**NON-UNIFORM HEAT SOURCE/SINK IMPACT ON CONDUCTING 3D FLOW OF HYBRID NANOFLUID OVER BIDIRECTIONAL PERMEABLE STRETCHING SURFACE**

**Nilanchala Sethy**

*Department of Mathematics, Siksha ‘O’ Anusandhan Deemed to be University, Bhubaneswar, Odisha 751030, India*

***1Email:*** *nsethy75@gmail.com* ***ORCID: 0000-0002-9928-7384***

***\*Corresponding author Email:*** *nsethy75@gmail.com*

**Abstract.** The combination of more than one solid nanoparticles in the host fluid particularly, the hybrid nanofluid, gained significant attention in various applications, such as thermal management systems, energy harvesting, etc., due to its superior heat transfer properties. The current analysis based on the influence of space/temperature-dependent heat source/sink on the combined approach of aluminium oxide (Al2O3) and Copper (Cu) nanoparticles. In the present flow over a bidirectional permeable elongating surface with the insertion of radiative heat in the energy distribution is also proposed. The mathematical model designed here for the assumption of various thermophysical models is transformed into their corresponding non-dimensional form for the suitable similarity rules. Moreover, numerical treatment is advisable for the solution of these models with appropriate boundary conditions available. The physical interpretation is briefly presented for each of the factors affecting the flow phenomena following a suitable validation.

**Keywords:** Hybrid nanofluid; Bi-directional stretching surface; Thermal radiation; Non-uniform heat source/sink; Numerical treatment.

**INTRODUCTION**

A heat transport phenomena of bi-hybrid nanofluid shows novel approach in enhancing heat transport properties. Therefore, nanofluid composed of more nanoparticles has the most momentous consequence on the heat movement. Furthermore, it give rise to greater impact for the higher generation rate of heat than hybrid and single-component nanofluids. Khan et al. [1] presented an axis-symmetric flow over a hybridized nanofluid containing the combined effect of copper-alumina dispersed in the traditional liquid water. Their investigation through the bi-axial permeable expanding/contracting surface under the action of hot air and Cattaneo-Christov heat conduction model. In general concept, the carbon nanotube (CNT) is a cylindrical nanostructure where the carbon atoms are arranged in a hexagonal manner and there are various use of CNT nanoparticle likely in material science, cooling of electronic devices, energy management, and biomedicine applications. Further, Amin et al. [2] reported the role of magnetization presenting the impact of Lorentz force combined with viscous dissipation, and solar radiation on kerosene oil and single-walled carbon nanotubes (SWCNTs) within a porous convergent/divergent channel. The integrating effect of nanoparticle conductivity, volume concentration, and mass flow rates play a diversified impact in augmenting the heat transport properties of nanofluids. The free convection of hybrid nanofluid flow of visco-elastic fluid via a vertically heated plate associated to sinusoidal surface temperature is reported by Roy et al. [3]. They also explored the flow of hybridized nanofluid with the behaviour of magnetic field along-side the role of thermal radiation. Sarfraj and Khan [4] conducted the thermal behaviors of tri-hybrid nanofluids composed of various materials likely silica, cadmium selenide quantum dots with copper dispersed in ethanol. They investigated these characteristics concerning Hiemenz and Homann's flows, which are typically associated with stagnation point flow. The study examines the flow over an infinite plate within a porous medium, moving at a constant velocity toward or away from a normal stagnation point flow. The flow dynamics are influenced by porosity and magnetic effects. Besides, heat transfer analysis employs the Cattaneo-Christov theory, incorporating factors such as Ohmic heating, nonlinear Roseland radiation, and heat sources /sinks. Kumar et al. [5] studied the significance of a third-grade radiative nanofluid via a porous medium over a Riga plate considering the convective boundary conditions. They have utilized the Cattaneo-Christov Double Diffusion model for the enrichment of the energy transport phenomenaon. Arif et al. [6] examined the flow over an oscillatory surface for the significance of solutal transfer impacts. Sohail et al. [7] explored the behaviour of peristaltic flow of a tangent hyperbolic nanofluid for the interaction of gyrotactic microorganisms. Moreover, the entropy analysis is one of the novel approaches for the hyperbolic flow of MHD and heat transport properties.

**Important analysis:** The vast literature survey led to filling up the gap focusing on;

* Analysis of Al2O3 with Cu in water presenting the hybrid nanofluid over a bidirectional permeable expanding surface.
* Impact of non-uniform heat source/sink on the energy equation enriching the flow phenomena.
* The role of thermal radiation along with permeable surface affects the bounding surface properties.

**Novelty:** The assumption of the study leads to the novel approach and these are;

* Hybrid nanofluid flow comprising of Al2O3 and Cu in water over a bidirectional stretching surface enhances the flow properties.
* The assumption of non-uniform heat source/sink on conducting fluid enhanced heat transport properties.
* The heat flux in association with the permeable surface reflects several industrial applications.

**MATHEMATICAL DESCRIPTION**

A three-dimensional transportation of a hybrid nanofluid contained with Al2O3 and Cu nanoparticles with H2O as base fluid passing a permeable biaxial stretching/shrinking sheet is demonstrated in FIGURE. 1. The sheet lies in plane and experiences stretchy/ shrunk in the y direction. The z axis is act orthogonal to the sheet, and the flow exist in the region. Along  axes the velocity components are. The velocities  and of the sheet are presented for and directions. Likewise, the constant mass flux velocity is given by . The wall temperature is considered to be , while the ambient temperature is 

Based on the premises mentioned above, the standard set of equations are

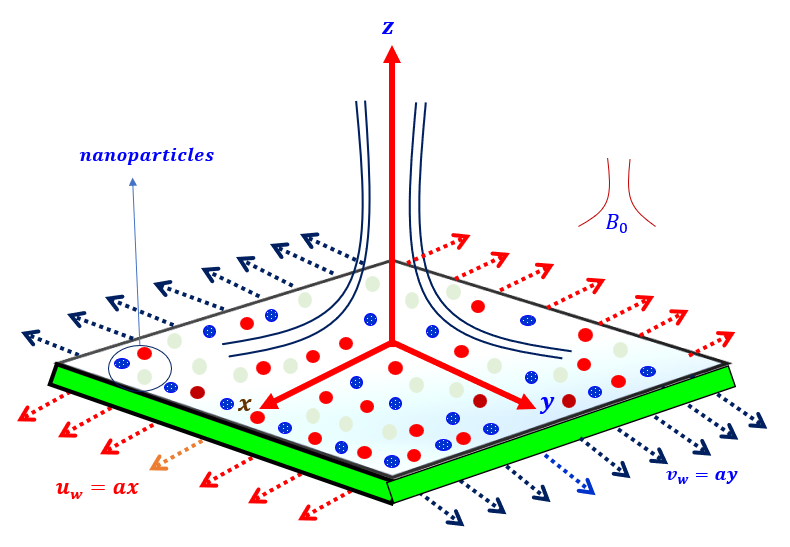


FIGURE. 1 Flow model

 (1)

 (2)

 (3)

 (4)

 (5).

With the surface conditions:

 (6)

Here,  shows the non-uniform heat source/sink described as

 (7)

Here indicates for space and temperature dependent heat source/sink.

In radiative heat flux indicates Stefan Boltzmann constant and  is the coefficient of mean absorption.

The thermophysical models for hybrid nanofluids are

, , ,

,



Here,  are the dynamic viscosity, density, thermal conductivity, and heat capacity of the hybrid nanofluid, respectively  is the nanoparticle volume fraction.

Introducing the similarity variables

 (8)

Incorporating the above similarity variables in the governing eq. (2-5) one can obtain

 (9)

 (10)

 (11)

 (12)

Modified surface conditions are

 (13)

Here the dimensionless terms are



The important quantities like skin friction coefficient and  and local Nusselt number are

 (14)

The dimensionless rate coefficients are

 (15)



**Table 1:** Thermophysical characteristics

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Thermophysical properties | Density | Thermal conductivity | Heat capacity | Electrical conductivity |
| Units |  |  |  |  |
| Al2O3 | 3970 | 40 | 765 |  |
| Cu | 8933 | 401 | 385 |  |
|  | 997.1 | 0.613 | 4179 |  |

**Table 2:** Comparison with previous work neglecting nonuniform heat source, magnetic parameter

|  |  |  |
| --- | --- | --- |
|  | Previous result (Wang [8]) | Present result |
|  |  |
| 0 | -1 | -1 |
| 0.2 | -1.0395 | -1.0395 |
| 0.4 | -1.0758 | -1.0759 |
| 0.5 | -1.0931 | -1.0931 |

**RESULTS AND DISCUSSION**

The three-dimensional flow of Al2O3 and Cu in the water over an expanding surface is reported in this case. The convective flow for the insertion of thermal radiation combined with non-uniform heat generation/absorption enriches the transport properties of the fluid. Moreover, the assumption of elongating sheet along both the direction of the flow over the surface encourages the transport properties. The thermophysical models such as thermal conductivity and viscosity are the important aspects of the investigation. Particularly, Table-1 displays the thermal attributes of the nanoparticles integrated with base liquid water within a temperature of 2980K collected from the particular reference. The standard numerical technique is adopted in handling the governing equations. The simulation is carried out by utilizing the built-in function bvp4c available in MATLAB. The validation of the current result with the earlier work of Wang [8] is depicted in Table-2. The correlating results show the convergence properties of the current methodology.

The process of computation for the velocity profile of the hybrid nanofluid is presented in two-distinct folds such as the Comparison between the Al2O3~water nanofluid and Al2O3+Cu~water hybrid nanofluid. In general, for the computation the particle concentration of Al2O3  and Cu  are considered by maximum 2% each. The permeability of the stretching surface lead to carry out the impact of suction/injection  that is significant on the velocity profile for the consideration of. The velocity ratio  which is depends upon the stretching ration along the axial and transverse direction of the flow is depicted for the numerical range of. all these factors are also affecting the transverse and the normal direction of the flow. Further, the temperature profile of the hybrid nanofluid presented by the inclusion of the thermal radiation  which is reported for the assumption of Rosseland approximation heat flux is conducted for the range of. The additional heat source/sink for the assumption of space and temperature dependent presented by the factors of A and B are considered as  and  respectively.

The velocity ratio impact on  is presented in **FIGURE.2**. The non-dimensional quantity is presented for the ratio between the ratio of the fluid velocity at the surface of each of the profile of axial and transverse direction. The positive variation and its increasing behaviour lead to resists the fluid motion since the axial direction stretching is more and dominates the fluid velocity throughout. However, the negative variation shows opposite impact on the fluid velocity and the fact is the transverse direction velocity is more pronounced than the axial velocity which lead to a significant deceleration in the fluid momentum. The comparison between the nanofluid and the hybrid nanofluid presented earlier also favors in enhancing the fluid velocity. **FIGURE. 3** explained the outcome of velocity ratio on . A larger velocity ratio suggests the wall motion is relatively stronger compared to the main flow direction. This induce stronger transverse flow components, lead to an elevation in . The scenario of hybrid nanofluid exhibits higher momentum as compared to the scenario of mono nanofluid. **FIGURE. 4** exhibits the interpretation of for different values of . The positive case of enhances the boundary layer and lead to the elevation in , meanwhile the negative case of declines the fluid momentum which slower the boundary layer growth, causing lower . It is disclosed that the hybrid case has a higher transport capability than nanofluid. **FIGURE. 5** visualizes the physical interpretation of on , which augments the thermal state of fluid. The increasing level of radiation dominates over convection and conduction, leading to temperature due to the additional radiative heat flux, which contributes to energy transport and augments the thermal conductivity. Hybrid nanofluid exhibit more heat accumulation as compared to nanofluid. **FIGURE. 6** and **FIGURE. 7** explains the role of space dependent  and temperature dependent heat source/sink on respectively. The space dependent heat source/sink refers to the heat generation/absorption varies with spatial coordinates, while the temperature dependent heat source/sink indicates the heat generation is a function of local fluid temperature. Both parameters enhance the thermal transport by incorporating the additional energy to the system, leading to growth in temperature of fluid. **FIGURE. 8** and **FIGURE. 9** explain the output of and on Nusselt number in relation to . A higher radiative thermal flow augments over all energy transportation. This strengthened energy exchanged to higher temperature difference near the surface, which hike the thermal efficiency rate. The heat transfer rate is controlled as the space dependent and temperature dependent heat source rises. These factors inject additional energy to the system, leading to a reduction in thermal gradient between the surface and the fluid, hence a lower thermal gradient decline the thermal efficiency rate. It is disclosed from the study is hybrid nanofluid exhibit superior heat transfer performance compare to single nanofluid.



FIGURE. 2for different  FIGURE. 3 for different 



FIGURE. 4 for different  FIGURE. 5 for different 



FIGURE. 6 for different  FIGURE. 7 for different 



FIGURE. 8  versus FIGURE. 9  versuswith 

**CLOSING REMARKS**

In conclusion, the proposed study demonstrates the simultaneous influence of Al2O3 and Cu in H2O for 3D transportation of hybrid nanofluid over an expanding surface considering thermal radiation and non-uniform heat source. The major outcome of the study is described below:

* The study confirms the synergistic interaction between metal and metal oxide nanoparticle, leading to better heat and momentum transportation.
* Radiation parameter, heat source parameters enhances the thermal state of fluid.
* Space-dependent and temperature-dependent heat sources lowering the heat transfer rate.
* The hybrid nanofluid shows a higher heat transfer rate compared to the single nanofluid due to the improved thermal conductivity.

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