**Waste Heat Recovery Systems Using Ethylene Glycol-Based Bi-Hybrid Nanofluid For Thermal Management In A Sun-Powered Energy Ship: Optimizing Analysis With A Statistical Approach**

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**Abstract:** Radiative flow of magnetohydrodynamic (MHD) Casson, a non-Newtonian fluid past an elongating surface with the effect of chemical reaction presents significant attention in current days since its diversified applications in biomedical engineering, polymer processing, thermal control systems, etc. The non-Newtonian viscoplastic model, the Casson fluid is used to describe the significant behavior of biological fluids i.e., blood, chocolate, particularly when subjected to magnetic and thermal fields. The significance of the proposed investigation is due to the coupled effects of thermal radiation and homogenous chemical reaction on the momentum, heat and solutal systems of an electrically conducting Casson fluid. Moreover, the chemical reaction plays a crucial role in enhancing species concentration with direct implication of chemical manufacturing, and medical diagnostics involving drug diffusion. The mathematical model proposed for the key parameter involved in the flow phenomena are designed are converted into standard form with the utility of particular similarity rules and then the proposed model is handle numerically utilizing the shooting-based “Runge-Kutta technique” with the impact of involved factors. The physical behavior of distinct parameters are presented graphically with the variation of these factor within certain range. The analysis is reported physically and designated concisely followed by corroboration of the analysis with the earlier investigation.

**Keywords:** Non-Newtonian Casson fluid; Magnetohydrodynamic; thermal radiation; chemical reaction; Numerical method.

# INTRODUCTION

The exploration of Casson nanofluids has attracted substantial attention in current years owing to their unique combination of non-Newtonian rheology and improved thermal characteristics through nanoparticle integration. Casson fluids, which possess a yield stress threshold, are commonly used to model complex fluids such as blood, ink, and certain food substances. When nanoparticles are introduced into these fluids, their thermal performance improves significantly, rendering them effective in biomedical heat regulation, microchannel cooling systems, and industrial heat exchangers. The foundational study of Nadeem et al. [1] scrutinized the flow characteristics of the proposed fluid through an elongating sheet for the adequacy of magnetization and thermal radiation. Their examination demonstrated that incorporating nanoparticles boosts the fluid’s thermal conductivity, while the Casson fluid parameter, representing yield stress, results in reduced flow velocity due to enhanced resistance to deformation. Reddy et al. [2] extended this by incorporating chemical reactions and internal heat generation, finding that these parameters significantly affect the thermal and concentration boundary layers, making such fluids ideal for reactive systems. Further, Krishna et al. [3] scrutinized the Newtonian heating property on the thermal dynamics within the transport of the Casson fluid. The governing equation PDEs were addressed using the “Laplace transform technique”. Their work presented detailed graphical interpretations of velocity, temperature, and concentration profiles. Due to its capacity to describe yield-stress fluid behavior, the proposed model is beneficial for several practical applications likely hydrological systems and the treatment of crude oil. The flow characteristics of their study was affected by oscillatory motion of the boundary and a uniformly applied magnetic field perpendicular to the flow direction. In a related analysis, Mustafa et al. [4] examined the thermal impact of Casson fluid past over an affecting flat surface. Using a “homotopy-based analytical technique”, they developed series-form solutions for the nonlinear system presenting the flow and temperature fields. Additionally, Baithalu et al. [5] studied the integrating transport of Casson liquid flowing through a diverging channel, considering the effects of Joule and Darcy’s resistance within a porous medium. By introducing appropriate similarity transformations, the dimensional model was reduced to a system of non-dimensional equations. These reduced equations were then solved numerically using MATLAB's bvp4c solver, allowing for a detailed understanding of the transport characteristics under the stated conditions. Wahid et al. [6] addressed nanofluid transportation of generating source via an exponentially stretched, packed with pores. In a separate study, Ali et al. [7] investigated the flow dynamics of an Al₂O₃–Cu/water hybrid nanofluid over a Riga plate under the impact of strong suction. Additionally, their work employs a two-phase model to accurately capture the nanofluid behavior, incorporating the Grinberg term to represent the wall-parallel Lorentz force induced by the Riga plate. The research emphasizes the significance of minimizing entropy generation to streamline the analysis, the governing partial differential equations are non-dimensionalized utilizing relevant parameters, and the assumption of strong suction aids in simplifying the model. Finally, the resulting system of equations is then solved numerically via the “Adams–Bashforth method”. In an alternative study, Ali et al. [8] conducted a comprehensive study on the rotational flow dynamics of a chemically reacting tri-hybrid nanofluid over a suddenly accelerated plate, incorporating advanced electromagnetic and thermal effects. Their model accounts for the influences of Hall and ion slip currents, which play a important role in modifying the Lorentz force distribution, thereby altering the fluid layers. In addition, the analysis considers Newtonian heating and mass flux conditions, enhancing the realism of thermal interactions at the surface. The use of tri-hybrid nanoparticles introduces a complex interplay between thermal conductivity enhancement and viscous resistance, offering improved heat transfer characteristics compared to conventional or hybrid nanofluids. Recent studies have extensively explored the complexities of non-Newtonian and microstructured fluid flows under various thermal and physical effects. Rehman et al. [9] studied the isothermal flow of non-Newtonian fluid within a converging channel, incorporating “Cattaneo–Christov heat and mass flux” models to address non-Fourier behavior in thermal and solutal transport. Baithalu and Mishra [10] focused on the magneto-micropolar fluids in the presence of radiation and reacting species, employing “response surface methodology (RSM)” to optimize the heat transfer rate, thereby providing deeper insight into thermally responsive magnetic fluids. Similarly, Pattnaik et al. [11] utilized RSM to maximize heat transfer in the micro-structured fluids, highlighting the significance of porous media and boundary effects in enhancing thermal performance. Collectively, these investigations underscore the value of advanced thermal models and optimization techniques in understanding and improving heat and mass transfer processes in complex fluid systems. A number of recent investigations have contributed significantly to the understanding of magnetohydrodynamic (MHD) and nanofluidic transport involving coupled heat and mass transfer under varying physical phenomena. Bejawada and Yanala [12] analyzed transient MHD flow along an inclined surface, accounting for Soret and Dufour effects through a finite element framework. Reddy et al. [13] presented the significance of magnetic and radiative parameters on nanofluid stagnation flow across a stretching surface, emphasizing key transport behaviors. In a related study, Kumar et al. [14] focused on Casson fluid dynamics over a porous vertical surface, incorporating thermal diffusion, mass diffusion, and reactive effects. Shankar Goud et al. [15] numerically examined nanofluid motion over a rotating inclined disk under heat absorption and chemical interaction influences. A further contribution by Bejawada et al. [16] examined the impact of electromagnetic radiation and viscous dissipation on nanofluid flow over an accelerating vertical plate. Moreover, Ul-Haq et al. [17] utilized artificial neural network modeling to assess natural convection in Casson fluids along curved stretching geometries, showcasing the effectiveness of AI-based approaches under dissipative heat conditions. Together, these works highlight the intricate roles of magnetic fields, radiation, diffusion, and machine learning techniques in analyzing complex fluid flow systems. Trihybridized fluids were studied theoretically by Manjunatha [18] using water as a base liquid in an elongating surface with convective constraints. By using a non-Newtonian base fluid, Nazir et al. [19] investigated the flow characteristics of hyperbolic fluids heightened by bi-trihybrid nanofluids. Mujahid et al. [20] investigated the characteristics of magnetised Casson fluid within curved corrugated walls. The motion of hybrid nanofluids through a curved pipe driven by thermal radioactive pressure was also analysed using molecular dynamics.

**Significance of the study:** The current investigation following the aforementioned literatures leads to the important observations and these are;

* To investigate the integration of radiative heat impact and the chemical reaction on the conducting Casson fluid over an expanding surface.
* To analyze the energy and concentration profiles for the role of magnetization, and other characteristics distinct parameters likely radiating heat etc.
* To obtained computational result, numerical method combining with shooting technique is utilized.
* To provide practical importance of the Casson fluid for the participation of distinct factors such as magnetization, radiation, and chemical reaction parameter.

**Novelty:** The proposed factors involved in the flow phenomena lead to various novel approaches and these are;

* Incorporation of thermal radiation and chemical reaction on the MHD flow of a Casson fluid over an expanding surface which has been widely addressed in earlier investigations.
* Modelling the flow using a realistic Casson fluid where the impact of yield stres is vital to biological and industrial fluids.
* The exploration of coupled thermal radiative and reactive transport properties under magnetic influence relevant to advanced thermal management systems and biomedical applications.
* The analysis serves as a benchmark for optimizing radiative and reactive flow conditions in polymer extrusion, blood flow in magnetic fields and thermal shielding. .

**Key research questions:** The analysis presented here with several aspects lead to various questioners those are significant in investigating in following future aspects.

How the incorporation of significant nanoparticles contributing higher thermal conductivity does useful in various real-life applications.

* What is the role of radiatng and chemical reaction affects the heat and solutal transport phenomena?
* How the reactive species inspirations the solutal profile and the properties of mass transfer characteristic over the stretching surface?
* Whether the methodology implemented here will adequate to find the complete solution of the model with the involvement of characterizing factors?

## MATHEMATICAL FORMULATION

The steady flow of Casson fluid through a stretchy material packed with porous substrate is deliberated in current analysis. The assumptions revels that the flow is along the surface z=0 where the sheet is stretching towards x direction with momentum and along y axis respectively. The coordinate axis for the 3D flow is with velocity components  (FIGURE. 1). Moreover, the applied magnetic field of strength is deployed towards the normal direction of flow combining with the interpretation of radiant heat which is assumed from the Roseland approximation and additional heat source.

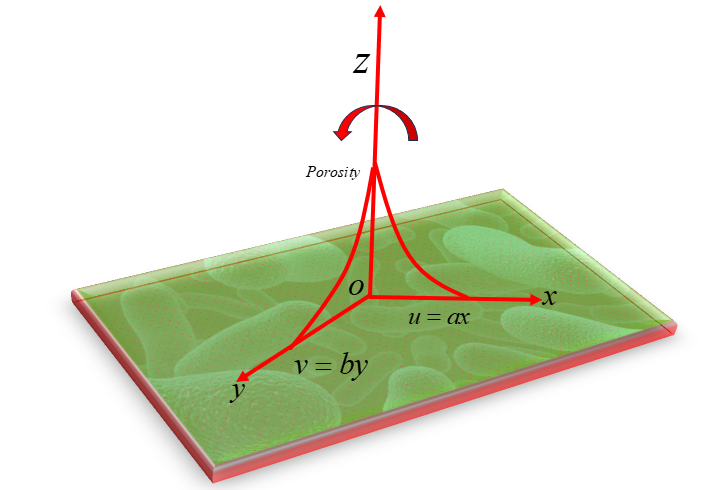


FIGURE. 1 Flow geometry

A Casson fluid's rheological standard equation is described as

 (2.1)

Where with a rate  is the  and  is presented.

The proposed models are formulated as;

 (2.2)

 (2.3)

 (2.4)

 (2.5)

 (2.6)

The imposed surface restrictions are

 (2.7)

is expressed through the following relation for radiative heat flux

 (2.8)

Breaking the nonlinearity of the radiative heat flux using Taylor series about lead to

 (2.9)

Moreover, Eq. (2.8) with excluded higher-order terms is

 (2.10)

On replacing Eq. (2.9) within Eq. (2.7), we arrive at

 (2.11)

Employing (2.10), Eq. (2.5) can be expressed as

 (2.12)

Introducing the following similarity functions are

. (2.13)

Applying the aforementioned similarity transformation;

 (2.14)

 (2.15)

 (2.16)

 (2.17)

The revised surface restrictions are

 (2.18)

Concerning the relevant physical characteristics, where

 (2.19)

The engineering coefficients are

 (2.20)

 (2.21)

 (2.22)

 (2.23)

Where & are the local Reynolds numbers.

**TABLE 1** Comparative evaluation of ****

|  |  |  |
| --- | --- | --- |
| Pr | Devi and Devi [22] | Present result |
| 2 | 0.91135 | 0.911354334 |
| 7 | 1.8945 | 1.894534556 |
| 20 | 3.3539 | 3.353956439 |

**RESULTS AND DISCUSSION**

The non-Newtonian characteristic of magnetized Casson fluid movement via a stretchy material for the contribution of radiative energy and chemical activity is reported in this present analysis. The mathematical model describing the role of proposed assumptions lead to characterizing the significant role of various factors obtained from the transformation of the model utilizing a proper similarity rule. Moreover, the analysis is carried out for the solution of the model by deploying the shooting-based numerical technique generally known as Runge-Kutta fourth-order for the implication of bvp4c in-house function in MATAB. Numerical consistency of the obtained for a chosen case is shown in **Table-1**, benchmarked against the study by Devi and Devi [22], affirming agreement. The simulations are deployed for the suitable choice of the constraints presented in the present flow phenomean collected from various literatures. Moreover, the following section is enthusiastic to the physical behavior of these factors affecting the flow pheneomena presented graphically.

**Variation of magnetization and Casson parameter:**

The ploted diagram of the several factors like magnetization, thermal radiation, Casson parameter along contribution of radiative energy and chemical consumption affecting the flow phenomena significantly. **FIGURE.2** caracterizes the analysis of magnetization for the utilization of magnetic force field on the Casson fluid circulating via an expanding material. To analyze the behavior of the numerical values are rpoted as presenting the lack of magnetism considering and its deploying The involvement of magnetization aspects producing a resistance which lead to the generation of Lorentz force and notably, the emergence of opposing forces dictates the momentum of the fluid, resulting in considerable slowing down. The energetic behavior is reported in two-distinct profiles such as the interpretation of non-Newtonian Casson parameter. The results of the variation of  is reported consideringand respectively. The factor  is presented that for the lower value of lead to the non-Newtonian behavior which is primarily dictated by the behavior of the higher value of  presenting the situation of Newtonian fluid. The pictorial diagram discloses that for the case of non-Newtonian fluid the yield stress augments the velocity profile more efficiently. Intrestingly, the absence of magnetization in both the cases portrays the greater strength tyhanm that of the occurance of magnetization. **FIGURE.3** explains the attribute of the magnetization combined with the variation of the Casson parameter on the transverse momentum. The graphical illustartion of the transverse velocity shows similar variation compared to the axial velocity. Again, the resultant analysis of the magnetization also affects the energy pattern of the Casson fluid in the presence of Casson parameter. **FIGURE.4** visualizes the results of the magnetization term upsetting the energy pattern of the fluid integrating with the non-Newtonian/ Newtonian behavior. The projected behavior of the magnetization that retards the velocity, accumulates energy in the lower region and this gathered energy elevates the thermal state of the non-Newtonian fluid. The accumulation of energy is due to the resistivity of the Lorentz force and the profile enehances significantly. The resulting behavior also favors in enhanciong the thermal boundary layer thickness, showing an assymptotic behavior of the profile. Moreover, the comparative analysis is reported considering the impact of Casson parameter. It is evident from the FIGUREure that, the non-Newtonian factor obtained due to the inclusion of Casson parameter projected a greater control over the fluid temperature and it lead to the significant deceleration in the pattern for the growing non-Newtonian characteristic of the profile which is prevailed by the Newtonian behavior of the fluid. Again, in reverse the role of Casson parameter on the axial momentum is reported through **FIGURE.5** and impartant properties are reported for the comparative result of magnetization. Here, the factor beta is repoprted considering the standard numerical results on belong to [1,4]. The graphical behaviour illustrates that the enhanced attenuates the profile and it is elucidating that the greater variation of the Casson parameter shows the Newtonian characteristic and it led to an attenuates in the profile temperature and in reverse the non-Newtonian factor shows greater hike in the fluid’s energy. However, a comparative result is depicted for the variation of the magnetization where signifies the absence and reported the resulting behaviour of the magnetization. As described earlier, the incorporation of the resistivity decelerates the velocity distribution, irrespective to the deviation of distinct values of the factors presenting the axial velocity. A similar observation is projected in **FIGURE.6** for the transverse velocity profile. The increasing magnetization provides the significant deceleration in the transverse velocity. **FIGURE.7** portrays the features of the both the termsand on the energy pattern of fluid. The clarification reveals from the numerical interpretation of the factor  is that with growing  the pattern temperature significantly enhances. Higher  that signifies the Newtonian property of the fluid and it dominates the non-Newtonian property of the fluid. Again, the properties for the magnetization with the regular variation of the factor also augment the fluid's thermal state.



**FIGURE.2** Change in due to distinct values of 



**FIGURE.3** Change in due to distinct values of 



**FIGURE.4** Change in due to distinct values of 



**FIGURE.5** Change in due to distinct values of 



**FIGURE.6** Change in due to distinct values of 



**FIGURE.7** Change in due to distinct values of 

**Significance of**  **and** 

The role of thermal radiation and the variation is reported in several distinct features of suction/injection **(FIGURE.8).** The property of radiating heat is featured with the numerical range of In particular, portrays the behaviour of the temperature profile without the property of  and the nonzero variation of the radiative heat transfer portrays the features of the radiative heat property. The increasing attributes of the radiation shows a substantial growth in the energy. This behaviour is reported for the illustration of the heat source/sink Here, portrays the case of heat sink and, in this case, the outer fluid comes into the system. Further, the positive variation i.e.  shows behaviour of source where the fluid present in the system moves away and finally, shows that the flow of fluid for the absence of heat source. The plotted results reveal that suction significantly alters the fluid's thermal profile, whereas injection produces a diminishing influence. It is interesting to observe that the case of suction shows retardation within the domain but injection shows opposite impact.



**FIGURE.8** Change in due to distinct values of 

**Behavior of Schmidt number and chemical reaction parameter**

The fluid concentration is influenced by the significant variation of the combined factor of the heavier species which is designated as Schmidt number and the chemical consumption term. **FIGURE.9** reports the resulting behaviour of the Schmidt number and chemical reaction parameter on the concentration profile. To execute the characteristic of , the numerical range is considered as Particularly, these values signifies the addition of gaseous elements into the system etc. Physically, the significant ratio of the kinetic diffusivity with the solutal diffusivity gives rise to the impact of Schmidt number. The increasing numerical variation deployed that the concentration profile retards significant for the retardation in the concentration distribution. In particular, a three-fold variation is reported for the interaction of chemical reaction Kr where the negative shows constructive and the positive indicates the constructive reaction and each case the profile retards throughout.



**FIGURE.9** Change in due to distinct values of 

**Flow pattern**

**FIGURE.10** describes the streamlines for the velocity utilizing the behaviour of the magnetization and the Casson parameter. Particularly, the graph shows the variation of and whereas the results are also depicted for the andrespectively. It represents the flow pattern of the velocity where with increasing magnetization the profile retards significantly and became flatter and this retardation give rise to thinner. Further, the increasing behaviour of also thinning the velocity boundary layer. This show a thinner in the profile thickness for the Newtonian situation more prominently rather that the non-Newtonian case.



**FIGURE.** 10 stream line for velocity profile for different values of Casson parameter

**Engineering coefficients**

The computation of engineering coefficients likely the shear rate along with the heat transfer rate are simulated for the variation of different characterizing factors. These conditions are vital those affect the entire flow behaviour depending upon the surface criteria. **FIGURE.11** illustrates the behaviour of shear stress with the variation of the magnetization and the Casson parameter. It is clear to observe that for the increasing resistivity which retards the fluid velocity, it urge to increase the velocity gradient and therefore, the shear stress retards. Moreover, the enhanced yield stress presented for the variation of the Casson parameter favors in retarding the shear rate significantly. However, **FIGURE. 12** outlines the pattern of heat transfer rate characterized by the Nusselt number under different levels of thermal radiation and heat source effects. The intensified thermal radiation that contributes to boosting the fluid’s thermal conductivity leads to an elevated heat transfer rate, thereby raising the Nusselt number.

**FIGURE.**11 Illustration of Magnetic parameter and Casson parameter on Shear rate

**FIGURE.**12 Illustration of Thermal radiation and heat source on Nusselt number

**CONCLUSION**

A computational scrutinization is presented for the circulation of the non-Newtonian Casson fluid, a conducting fluid via a permeable expanding material. The analysis is depicted for the integrating effect of the radiating energy and the chemical species flow behaviour of the proposed fluid. The model presenting the aforementioned assumptions is transformed to ordinary and then shooting combined with the traditional numerical technique is adopted to handle the solution. The physical behaviour of the factors are reported graphically deployed and elaborated. The important outcomes are;

* The comparative results presented numerically for all the three profiles in different conditions show the earlier investigation is well equipped with the present methodology.
* This further confirms and demonstrates the consistency of the approach employed in solving the present model.
* The magnetization property of the Casson model exhibits a resistive force which controls the velocity of the fluid and in reverse the fluid temperature enhances.
* The radiative heat presents a significant augmentation in the heat transport property as well as the heat transfer rate phenomenon.
* The heavier species presents the Schmidt number favors in retarding the concentration profile and the heat source presents a significant enhancement in the temperature distribution.

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