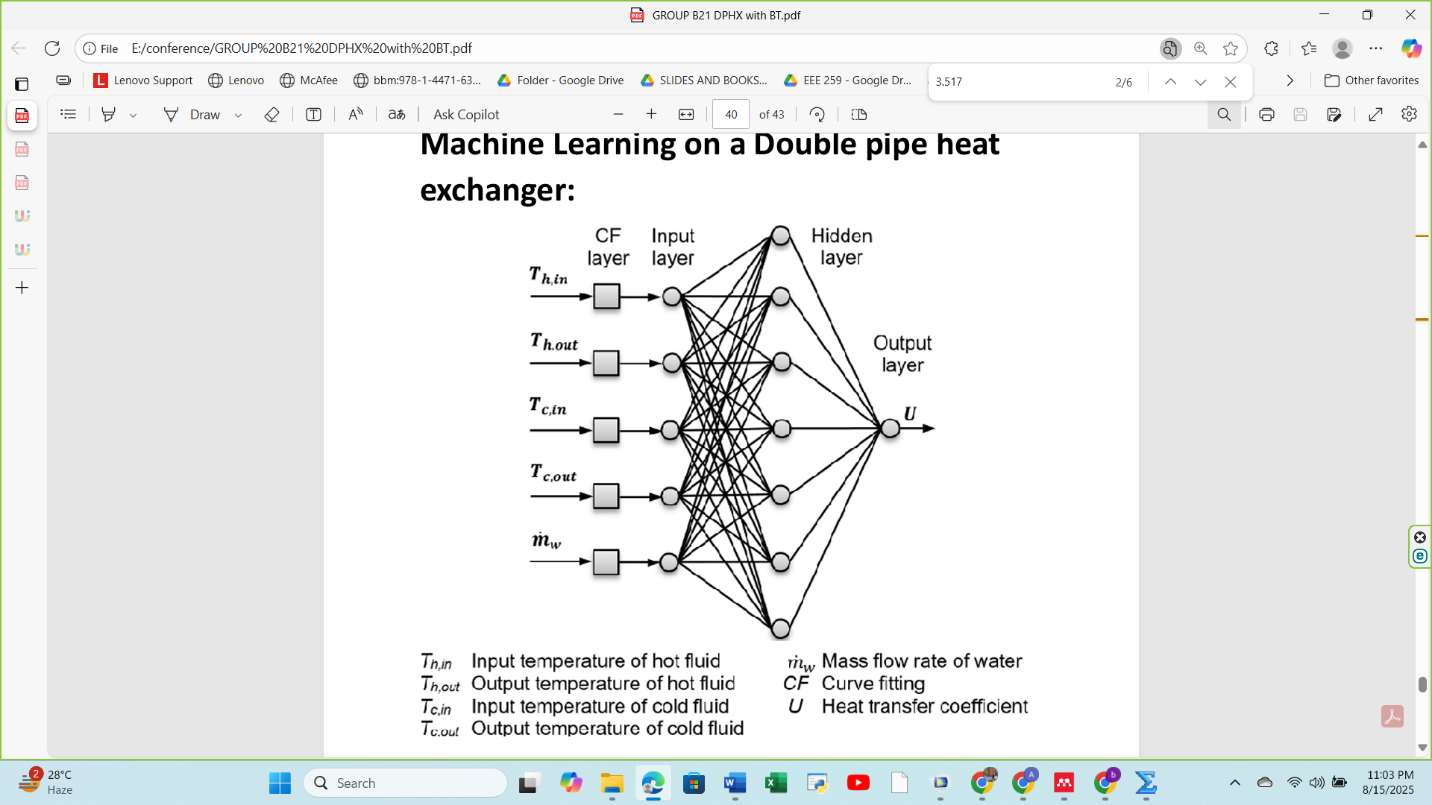
**FIGURE 12.** *Variation of (a) Nusselt number (Nu), (b) friction factor (f) with Reynolds number (Re) across ball diameter ratio (BDR).*

Fig. 11(a) demonstrates that annular pressure drop rises nonlinearly with annular mass flow rate, with the BT case exhibiting a significantly steeper increase. This is due to intensified turbulence and form drag. Similarly, Fig. 11(b) shows that pipe pressure drop increases rapidly with velocity in the BT case, again due to additional blockage and vortex formation. These trends emphasize the need to balance heat transfer gains with pumping power requirements.

Figure 12(a) illustrates that the Nusselt number (*Nu*) rises with increasing Reynolds number (*Re*) for each BDR value, indicating that higher flow velocities intensify turbulence, reduce the thermal boundary layer thickness, and thereby enhance heat transfer. While all configurations show similar growth trends, BDR = 0.75 achieves the highest Nusselt numbers, followed closely by PBDR = 0.67 and BDR = 0.50. On the contrary, from Fig. 12(b), for all BDR values, the friction factor (*f*) decreases with increasing *Re*, consistent with the transition from higher relative viscous resistance at low flow rates to more inertia-dominated flow at higher Reynolds numbers. Among the configurations, BDR = 0.50 consistently shows the lowest friction factor, indicating reduced blockage effect and lower hydraulic resistance. Conversely, BDR = 0.75 exhibits the highest friction factor across the Reynolds number range due to its larger flow obstruction, which promotes turbulence but increases pressure drop. The PBDR = 0.67 case lies between these two extremes. This trend highlights the trade-off between turbulence generation and hydraulic penalty when selecting BT dimensions. The prescribed ball diameter ratio, PBDR = 0.67 provides a compromise between the two.

**4.4 Machine-Learning-Assisted Prediction**

To complement the experimental and CFD analyses, a Decision Tree Regressor was trained on synthetic operating data generated from the energy balance equation.



Gaussian noise was added to mimic measurement variability. The model achieved a low MSE and high the proportion of variance (*R2*) on the test set, indicating reliable predictive accuracy. The ML model offers rapid estimation of heat duty for given operating points without running new CFD simulations or experiments. Multi-objective optimization to identify flow rates and temperature differences that maximize 𝑈0 while limiting pressure drop. With real plant data, this predictive framework can be integrated into an online decision-support system for process optimization.

**5. CONCLUSION**

This present study examined the thermo-hydraulic behavior of a double-pipe heat exchanger (DPHX) both with and without ball turbulators (BTs), using analytical formulations, CFD simulations in “Ansys 2023 R2: Ansys Fluent,” and predictive modeling supported by machine learning. The main findings are summarized as follows:

1. The insertion of BT increased the overall heat transfer coefficient from *U0* = 606.62 W·m⁻²·K⁻¹ to *U0* = 966.18 W·m⁻²·K⁻¹, representing an improvement of approximately 60%. This enhancement was confirmed by CFD analysis in case of DPHX without ball turbulator.
2. The improvement in heat transfer was accompanied by significant increases in pressure drop. The pipe and annulus pressure drops rose by factors of 3.60 and 3.14, respectively. These increases are attributed to additional flow blockage and shear induced by the BT elements.
3. A Decision Tree Regressor trained on synthetic operating data demonstrated high predictive accuracy for heat transfer rate, indicating that ML models can serve as effective surrogates for rapid operating point evaluation and multi-objective optimization.
4. The proposed design demonstrates particular suitability in systems where compact exchangers are needed, and the added pumping requirement can be justified by the improved heat transfer. For energy-sensitive systems, optimization of BT geometry and spacing, supported by CFD and ML tools,can balance thermal performance with acceptable hydraulic losses.

Future work should focus on exploration of alternative BT configurations, and integration of real operational data into the predictive framework to enable robust, real-time process control.

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**NOMENCLATURE**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Roman Symbols*** | | | ***Greek Symbols*** | |
| *cp* | Specific heat of the fluid (J/kg.K) | | *μ* | Dynamic viscosity (Pa/sn-2) |
| *D* | Inner diameter of tube (m) | | *ρ* | Density (kg/m3) |
| *Dh* | Annular inner diameter (m) | |  |  |
| *Dh* | Hydraulic diameter of flow passage (m) | |  |  |
| *Dh* | Outer diameter of the pipe (m) | |  |  |
| *f* | Darcy–Weisbach friction factor | | ***Subscript*** | |
| *H* | Pitch spacing between ball turbulators (m) | | *a* | Annulus (outer passage) |
| *ha* | Convective heat transfer coefficient in annulus (W·m⁻²·K⁻¹) | | *c* | Cold |
| *hp* | Convective heat transfer coefficient in inner pipe (W·m⁻²·K⁻¹) | | *h* | Hot |
| *k* | Thermal conductivity of fluid (W·m⁻¹·K⁻¹) | | *m* | Mesh |
| *L* | Effective heat transfer length of the tube (m) | | *in* | Inlet condition |
| *ṁh* | Mass flow rate of hot fluid (kg·s⁻¹) | | *out* | Outlet condition |
| *ṁc* | Mass flow rate of cold fluid (kg·s⁻¹) | | *p* | Pipe (inner tube) |
| *n* | Exponent in Nusselt number correlation | |  |  |
| *Nu* | Nusselt number | |  |  |
| *Pr* | Prandtl number | |  |  |
| *Q* | Heat transfer rate (W) | |  |  |
| *Re* | Reynolds number | |  |  |
| *Rf* | Fouling factor (m²·K·W⁻¹) | |  |  |
| *Th,in* | Inlet temperature of hot fluid (°C or K) | |  |  |
| *Th,out* | Outlet temperature of hot fluid (°C or K) | |  |  |
| *Tc,in* | Inlet temperature of cold fluid (°C or K) | |  |  |
| *Tc,out* | Outlet temperature of cold fluid (°C or K) | |  |  |
| *U0* | Overall heat transfer coefficient (W·m⁻²·K⁻¹) | |  |  |
| *V* | Mean velocity of fluid (m·s⁻¹) | |  |  |
|  |  | |  |  |
|  | **Abbreviations** |
| *BT* | Ball Turbulator |
| *BDR* | Ball Diameter Ratio |
| *DPHX* | Double Pipe Heat Exchanger | |  |  |
| *MSE* | Mean Squared Error | |  |  |
| *PBDR* | Prescribed Ball Diameter Ratio | |  |  |