**Influence of Chemical Reaction on Unsteady MHD Flow of a Viscoelastic Micropolar Fluid Through a Porous Medium in Presence of Thermal Radiation and**

**Hall Current**

J. Barik1,a), S.N. Sahoo2,b), C.R. Dwivedy3,c)

1Dept of Math., FET (ITER), S ‘O’ A University, Bhubaneswar, PIN-751030, Odisha, INDIA

2Department of Math., S.C.S. (A) College, Puri, PIN-752001, Odisha, INDIA

3Department of Physics, S.C.S. (A) College, Puri, PIN-752001, Odisha, INDIA

*a)Corresponding author:* [*jitumathematics@gmail.com*](mailto:jitumathematics@gmail.com)

*b)[sachimath1975@gmail.com](mailto:sachimath1975@gmail.com)*

*c)crdwivedy1968@gmail.com*

**Abstract.** The present work explores how chemical reactions influences the unsteady motion of a viscoelastic micropolar fluid past an infinite, moving, and permeable plate embedded in a porous medium. The study considers the effect of a transverse magnetic field, thermal radiation, and Hall current. The dimensionless governing equations are solved analytically using a perturbation method to obtain expressions for velocity, microrotation, temperature, and concentration profiles. The impacts of key flow parameters are illustrated graphically, while wall shear stress, couple stress, and heat and mass transfer rates are presented in tabular form. The results highlight that Lorentz force enhances fluid velocity in the presence of chemical reactions, and both endothermic and exothermic reactions intensify velocity as the chemical reaction parameter increases. Furthermore, the thermal boundary layer becomes thinner with a rise in the Prandtl number.

**Keywords:** MHD flow,Micropolar fluid,Thermal radiation, Chemical reaction, Hall current

**INTRODUCTION**

Heat and mass transfer through porous media in different geometrical configurations is critical to many engineering and geophysical applications. Examples include drying porous solids, designing thermal insulations, cooling nuclear reactors, extracting crude oil, and underground energy transport. Micropolar fluids, composed of randomly oriented particles suspended in a viscous medium, are characterized by the ability of particles to undergo microrotation. This feature distinctly influences the hydrodynamics of such flows compared to Newtonian fluids. Since Eringen [1] introduced micropolar fluid theory, it has attracted considerable attention for modelling complex fluids such as colloidal suspensions, polymeric solutions, and biological fluids like blood. These fluids play a vital role in several natural processes and industrial applications, including alloy solidification, astrophysical flows, geophysical circulation, and oceanographic phenomena.

In the presence of a strong magnetic field, Hall current effects must also be considered, as they induce cross-flows and render the system three-dimensional. Numerous studies [2–10] have examined MHD micropolar fluid flows under such conditions. For example, Rakesh [11] analysed unsteady viscoelastic fluid flow across a vertical porous plate under slip and Hall effects, while Zakaria [12] studied electromagnetic free convection of micropolar fluids with relaxation time through porous media. Abo-Eldahab and El Aziz [13] investigated heat transfer in micropolar fluids over a stretching surface in a porous medium, and Kim and Lee [14] analysed oscillatory MHD flow across vertical porous plates. Other researchers [15–21] extended these investigations to incorporate radiation, chemical reactions, drag forces, and nonlinear sheet expansions.

Chemical reactions significantly alter fluid characteristics, thereby influencing velocity, temperature, and concentration distributions. Examining unsteady MHD flows of viscoelastic micropolar fluids with thermal radiation and Hall effects has practical relevance to high-temperature industrial processes involving heat and mass transfer. While prior research has largely addressed viscous micropolar fluids, limited attention has been given to viscoelastic micropolar fluids. To bridge this gap, the present study extends the work of Olajuwon et al. [15] by incorporating chemical reactions to analyse their effect on unsteady MHD flows in porous media.

**FORMULATION AND SOLUTION OF THE PROBLEM**

The governing configuration considers a viscoelastic micropolar fluid flowing past an infinite permeable plate subject to thermal radiation and Hall current. The x\*-axis lies along the plate in the upward direction, while the y\*-axis is normal to it.

*y*\*

*x\**

*z*\*

 

 



Figure 1. Flow geometry

A uniform transverse magnetic field of strength B0 is imposed along the y-axis, and the induced magnetic field is neglected due to its insignificance compared with the applied field. Since the plate is considered infinite, the flow variables depend only on y\* and time t\*. A constant suction velocity v0 is applied at the plate surface.

Under the traditional Boussinesq approximation the fundamental equations governing the flow are given by

 (1)

 (2)

 (3)

 (4)

 (5)

 (6)

 (7)

The relevant Boundary Conditions (B.C.s) are

 (8)

The microrotation parameter (n), associated with the microgyration vector and shear stress, governs particle motion near the wall. For n=0, the microelements close to the wall cannot rotate, representing a high concentration of suspended particles. At n=0.5, the antisymmetric part of the stress tensor vanishes with minimal microelement concentration. The case n=1 is typically used to model turbulent boundary layers. In this study, the cases n=0 and n=0.5 are examined in detail.

Utilizing the Rosseland approximation, the radioactive heat flux is expressed as

 (9)

Assuming that the fluctuations in temperature within the flow enable us to represent  as a linear combination of temperature, we can further develop  through a Taylor series expansion centred at , as detailed below:

 (10)

and neglecting higher order terms beyond the first degree in , we have

 (11)

Differentiating eqn (9) w.r.t.  and using eqn (11), we obtain

 (12)

Introducing the below nondimensional variables

, , ,,, 

eqns (2)-(8) yield the below dimensionless eqns:

 (13)

 (14)

 (15)

 (16)

 (17)

 (18)

Also, the B.C.s become

 (19)

Introducing  and , equations (13)-(16) are reduced to the followings:

 (20)

 (21)

The relevant B.C.s are

 (22)

In order to solve eqns (20), (21), (17) and (18) subject to the B.C.s (22), we presume a perturbation method of the following form:

 (23)

Substituting eqn (23) into equations (20), (21), (17), and (18) we obtain the following set of eqns respectively:

 (24)

 (25)

 (26)

 (27)

where 

.

The relevant B.C.s can be expressed as

 (28)

The solution of (24)-(27) satisfying the B.C.s are

, (29)

, (30)

, (31)

 (32)

At the wall, the coefficient of the skin friction (*Cf* ) is

, (33)

where the shear stress of the wall () is

 (34)

At the plate, the coefficient of the couple stress is

 (35)

where the couple stress of the wall (M*w*) is

 (36)

The rate of heat transfer at the surface can be represented by the Nusselt number as

 (37)

 (38)

At the surface, the rate of mass transfer can be expressed using the Sherwood number as

 (39)

 (40)

**RESULT AND DISCUSSION**

The effect of a chemical reaction on the unsteady flow of a viscoelastic micro-polar fluid over an infinite moving permeable plate within a saturated porous medium has been investigated in the presence of a transverse magnetic field, thermal radiation, and Hall current.

**Figure 2** presents the velocity profile U as a function of the transverse coordinate y for different values of the governing parameters M, Nr, and m. In all cases, the velocity rises sharply near the boundary, reaches a maximum at approximately y = 0.5, and then decreases gradually towards zero as y increases, reflecting the boundary layer decay. Among the profiles, Curve II (M = 1.0, Nr = 0.5, m = 0.4) shows the highest peak, indicating that increasing the magnetic parameter MM enhances the velocity. Curve III (M = 0.5, Nr = 0.7, m = 0.4) also lies above Curve I, suggesting that higher radiation parameter Nr increases the velocity distribution. In contrast, Curve IV (M = 0.5, Nr = 0.5, m = 0.6) exhibits the lowest velocity profile, signifying that an increase in the viscosity-related parameter mm suppresses the fluid velocity. Overall, the figure highlights the competing influences of M, Nr, and m on the flow field, where magnetic and radiation parameters tend to accelerate the fluid motion, while the viscosity parameter tends to retard it.



Figure 2. Velocity profile for different values of M, Nr and m



Figure 3. Velocity profile for different values of Kc



Figure 4. Velocity profile for different values of K, h and a

**Figure 3** shows the velocity profile U as a function of the transverse coordinate y for different values of the chemical reaction parameter Kc. For all cases, the velocity increases rapidly near the boundary, reaches a maximum around y≈0.5, and then decreases monotonically towards zero as y increases, reflecting the boundary layer nature of the flow. The curve corresponding to Kc = 0 lies at a higher position than those for positive values of Kc, indicating that in the absence of a chemical reaction, the velocity is greater. For positive values of Kc (0.2, 0.5, 1.0), the velocity profile is suppressed, showing that an increase in the chemical reaction parameter reduces the velocity field. In contrast, for negative values of Kc (-0.2, -0.5), the velocity profile is enhanced, attaining larger peak values compared to the case Kc=0. Thus, the figure highlights that a destructive chemical reaction (Kc>0) retards the flow, whereas a generative chemical reaction (Kc<0) accelerates the fluid motion.

**Figure 4** illustrates the variation of the velocity profile U with respect to the transverse coordinate y for different values of the parameters K, h, and a. Four distinct curves are presented, labelled I to IV, with their corresponding parameter values provided in the legend. All curves begin from zero at y = 0, rise to a maximum velocity, and then gradually decay to zero as y increases. Curve I, corresponding to K = 0.5, h = 0.2, a = 0.2, exhibits the highest peak velocity, reaching approximately 1.2, before decaying. Curve II, for which K = 0.7, shows a significant reduction in velocity magnitude, with its peak being the lowest among all curves, indicating that an increase in K suppresses the flow velocity. Curve III, obtained for higher h = 0.4, demonstrates a lower peak velocity compared to Curve I but higher than Curve II, suggesting that increasing h decreases the velocity moderately. Curve IV, corresponding to a higher a = 0.4, lies between Curves I and III in terms of peak velocity. Overall, the figure reveals that increasing K leads to a strong reduction in velocity, while increasing h and a result in a moderate decline. In all cases, the velocity profiles decay smoothly to zero at large y, confirming the boundary layer nature of the flow.



Figure 5. Microrotation profile for different values of M, Nr and m

**Figure 5** depicts the variation of the microrotation profile P with respect to the transverse coordinate y for different values of the parameters M, Nr, and m. Four curves (I–IV) are shown, with the corresponding parameter values listed in the legend. All curves start with maximum values near the wall at y = 0 and decay monotonically to zero as y increases, representing the thermal boundary layer behaviour. Curve I, corresponding to M=1.0, Nr=0.5, m=0.4 shows the highest temperature profile among all cases. Curve II, with an increased magnetic parameter (M = 2.0), exhibits a significant reduction in the temperature distribution compared to Curve I, indicating that a stronger magnetic field suppresses thermal transport. Curve III, with a higher radiation parameter (Nr=0.7), also shows a reduction in the temperature profile relative to Curve I, though less pronounced than the effect of increasing M. Curve IV, corresponding to a larger m = 0.6, presents a moderate decrease in temperature compared to Curve I, lying between Curves I and III. Overall, the figure highlights that increasing the magnetic parameter M, radiation parameter Nr, and exponent m all tend to reduce the temperature distribution, with the influence of the magnetic parameter being the most significant. In every case, the temperature profiles decay asymptotically to zero at large y, consistent with physical expectations of the boundary layer.



Figure 6. Microrotation profile for different values of K, a and h

**Figure 6** shows the variation of the microrotation profile P with respect to the transverse coordinate y for different values of the parameters K, a, and h. Four curves (I–IV) are presented, with the corresponding parameter values given in the legend. In all cases, the temperature starts with a maximum value at y = 0 and decreases exponentially to zero as y increases, reflecting the expected boundary layer behaviour. Curve I, for K = 0.5, a = 0.2, h = 0.2, represents the reference case and shows a moderate decay of temperature with distance. Curve II, with an increased K = 0.6, exhibits a noticeable reduction in the temperature profile, indicating that a higher thermal conductivity parameter enhances heat transfer and reduces the thermal boundary layer thickness. Curve III, obtained for a = 0.4, shows a lower temperature distribution compared to Curve I, implying that increasing aa leads to faster decay of temperature. Curve IV, for which h=0.4, shows the highest temperature distribution among all cases, indicating that higher values of h enhance the temperature profile significantly. Overall, the figure highlights that increasing K and a decrease the temperature distribution, while increasing h enhances it. In every case, the temperature diminishes asymptotically to zero at larger values of y, which is consistent with the thermal boundary layer characteristics.

**Figure 7** presents the influence of the chemical reaction parameter Kc on the microrotation profile P with respect to the transverse coordinate y. All curves start with a maximum temperature value at the wall (y=0) and decay exponentially to zero as y increases, representing the behaviour of the thermal boundary layer. The reference curve with Kc=0.0 lies in between the others. For negative values of the chemical reaction parameter (Kc=−0.1, −0.2), the temperature profiles lie above the reference curve, showing that destructive (or consuming) reactions tend to enhance the temperature distribution and slow down the rate of decay. Conversely, for positive values of Kc (Kc=0.1,0.2), the temperature profiles shift downward, leading to a significant reduction in the thermal field. This indicates that generative chemical reactions intensify heat transfer and thin the thermal boundary layer. Overall, the figure clearly demonstrates that negative Kc increases the temperature profile, while positive Kc decreases it, with all cases ultimately approaching zero asymptotically as y becomes large.



Figure 7. Microrotation profile for different values of Kc



Figure 8. Temperature profile for different values of Pr and Nr

**Figure 8** illustrates the variation of the temperature profile T with respect to the transverse coordinate y for different values of the Prandtl number (Pr) and the radiation parameter (Nr). In all cases, the temperature starts from its maximum at the wall (y = 0) and decreases monotonically to zero as y increases, representing the thermal boundary layer behaviour. For Pr = 0.3 and Nr = 0.5, the temperature distribution is higher compared to the case of Pr = 0.3 and Nr = 0.2, indicating that increasing the radiation parameter enhances the temperature field. On the other hand, for constant Nr = 0.2, an increase in the Prandtl number reduces the temperature profile significantly. For instance, when Pr increases from 0.3 to 0.7, and further to 7.0, the temperature field decreases sharply, showing that larger Prandtl numbers (associated with fluids of lower thermal diffusivity, such as oils) result in thinner thermal boundary layers. Overall, the figure demonstrates that higher values of Nr increase the temperature distribution, while higher values of Pr decrease it. In all cases, the temperature profiles decay asymptotically to zero for large values of y.



Figure 9. Concentration profile for different values of Sc and Kc

**Figure 9** shows the influence of the Schmidt number (Sc) and the chemical reaction parameter (Kc) on the concentration profile C as a function of the transverse coordinate y. In all cases, the concentration starts from a maximum value at the wall (y = 0) and decreases exponentially to zero as y increases, which is typical for concentration boundary layers. Curves I, II, and III correspond to Kc=0.1 with increasing Schmidt numbers (Sc = 0.5, 1.0, 1.5, respectively). As Sc increases, the concentration profile decreases more rapidly, indicating that higher Schmidt numbers (associated with fluids of lower mass diffusivity) reduce the concentration boundary layer thickness. Curve IV, with Sc=0.5, Kc=0.2 shows a slightly lower concentration compared to Curve I, signifying that increasing the positive chemical reaction parameter enhances mass transfer and decreases concentration. Curve V, with Kc=0.5, shows an even stronger reduction in the concentration profile, further confirming this effect. On the other hand, Curve VI, corresponding to a negative chemical reaction parameter (Kc=−0.2), lies above all other curves, showing that destructive (negative) chemical reactions tend to increase the concentration distribution. Overall, the figure demonstrates that increasing the Schmidt number and the positive chemical reaction parameter reduces the concentration field, while a negative chemical reaction parameter enhances it. In all cases, the concentration profiles decay asymptotically to zero as y increases, consistent with the expected physical behaviour of concentration boundary layers.

**Table 1.** Effect of n, a, m, Nr and Kc on C*f*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| n | a | m | Rd | Kc | C*f* |
| 0.0 | 0.2 | 0.2 | 0.5 | 0.3 | 7.7554 |
| 0.5 | 0.2 | 0.2 | 0.5 | 0.3 | 10.9792 |
| 1.0 | 0.2 | 0.2 | 0.5 | 0.3 | 13.1631 |
| 0.5 | 0.5 | 0.2 | 0.5 | 0.3 | 45.1586 |
| 0.5 | 1.0 | 0.2 | 0.5 | 0.3 | 50.0972 |
| 0.5 | 0.2 | 0.3 | 0.5 | 0.3 | 8.3647 |
| 0.5 | 0.2 | 0.5 | 0.5 | 0.3 | 6.0095 |
| 0.5 | 0.2 | 0.2 | 0.7 | 0.3 | 9.6377 |
| 0.5 | 0.2 | 0.2 | 1.0 | 0.3 | 8.2432 |
| 0.5 | 0.2 | 0.2 | 0.5 | 0.4 | 10.9795 |
| 0.5 | 0.2 | 0.2 | 0.5 | 0.5 | 10.9796 |

**Table 2.** Effect of Pr and Nr on Nusselt number (Nu)

|  |  |  |
| --- | --- | --- |
| Pr | Rd | Nu |
| 3.0 | 0.2 | 1.4890 |
| 4.0 | 0.2 | 1.4962 |
| 5.0 | 0.2 | 1.5007 |
| 3.0 | 0.3 | 1.4752 |
| 3.0 | 0.4 | 1.4623 |
| 3.0 | 0.5 | 1.4501 |

**Table 3.** Effect of Sc and Kc on Sherwood number (Sh)

|  |  |  |
| --- | --- | --- |
| Sc | Kc | Sh |
| 2.0 | 0.3 | 3.2702 |
| 3.0 | 0.3 | 4.7791 |
| 4.0 | 0.3 | 6.2839 |
| 2.0 | 0.4 | 3.3461 |
| 2.0 | 0.5 | 3.4181 |

**Table 1** illustrates the influence of the microrotation parameter (n), viscoelastic parameter (a), Hall current parameter (m), thermal radiation parameter (Nr), and chemical reaction parameter (Kc) on the skin-friction coefficient (C*f*). It is evident from the table that as n, a, and Kc increase, the skin-friction coefficient also rises; however, an opposite trend is noted with the increase in m and Nr. Upon closer examination, it is observed that the increase in the chemical reaction parameter leads to a very moderate rise in the skin-friction coefficient. This phenomenon can be attributed to the viscoelastic characteristics of the fluid. **Table 2** depicts the influence of the Prandtl number (Pr) and the thermal radiation parameter (Nr) on the Nusselt number (Nu). It is noted that as Pr increases, the Nusselt number also rises; conversely, an increase in Nr leads to a decrease in the Nusselt number. **Table 3** presents the impact of the Schmidt number (Sc) and the chemical reaction parameter (Kc) on the Sherwood number (Sh). The data indicates that the Sherwood number increases with rising values of both Sc and Kc.

**CONCLUSION**

The study presented above leads to the following conclusions:

1. The Lorentz force tends to increase fluid velocity in the presence of a chemical reaction.
2. In both endothermic and exothermic reactions, an enhancement in the chemical reaction parameters result in an increase in fluid velocity.
3. Fluid velocity rises with an increase in permeability and viscoelastic parameters.
4. As the slip flow parameter increases, the microrotation of fluid particles is enhanced.
5. The thickness of the thermal boundary layer diminishes as the Prandtl number increases.
6. Heavier diffusing species exert a greater retarding effect on the concentration distribution within the flow field.

**APPENDIX**

, , , , ,

, ,  ,

, , ,

, 

, 



**NOMENCLATURE**

u\*, v\*, w\*: Components of fluid velocity along x\*, y\*, and z\* axes.

N1\*, N2\*: Components of microrotation (rotation of fluid particles) along x\* and z\* axes.

: Kinematic viscosity (measure of fluid's resistance to shear stress).

 : Kinematic microrotation viscosity (measure of fluid's resistance to microrotation).

K0: Limiting viscosity (viscosity at very high shear rates).

 : Density of the fluid.

: Coefficient of thermal expansion (change in volume due to temperature change).

: Coefficient of concentration expansion (change in volume due to concentration change).

T : Dimensional temperature of the fluid.

Tw\*, : Temperature at the plate and temperature far away from the plate.

: Dimensional concentration of the solute.

,: Concentration of the solute at the plate and concentration of the solute far away from the plate.

 : Permeability of the porous medium (measure of how easily fluid can flow through).

*k* : Thermal conductivity of the medium.

: Electrical conductivity.

m: Hall current parameter (related to the interaction between electric and magnetic fields).

 : Micro-inertia density or micro inertia per unit mass (related to the rotation of fluid particles).

 : Spin gradient viscosity (related to the rotation of fluid particles).

 : Characteristics length (a reference length scale).

 : Dimensional frequency of oscillation.

D: Molecular diffusivity (measure of how easily molecules can diffuse through the fluid).

: Radiative heat flux.

g : acceleration due to gravity

**REFERENCES**

1. Erigen AC. Theory of micropolar fluids, *J. Math. Mech.* 16 (1966) 1-18.
2. Chaudhary RC, KJ Abhay. *Appl. Math. Mech. Engl. Ed*. 29 (9) (2008) 1179-1194.
3. Eldabe NT, ME Ouat.*,* *Appl. Math. Comput.* 177 (2006) 561-571.
4. Keelson NA, A Desseaux*Int. J. Eng. Sci.* 39 (2001) 1881-1897.
5. Mahmoud MAA., *Phys. A.* 375 (2007) 401- 410.
6. Magdy AC., *J. Appl. Math.* 2 (70.4) (2005) 271-292.
7. Modather M, AM Rashad, AJ Chamkha., *Turkis J. Zeng. Env. Sci.* 33 (2009) 245-257.
8. Patil PM, PS Kulkarni., *Int. Therm. Sci.* 4 (2008) 1043-1054.
9. Roslinda N, I Anuar, P Ioan. *Int. J. Eng. Appl. Sci.* 4 (2008) 7.
10. Rehbi AD, AA Tariq, AS Benbella, AA Mahoud., *Turkish J. Eng. Environ. Sci.* 31 (2007) 225-233.
11. Rakesh K, C Khem., *Int. J. Eng. Sci. Technol.* 3 (4) (2011) 3124-3133.
12. Zakaria M.*, Applied Mathematics and Computation*, 152 (2004) 601-613.
13. Abo-Eldahab EM, MA El Aziz.*, Applied Mathematics and Computation*, 162 (2005) 881-899.
14. Kim, YJ, JC Lee., *Surface and Coating Technology,* 171 (2003) 187-193.
15. Olajuwon BI, JI Oahimire, M Ferdow. *Engineering Science and Technology, an International Journal*, 17 (2014) 185-193.
16. Sahoo SN, PK Rout, GC Dash., *Songklanakarin Journal of Science and Technology*, 44(3) (2022) 698-707.
17. Sahoo SN, PK Rout, GC Dash., *International Journal of Ambient Energy,* 43(1) (2022) 7977-7986**.**
18. Prusti KK,SN Sahoo, SR Mishra. *The Open Chemical Engineering Journal*, 16 (2022)1-8.
19. Sahoo SN, GC Dash., *AIP Conf. Proc.* 2819(1) (2023) 1-10.
20. Baitharu AP, SN Sahoo, GC Dash, *Proceedings of National Academy of Sciences, India, Section* *A: Physical Sciences*,94 (2024) 325–333.
21. Sahoo SN,KK Prusti, SR Mishra, PK Pattnaik. *Journal of Nanomaterials Nanoengineering and Nanosystems*, <https://doi.org/10.1177/23977914241286353>