Thermal Performance of Hybrid Nanofluid Flow over a Stretching Sheet Embedded in a Porous Medium under Heat Source/Sink Effects

Prasanta Parida,1, a) Hibah Islahi1,b) and Kharabela Swain2, c)

1Institute of Applied Sciences, Mangalayatan University, Aligarh-202146, India

1Dept. of Mathematics, GIFT Autonomous, Bhubaneswar- 752054, Odisha, India

c) Corresponding author: kharabela1983@gmail.coma) 20230218\_prasanta@mangalayatan.edu.in

b) hibah.islahi@mangalayatan.edu.in

**Abstract.** The current work examines the heat transfer characteristics of hybrid nanofluid flow over a stretching sheet embedded in a porous medium, incorporating the effects of Joule heating and viscous dissipation. Hybrid nanofluids comprising multiple nanoparticles suspended in a base fluid exhibit superior thermal properties compared to conventional fluids, making them highly effective for advanced heat transfer applications. In this work, water (H₂O) is considered as the base fluid, while copper (Cu) and ferrous ferric oxide (Fe₃O₄) serve as the nanoparticles. The governing momentum and energy equations are formulated by accounting for viscous dissipation, which converts mechanical energy into heat, and Joule heating induced by electrical conductivity of the fluid. A similarity transformation technique is employed to reduce the governing partial differential equations to ordinary differential equations, which are then solved numerically using an efficient computational scheme. The influence of key physical parameters such as nanoparticle volume fraction, porosity of the medium, applied electric field intensity, and fluid viscosity on the velocity and temperature fields is analyzed. The results reveal that both Joule heating and viscous dissipation markedly elevate the fluid temperature, while hybrid nanofluids demonstrate superior heat transfer performance compared to conventional fluids. These findings provide valuable insights for optimizing thermal systems in industrial applications, including heat exchangers, cooling technologies, and thermal management of electronic devices.

# INTRODUCTION

A new category of nanofluids, referred to as hybrid nanofluids, have been the subject of much recent research. Due to the combined effect of multiple components, hybrid nanofluids have improved thermal characteristics. Due to its several potential advantages over nanofluids, this recent development which was brought about by the addition of different nanoparticles to the working liquid has gained widespread attention from researchers. Nowadays, a lot of studies have tried hybrid nanofluids or mixed, which are made up of two different kinds of nanoparticles. The main goal of using hybrid nanofluids is to improve their thermo physical characteristics, especially the heat switch charge, in comparison to single-particle or conventional nanofluids. The procedure of evaluating hybrid nanofluids' performance stays in the improvement phase because they are a novel class of nonliquids. High heat transfer performance is anticipated when hybrid nanofluids are used. Researchers have recently focused on extended classes of nanofluids, which can create a range of nanoparticles, including carbon nanotubes, non-metal particles, and metal particles.

Stretching (elongating)/shrinking (contracting) sheets are used to conduct research on the fluid's momentum and heat transfer, leading to a wide range of applications, such as heat conservation, the design of complex synthetic structures, and transport systems. Because of how quickly heat replaces itself and how quickly velocity gradients change, industrialized applications of heat transfer and momentum analysis are essential to the development of a refined product for consumption (see ref. [1] - [3]).

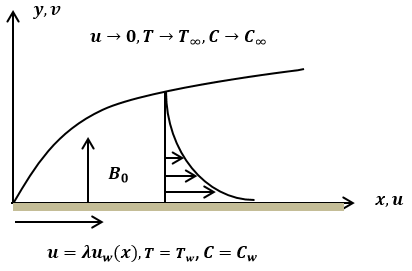
Hybrid nanofluid (HNF) is a new type of heat transfer fluids with the potential to outperform both ordinary fluids (water and ethylene glycol etc.) and nanofluids containing single type of nanoparticle. Hybrid nanoparticles have been shown a significant role in various industrial and engineering applicationa (see ref. [4]–[7]). HNF flow across a porous shrinking surface was examined by Waini et al. [8]. Devi & Devi [10] and Ahmad et al. [9] investigated the flow of HNF in a porous matrix containing nanoparticles of alumina and copper. Rawat et al. [12] have observed the effects of copper and silver nanoparticles passing a vertical Riga plate.

Thermal radiation plays a vital role in various industrial and engineering fields, such as solar thermal energy production, furnaces, and combustion processes. It is also essential for spacecraft insulation, electronics cooling, and heating in buildings. Moreover, it finds use in material treatments like drying, annealing, and welding operations. Ghadikolaei et al. [23] developed a modeling on the effect of radiative squeezing flow of Eyring Powell fluid in the central of two similar plates and accomplished that the radiative parameters dwindles the temperature contour. Khan et al. [24] and Hasona et al. [25] studied the Williamson nanofluid over a stretching sheet and peristaltic motion of Carreau nanofluids for the cancer treatment respectively. Zainal et. al. [26] have illustrated radiative influence on Maxwell hybrid nanofluids at the stagnation point flow. In the two-dimensional case, explore on hybrid nanofluids with MHD, radiation, and Joule heating has been rigorous studies (see [27] – [29]).

The existing literature on the heat transfer analysis of hybrid nanofluid flow over a stretching sheet in a porous medium lacks comprehensive studies that address the combined effects of Joule heating, viscous dissipation, nanoparticle interactions, and porous medium properties. There is a need for more detailed theoretical, and numerical studies to explore these gaps and provide a more accurate solution of hybrid nanofluid heat transfer in such complex scenarios. This paper aims to study the heat transfer and fluid flow characteristics of a hybrid nanofluid over a stretching sheet within a porous medium. The research explores the effects of Lorentz force, heat source/ink, and the contributions of viscous and Joule dissipation on the energy equation due to energy losses in industrial applications. The study finds that the rate of heat transfer is more significantly enhanced by HNF compared to single nanofluids and base fluids. Numerical solutions for the transformed ordinary differential equations (ODEs) are obtained using the bvp4c solver in MATLAB. The results, including the influence of various operating parameters, are presented through tables and graphical representations.

# FORMULATION OF THE PROBLEM

A steady 2D incompressible flow of HNF over a stretching sheet is studied. In x direction, the sheet is kept and the *y*-axis is perpendicular to the sheet and uniform magnetic field strength is applied in it as shown in Fig. 1 with ( is constant) is the stretching velocity. Let and represent the surface and ambient temperatures respectively.



**FIGURE 1** Flow geometry

The leading equations following [9] and [12] are:

(1)

(2)

(3)

(4)

(5)

where and are represent the magnetic field strength, heat source coefficient, stretching parameter and Brownian motion coefficient respectively.

Consider the following dimensionless variables

and .

(6)

(7)

(8)

(9)

where (magnetic parameter), (Prandtl number), (radiation parameter), (Eckert number), (heat source/sink parameter) , (Schmidt number) and (stretching parameter).

The friction coefficient , Nusselt number and Sherwood number are defined as

which gives respectively.

Here, shear stress , heat flux , mass flux and is the local Reynolds number.

Moreover, describes the base fluid (water). Tables 1 and 2 represent the thermophysical properties of NF and HNF.

**TABLE 1** Thermophysical properties of NF and HNF (Waini et al. [8])

|  |  |  |
| --- | --- | --- |
| Properties | Nanofluid | Hybrid nanofluid |
| Density |  |  |
| Dynamic viscosity |  |  |
| Thermal conductivity |  |  |
| Heat capacity |  |  |
| Electrical conductivity |  |  |

**TABLE 2** Thermo-physical properties of water and nanoparticles (Devi and Devi [22]).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Properties |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

**TABLE 3** Comparison of for different values of

when and

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Devi and Devi [12] | Khan and Pop [1] | Ahmad et al. [9] | Present study |
| 2 | 0.91135 | 0.9113 | 0.91045 | 0.9106823 |
| 7 | 1.89540 | 1.8954 | 1.89083 | 1.8949270 |
| 20 | 3.35390 | 3.3539 | 3.35271 | 3.3535494 |
| 70 | ---- | 6.4621 | 6.47814 | 6.4619285 |

# RESULTS AND DISCUSSION

The transformed system of non-linear ODEs (7)–(9) is solved numerically in MATLAB using the bvp4c solver. Convergence, stability, and accuracy of the results were verified by employing a step size and error tolerance. Further, to validate the numerical code, we comparing our results to that of [1, 9, 12] (see Table 3).



Fig. 2 Variation of with for

Fig. 2 shows the variation of horizontal velocity component of the base fluid (water), matalic nanofluid (Cu-water), metallic oxide nanofluid (water-) and hybrid nanofluid (Cu--water) under the influence of magnetic field strength. The velocity profile fails to obey the boundary layer structure since the potential flow velocity is set to zero i.e. . On careful observation it is seen that the flow of base fluid assumes higher velocity distribution than the velocity of metallic oxide nanofluid (water-). The velocity is further reduced in case of hybrid nanofluid (Cu--water). It is interesting to note that further reduction is observed in case of metallic nanofluid i.e. (Cu-water). Thus, it is concluded that maximum reduction in velocity occurs in case of metallic nanofluid (Cu-water). The physical reasoning of the reduction of the velocity may be attributed to the inertia force due to higher density of the metallic nanoparticle that slows down the momentum transport.

Fig. 3 shows the transverse velocity component distribution. The velocity distribution shows boundary layer structure depicting same physical attributes as in Fig. 2. The striking difference is that the horizontal velocity component shows an asymptotic behaviour i.e. when horizontal velocity variation . This encompasses three cases. The profiles show symmetric velocity distribution about the impermeable bounding surface at . The fall of velocity is seen across the flow domain to attain ambient state (stretching, . No fluid motion along the horizontal direction (without stretching, . The inverted profiles with negative sign indicates the flow reversal in case of shrinking.

Fig. 4 depicts the thermal boundary layer distribution that remains unaffected due to the stretching/shrinking, unlike velocity boundary layer where the velocity boundary layer structure gets inverted depending upon the stretching or shrinking of the bounding surface. It is seen that the shrinking of the plate leads to higher heat transfer of the nanofluid than stretching of the plate in the entire flow domain.

From Fig. 5 it is to note that the reverse effect of magnetic parameter is also seen depending upon stretching/shrinking (/. Thus, it is concluded that when the plate under goes shrinking, an increase in magnetic parameter leads to decrease the temperature level. But in case of stretching, opposite effect is observed. Therefore, it is concluded that the strength of the applied transverse magnetic field is to be regulated to produce the electromagnetic force as per the design requirements.

Fig. 6 shows the effect of an important parameter ( which is a measure of the heat energy dissipation in the flow, which contains square of the velocity in the numerator, therefore, it may be neglected for slow fluid motion. It is further seen that an increase in leads to increase in temperature. This shows that the present flow model of the nanofluid flow leads to higher temperature on increasing the value of Eckert number (.

Fig. 7 shows the effect of on temperature distribution. It is evident that an increase in the strength of the volumetric heat source leads to increase the temperature. Further an increase in sink strength leads to decrease the temperature. The physical reasoning may be attributed as: the higher amount of heat absorption leads to lower down the temperature.

Fig. 8 shows that significantly influence the concentration distribution. An **increase in**  implies lower mass diffusivity, which results in a **thinner concentration boundary layer** and a **steeper concentration gradient near the wall.** This leads to reduced solute diffusion into the fluid and is particularly important in controlling species transport in chemical processing systems. In contrast, **lower**  values allow for higher mass diffusion, leading to a more uniform concentration profile.



Fig. 3 Variation of with for



Fig. 4 Variation of with for



Fig. 5 Variation of with for



Fig. 6 Variation of with for



Fig. 7 Variation of with for



Fig. 8 Variation of with for

Table 4 shows the variation of and ; the important flow characteristics at the solid boundary which affects the transport phenomena in the flow field. In the present flow model the effect of and remain same the on skin friction (shearing stress) and Nusselt number at the bounding surface. It is seen that when increases, modulus of and at the boundary increases. An increase in yields the same effects as of . Further increasing in magnetic parameter, increases the increases but decreases. Thus, the effect of the transverse magnetic field plays an important role in enhancing momentum transport but reducing the thermal energy in the flow field. The findings suggest that volume fraction of nanofluid and hybrid nanofluid act in unison at the solid bounding surface but the magnetic field enhances but decreases .

**TABLE 4** Computational values of and when

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| 0 | 0 | 0 | 1 | 0 | 0 | -1 | 1.7704535 |
| 0.05 |  |  |  |  |  | -1.2616434 | 1.8498353 |
| 0.1 |  |  |  |  |  | -1.5296353 | 1.9332244 |
| 0.2 |  |  |  |  |  | -2.1288436 | 2.1168234 |
|  | 0.05 |  |  |  |  | -2.3004139 | 2.2397272 |
|  | 0.1 |  |  |  |  | -2.4937649 | 2.3668073 |
|  | 0.2 |  |  |  |  | -2.9643380 | 2.6399635 |
| 0.05 | 0.05 | 0.5 | 1 | 0.1 | 0.1 | -1.6853814 | 1.5405606 |
|  |  | 1.0 |  |  |  | -1.9252131 | 1.3479594 |
|  |  | 5.0 |  |  |  | -3.2614582 | 0.3165558 |
|  |  | 0.5 | 1.5 |  |  | -2.9344477 | 1.2886593 |
|  |  |  | 2 |  |  | -4.3880858 | 0.6322937 |
|  |  |  | 3 |  |  | -7.8157961 | -2.0086176 |
|  |  |  | 1.5 |  |  | -2.9344477 | 1.2479347 |
|  |  |  |  |  |  | -2.9344477 | 1.2104245 |
|  |  |  |  |  |  | -2.9344477 | 1.1435350 |
|  |  |  |  | 0.2 |  | -2.9344477 | 0.2060679 |
|  |  |  |  | 0.3 |  | -2.9344477 | -0.8765224 |
|  |  |  |  | 0.5 |  | -2.9344477 | -3.0417043 |
|  |  |  |  | 0.1 | 0.2 | -2.9344477 | 1.4712248 |
|  |  |  |  |  | 0.3 | -2.9344477 | 1.6423656 |
|  |  |  |  |  | -0.1 | -2.9344477 | 0.8795082 |
|  |  |  |  |  | -0.3 | -2.9344477 | 0.3850323 |

# CONCLUSION

* The presence of magnetic field reduces the horizontal component of velocity field in all cases i.e. flow of base fluid, nano and hybrid nanofluid due to resistive electromagnetic force but it is important to note that the flow of only base fluid assumes higher horizontal velocity than nano and hybrid nanofluid being the lowest.
* Nanofluid with metal species assumes lower velocity than metallic oxide .
* One striking result is that both the horizontal and transverse components attain their boundary assigning value after a certain layer, (Fig. 3).
* Stretching and shrinking of boundary surfaces produce boundary layer and inverted boundary layer structure. Thus, flow reversal is indicated in case of squeezing (Fig. 4).
* It is to note that fluid temperature increases with the increase in strength of volumetric heat source (Fig. 7).
* An increase in the Schmidt number corresponds to lower mass diffusivity. As a result, the concentration boundary layer becomes thinner, producing a steeper concentration gradient near the wall. This suppresses solute diffusion into the bulk fluid, which plays a crucial role in regulating species transport in chemical and industrial processing systems (Fig. 8).
* One noteworthy finding is that skin friction (shearing stress at the bounding surface) remains negative throughout i.e. for flow of base fluid, nanofluid and hybrid nanofluid irrespective of presence/absence of external body force or thermal condition including Joule heating, thermal radiation and Nusselt number remains positive throughout. This shows that the heat flows from the plate to the fluid mass.

# References

1. W. A. Khan, and I. Pop, International Journal of Heat and Mass Transfer **53**, 2477-2483 (2010).
2. K. Swain, I. L. Animasaun, and S. M. Ibrahim, International Journal of Ambient Energy **43**, 4045-4057 (2021).
3. T. Hayat, and S. Nadeem, Results in Physics **7**, 2317-2324 (2017).
4. K. Swain, and S. Mishra, International Journal of Ambient Energy **43**, 5810-5824 (2022).
5. W. Al-Kouz, K. Swain, B. Mahanthesh, *et al.*, Heat Transfer **50**, 4086-4102 (2021).
6. K. Swain, F. M. Oudina, and S. M. Abo-Dahab, Journal of Thermal Analysis and Calorimetry **147**, 1561-1570 (2022).
7. M. Ramzan, A. Dawar, A. Saeed, *et al.*,  PLOS ONE **16**,e0260854 (2021).
8. Waini, A. Ishak, and I. Pop, Mathematics, **9**, 538 (2021).
9. S. Ahmad, K. Ali, W. Rizwan, *et al.*, Case Studies in Thermal Engineering **25**, 100932 (2021).
10. S. P. A. Devi, and S. S. U. Devi, Int. J. Nonlinear Sci. Numer. Stimul. **17**, 249-57 (2016).
11. P. Gumber, M. Yaseen, S. K. Rawat, *et al*., Partial Differential Equations in Applied Mathematics **5**, 100240 (2022).
12. S. K. Rawat, A. Mishra, and M. Kumar, Multidiscip Model Mater Struct **15**, 714-736 (2019).
13. M. R. Eid, K. L. Mahny, A. Dar, *et al.*, Physica A **540**, 123063 (2020).
14. K. Triveni, and B. Mahanthesh, Appl. Math. Mech. **42**, 885-900 (2021).
15. H. Upreti, A. K. Pandey, and M. Kumar, Alexandria Eng J. **57**, 1839-1847 (2018).
16. H. Babazadeh, Z. Shah, I. Ullah, *et al*., Journal of Thermal Analysis and Calorimetry **143**, 1129-1137 (2020).
17. S. Nasir, A. S. Berrouk, A. Aamir, *et al*.. Z. Sci. Rep. **13,** 2006 (2023).
18. B. Mahanthesh, W. Al-Kouz, K. Swain, *et al*., Heat Transfer **50**, 6703-6718 (2021).
19. M. Radhika, and Y. Dharmendar Reddy, Radiation Effects and Defects in Solids **179**, 1656-1682 (2024).
20. K. Swain, and B. Mahanthesh, Arabian Journal for Science and Engineering **46**, 5865-5873 (2021).
21. M. Yaseen, M. Kumar, and S. K. Rawat, Partial Differ Equations Appl Math. **4**, 100168 (2021).
22. S. P. A. Devi, and S. S. U. Devi, IJNSNS **17**, 249-257 (2016).
23. S. S. Ghadikolaei, Kh. Hosseinzadeh, and D. D. Ganji, Case Stud. Therm. Eng. **10**, 579-594 (2017).
24. M. Khan, M.Y. Malik, and T. Salahuddin, Results Phys., **7**, 2512-2519 (2017).
25. W. M. Hasona, N. H. Almalki, A. A. ElShekhipy, *et al*., Heat Transf. Asian Res. **48**, 938-956, 2019.
26. N. A. Zainal, R. Nazar, K. Naganthran, *et al.*, Nanomaterials (Basel) **12**, 1109 (2022).
27. O., Prakash, P. S. Rao, R. P. Sharma, *et al*., Pramana - J Phys **97**, 64 (2023).
28. N. S. Khashi’Ie, N. M. Arifin, R. Nazar, *et al.*, Chin. J. Phys. **64**, 251-263 (2020).
29. T. Sankar Reddy, P. Roja, S. M. Ibrahim, *et al.*, Mathematical Modelling of Engineering Problems **9**, 325-335 (2022).