Magnetized Williamson Nano Fluid Flow Over a Permeable Stretching/Shrinking Surface with Neild’s conditions

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**Abstract.**

This research presents a computational analysis of unsteady magnetohydrodynamic (MHD) motion of a Williamson nanofluid, modeled as a third-grade fluid, across a permeable stretching surface embedded in a porous medium. The formulation incorporates complex physical effects such as thermal radiation, Brownian motion, thermophoresis, chemical reactions, internal heat generation and the influence of a magnetic field. The governing nonlinear differential equations are derived through similarity transformations and addressed with the BVP4C approach in MATLAB. The effect of different dimensionless factors on the distributions of velocity, temperature, and concentration is thoroughly examined. Weissenberg number We, magnetic field strength M, porous medium permeability K and suction parameter γ significantly influence both the velocity and temperature boundary layers. For example, a rise in We from 0.00 to 0.03 elevates the skin friction coefficient from 3.9247 to 5.4227, reflecting stronger elastic effects, while an increase in M from 0.0 to 0.3 lowers the skin friction coefficient from 6.1311 to 3.1441, indicating magnetic damping of fluid motion. In terms of thermal behavior, raising the thermal radiation parameter Rd from 0 to 3.0 results in a noticeable enhancement of the local Nusselt number from 8.5965 to 9.3164, demonstrating improved heat transfer. Likewise, increasing the thermophoresis parameter Nt from 0.1 to 0.4 boosts the concentration gradient, decreasing the Sherwood number from –1.1381 to –3.4194, indicating stronger nanoparticle transport toward the surface.

**Keywords:** Williamson Nano fluid, Thermal radiation, Porous medium, stretching/shrinking surface, Chemical reaction, Heat generation.

***Symbol/variable/parameter***

|  |  |
| --- | --- |
|  | : Reynolds number |
| Cp | : Specific heat (J kg−1 K−1) |
| C | : Particle concentration |
|  | : Stretching surface |
| Le | : Lewis number |
|  | : Density (kg m−3) |
|  | : Kinematic viscosity (m2 s−1) |
|  | : Brownian diffusion (m2s−1) |
|  | : Sheet distance |
| K | : Thermal conductivity (W m−1K−1) |
|  | : Thermophoresis diffusion (m2 s−1) |
| (u, v) | : Velocity components (m s−1) |
| G | : Force (m s−2) |
| T | : Temperature (K) |
| K : Porosity parameter  M : Magnetic Parameter  We : Williamson fluid Parameter  Pr : Prandtl number  Rd : Thermal Radiation  Nt : Thermophoresis  Nb : Brownian motion  Q : Heat generation/Absorption  Sc : Schmidt number  : Chemical reaction parameter  : Temperature difference parameter  E : Activation energy parameter  : Stretching/Shrinking parameter  : Suction parameter | |

# INTRODUCTION

Studying the flow within the viscous layer formed over a stretched sheet is of significant scientific interest because of its broad applications across engineering and manufacturing operations, such as polymeric material processing, wire alignment, fiber spinning, and heat exchanger systems. Sakiadis [1] laid the foundation for theoretical studies of viscous flow near a deforming surface, with subsequent experimental confirmation provided by Tsou et al. [2]. This study laid the foundation for numerous investigations exploring the effects of magnetic fields, radiative heat transfer, thermophoresis, as well as random particle motion on the nanofluid movement across an extending interface.

Fang et al. [3] expanded a classical problem by considering the effects of variable sheet thickness and power-law surface velocity, utilizing similarity transformations and providing analytical solutions for special cases. Khan and Pop [4] introduced nanofluid effects by incorporating thermophoresis and Brownian motion, and observed that a higher Brownian motion parameter (Nb), Lewis number (Le), and thermophoresis parameter (Nt) leads to a decrease in the rate of heat transfer. Ishak [5] investigated how thermal radiation impacts the movement of a viscoelastic substance across an extensible boundary, showing that greater radiation intensity enhances surface heat dissipation. Furthermore, Salleh et al. [6] analyzed Newtonian heating conditions, comparing on constant wall heat flux (CWHF) and constant wall temperature (CWT) subject to surface conditions and highlighting their effects on thermal flow behavior. Hassani et al. [7] investigated the impact of magnetic interactions, velocity slip, and radiative heat transfer on the flow dynamics, thermal profile, and nanoparticle distribution over a permeable stretching sheet. They demonstrated how increasing Nt and Nb intensifies the thermal boundary layer, while the concentration boundary layer shrinks with rising Nb. Similarly, Ibrahim et al. [8] investigated the impact of magnetic interactions, velocity slip, and radiative heat transfer on the flow dynamics, thermal profile, and nanoparticle distribution over a permeable stretching sheet. Their numerical study demonstrated how MHD interactions influence heat and mass transfer dynamics.

Makinde et al. [9] conducted an in-depth study on convective boundary conditions for the movement of nanoparticle-laden fluids across a stretchable surface, considering random thermal diffusion and temperature-gradient-driven particle migration in the transport and distribution processes. They concluded that elevated Brownian motion coupled with thermophoresis boosts thermal transport, thickens the thermal boundary layer and simultaneously lowers nanoparticle concentration as a result of their effect on the Lewis number. Nadeem [10] et al. investigated how convective energy transport, radiative thermal effects, and magnetohydrodynamics (MHD) influence the behavior of nanofluids in motion. Their study revealed that increasing the Prandtl number (Pr) lowers thermal conductivity as well as thermal energy transfer rates, while raising the surface temperature. Ishfaq et al. [11] derived an analytical solution for nanofluid flow inside the boundary region over a substrate, considering the condition involving zero nanoparticle flux. They noted that Brownian motion contributes insignificantly to the Nusselt number, but results in an overall enhancement of heat transport. Goyal [12] et al. performed a computational study regarding non-Newtonian nanofluid movement along an extending surface, emphasizing the impact that thermal transport characteristics have upon flow dynamics and velocity slip. Reddy [13] studied the combined effects of a magnetic zone, heat emission, slip velocity and advective thermal boundary conditions by comparing scenarios with fixed temperature and fixed heat flux. Aly et al. [14] formulated a transformation method that converts unbounded boundary conditions into bounded ones, allowing for more accurate numerical solutions in nanofluid studies. Abo-Eldahab et al. [15] explored how nanofluid flow near a stretching boundary is affected by thermophoresis, microscopic particle movement, and magnetic field forces. Their analysis provided insights into how these factors influence on mass and heat transfer with the various parametric conditions. Additionally, studies incorporating chemical reactions, activation energy, and hybrid nanofluids have further advanced the understanding of complex nanofluid behavior.

Shakhaoath Khan et al. [16] examined nanofluid behavior and thermal transport along a stretching surface impacted by magnetic and radiative thermal effects. Their study showed that radiation parameters elevate the temperature profile, while higher Lewis numbers and Brownian motion reduce concentration levels. Sarif et al. [17] investigated the fluid motion within the near-surface region along an extending sheet under Newtonian thermal conditions, where the extent of thermal transport through surface heat exchange is influenced by the behavior in the thermal boundary layer. Their results showed that increasing the Prandtl number decreases the depth of the thermal layer, whereas stronger heating boosts both surface heat transfer and the expansion of the thermal region. Abel et al. [18] examined the behavior of nanofluids flowing past an elongating surface under the influence of magnetic fields and Newtonian heating conditions. They found that the impact of thermophoresis, along with random particle motion, enhanced both thermal and concentration profiles, while convective heating led to a thicker thermal boundary layer Hayat et al. [19] investigated the flow of nanofluid over a stretching surface while considering the influences of velocity, magnetic forces, and thermal boundary conditions. Their study demonstrated that magnetic forces reduce velocity but increase the thermal distribution. Sandeep et al. [20] investigated the nanofluid movement over a stretching or shrinking surface subjected to the effects of thermal radiation, magnetic forces, and heat generation due to viscosity. Their results revealed that stronger magnetic fields increase the depth of the thermal interface region, while internal friction heating raises the surface temperature. Mansur et al. [21] studied the impact of slip boundary conditions on the movement of nanofluids over an expanding surface under the influence of a magnetic field. They were discovered that magnetic strength as well as slip conditions significantly influence the flow, thermal, and concentration transfer rates. Babu et al. [22] investigated the behavior of fluid flow within the nanofluid’s boundary layer over a stretching surface embedded in a permeable medium, considering the effects of thermal radiation as well as energy dissipation due to viscosity. Their findings indicated that higher thermal radiation and dissipation parameters raised both velocity and heat distributions. Building on prior work, Sarif et al. [23] examined the behavior of nanofluids along an extending sheet under heat transfer constraints at the boundary, showing that elevated Prandtl values lead to a thinner thermal boundary layer, while enhanced convection-induced heating intensifies the heat transfer rate at the interface. Finally, Hayat et al. [24] investigated the behavior of a magnetohydrodynamic (MHD) nanofluid flowing across a stretchable porous substrate subjected to convective heat transfer conditions. Their findings indicated that factors such as magnetic field intensity, Brownian motion and thermophoresis notably impact the profiles of velocity, temperature and concentration.

Although extensive work has been done on nanofluid flow over stretching surfaces, limited studies have considered the combined influence of:

* Magnetized Williamson nanofluids with shear-thinning behavior.
* Convective boundary conditions (Nield’s conditions) with surface permeability.
* The joint influence exerted by Brownian motion, thermophoresis, and thermal radiation acting simultaneously significantly impacts the system’s behavior.

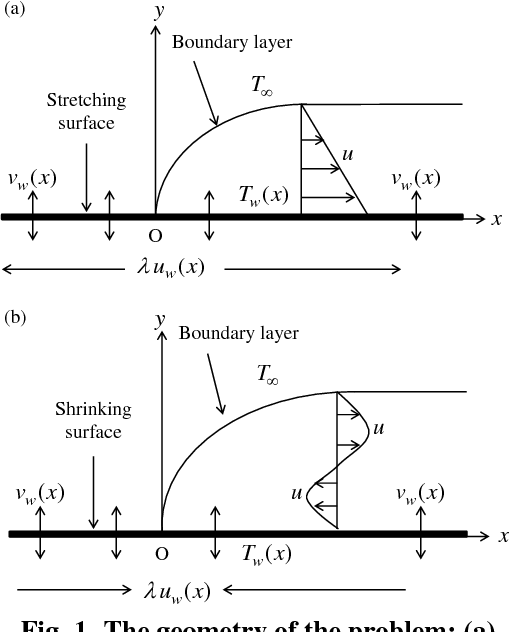
This research aims to conduct a computational study of magnetized non-Newtonian Williamson nanofluid motion over a porous stretching/shrinking surface under Nield’s boundary conditions using MATLAB’s BVP4C solver. While previous studies have explored individual factors such as magnetic fields, nanofluid properties, and convective boundary conditions, the coupled influence of shear-thinning, Brownian motion, thermophoresis, and surface permeability has received limited attention.

The focus is on:

* Investigates how magnetic field intensity, permeability, and the characteristics of Williamson fluid influence flow behavior and temperature distribution.
* Examine how thermophoresis, Brownian motion and thermal radiation affect the rates of energy and material transport.
* Evaluate the surface shear stress and convective heat transfer rate under varying physical parameters.
* Providing physical insights along with practical implications for industrial and engineering applications.

## MATHEMATICAL FORMULATION

The computational regime is considered as a stretching sheet and is depicted in Fig. 1. In this study, the laminar non-Newtonian Williamson two dimensional nanofluid, thermal radiation, magnetohydrodynamic, activation energy effects and heat generation/absorption are considered. A stretching sheet is considered to lie along the X-axis, with its velocity assumed to vary linearly with the distance from the origin, expressed as , where c is a constant. A magnetic field is considered to act at a right angle to the sheet. The  and  are represented by the ambient concentration and ambient temperature at y tends to infinity.



## Fig. 1. The Computational regime (a) stretching surface and (b) shrinking surface.

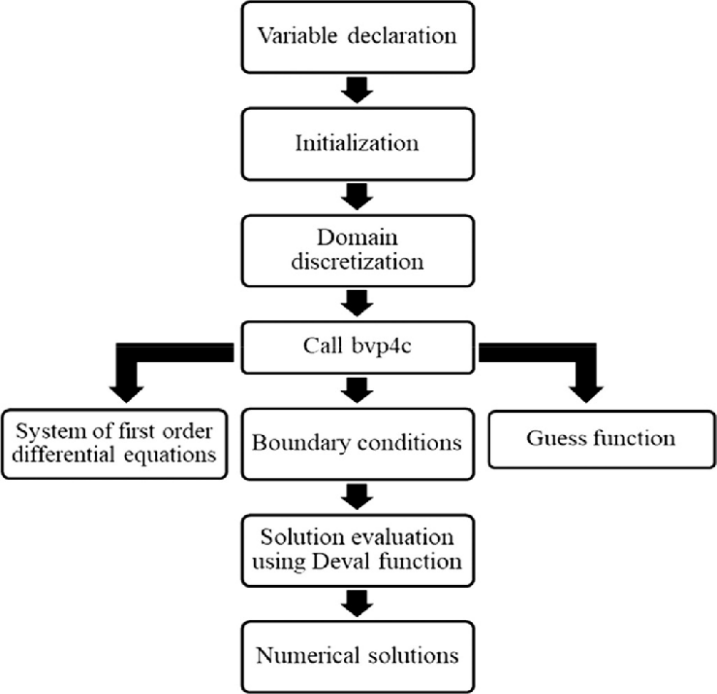
Thermophoresis and Brownian motion of nanoparticles are analyzed using Buongiorno’s model in nanofluids. Based on these assumptions, the dimensional form of the mathematical model is expressed through these equations

### (1)

### (2)

### (3)

### (4)



## Fig. 2. Flowchart of numerical procedure by MATLAB (bvp4c).

Boundary constraints linked to the governing equations are as follows:

### (5)

Similarity variables for this study are defined as follows:

### (6)

is a representation of this streamline function and is described by specific characteristics

### and (7)

These equations, numbered (1) through (4), are converted into a set of non-partial differential expressions through a transformation based on equation (4), and the final resulting equations are presented below:

### (8)

### (9)

### (10)

The transformed boundary conditions of the above equations are:

### (11)

### (12)

The is refers a Suction parameter, further the important engineering key parameters, including local skin friction factor , Nusselt number  and Sherwood number  play a role in interpreting the dynamics of shear stress, rate of heat and mass transfer. These parameters are mathematically defined as follows:



where, 

In this context  denotes the surface shear force,  indicates thermal energy transfer and  corresponds to the mass transfer rate.

### 

Where the Reynolds number .

|  |  |
| --- | --- |
|  |  |
| (a) | (b) |
|  | |
| (c) | |
| **FIGURE 3** The local Weissenberg number (*We*) effect on velocity, temperature, and concentration profiles. | |

## METHOD OF THE SOLUTION

Through the use of suitable similarity variables, the fundamental partial differential equations describing the behavior of a magnetized Williamson nanofluid flowing past a permeable surface undergoing stretching or shrinking—subject to Nield’s boundary constraints—are converted into a set of nonlinear ordinary differential equations. Computational techniques are then utilized to solve the transformed set using MATLAB’s BVP4C solver, which implements a finite difference collocation approach with adaptive mesh refinement to ensure precise results.

The system of third-order and second-order nonlinear coupled ODEs is transformed into an equivalent first-order system by introducing auxiliary terms:

Momentum equation (third-order ODE)



Convert to first-order system:



|  |  |
| --- | --- |
|  |  |
| (a) | (b) |
|  | |
| (c) | |
| **FIGURE 4** The magnetic number (M) effect on velocity, temperature and concentration profiles. | |

Energy equation (second-order ODE)



Convert to first-order system:

Concentration equation (second-order ODE)



Convert to first-order system:

The flow constraints are imposed on both the interface and the surrounding region:

At the surface 𝜂 = 0:



At infinity 𝜂 → ∞:

The numerical solution is computed with the MATLAB’s BVP4C. The initial approximation adopted for solving the system is provided using the bvpinit () function with a uniformly spaced mesh. The adaptive mesh refinement in BVP4C ensures accuracy, especially Close to the surface and throughout the boundary layer zone. A comprehensive flow diagram has been incorporated to enhance the understanding of the current approach utilizing the bvp4c technique. (Refer to Fig. 2 for a visual representation.

Numerical technique is validated with the existing work. The Table1 shows that the current numerical results compared with the previous research works who were done by Nazar [25] et al., Pop [26] et al., and Khan and Pop [27]. A comparison of highlights between present and existing results, demonstrates a significant agreement. The present results are simulated by the bvp4c MATLAB solver.

## TABLE. 1 The skin friction coefficient is compared for different parameter valueunder the conditions γ = M = We = K = Rd = σ = 0 and Pr = 1.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Nazar et al. [26]** | **Pop et al. [25]** | **Khan and Pop [27]** | **Present study** |
| 0.1 | -0.9694 | -0.9694 | -0.9694 | -0.9694 |
| 0.2 | -0.9181 | -0.9181 | -0.9181 | -0.9181 |
| 0.5 | -0.6673 | -0.6673 | -0.6673 | -0.6673 |
| 1.0 | - | - | 0 | 0 |
| 1.5 | - | - | - | 0.9095 |
| 2.0 | 2.0176 | 2.0174 | 2.0175 | 2.0175 |
| 3.0 | 4.7296 | 4.729 | 4.7293 | 4.7292 |
| 3.5 | - | - | - | 6.3006 |

## RESULTS AND DISCUSSION

The investigation numerically analyzes the behavior of magnetized Williamson nanofluid influenced by a permeable medium undergoing extension or contraction, under nonlinear constraints related to temperature as well as concentration. The study incorporates significant effects such as magnetic field, thermal radiation, porous medium permeability, suction/injection, random particle diffusion, thermally induced particle migration, and internal thermal energy variation. For the resolution of the highly nonlinear boundary value problem, the BVP4C technique was employed. Graphical results are presented to illustrate how various control factors influence the flow speed, thermal profile and concentration distribution. The parametric variations and their physical implications such as boundary layer thickness, thermal diffusion and nanoparticle distribution are effectively captured, offering deeper insight into the transport phenomena governed by the interaction between non-Newtonian fluid behavior and nanofluid characteristics.

|  |  |
| --- | --- |
|  |  |
| (a) | (b) |
|  | |
| (c) | |
| **FIGURE 5** The permeability of porous medium (*K*) effect on velocity, temperature and concentration profiles. | |

Figure 3 illustrates how changes in flow speed, thermal distribution, and solute concentration respond to the local value of the Weissenberg number. As illustrated in Figure 3(a), with increasing Weissenberg number (We), velocity rises near the boundary owing to the shear-thinning behavior of the Williamson fluid, which reduces effective viscosity and promotes fluid motion. However, further the velocity diminishes away from the surface with increasing elastic effects dominate and resist further deformation. Figure 3(b) illustrates That the temperature distribution declines as increases We, as reduced viscosity improves the heat conductivity, leading to enhanced heat dissipation. Similarly,

Figure 3(c) illustrates that concentration decreases as the Weissenberg number (We) increases, indicating that the non-Newtonian nature of the fluid enhances species transport and minimizes accumulation near the surface.

Figure 4 illustrates how variations in the magnetic number (M) influence the flow field, thermal distribution, and species concentration. In Figure 4(a), increasing M significantly reduces velocity due to the imposition of Lorentz forces, which resist motion and cause the momentum region to become thicker. In Figure 4(b), a rise in the magnetic parameter MMM results in intensified Joule heating due to magnetic resistance, which in turn elevates the temperature and enhances thermal energy retention. Figure 4(c) demonstrates that the concentration profile decreases with M, as the suppressed fluid motion limits mass diffusion and increases solute accumulation near the surface. In Figure 5, variations in permeability (K) within the porous medium reveal their effect on flow characteristics. Figure 5(a) shows that increasing K enhances velocity, as higher permeability allows fluid to penetrate the medium more easily, reducing flow resistance. Figure 5(b) shows higher temperature values with larger K, as greater fluid mobility enhances thermal conduction and convective heat transfer. Figure 5(c) depicts an increase in the concentration profile with K, indicating that enhanced permeability promotes mass transport and diffusion, reducing concentration gradients within the flow.

In Figure 6, changes in the suction parameter (γ) affect the velocity, temperature, and concentration fields. Figure 6(a) illustrates a decrease in velocity with increasing γ, as fluid is withdrawn at the surface, which suppresses the flow and reduces the boundary layer thickness. Figure 6(b) demonstrates on the distribution of temperature decreases as stronger suction enhances cooling efficiency, effectively removing thermal energy from the viscous layer. Figure 6(c) demonstrates that increasing γ leads to a decrease in the concentration profile, as suction limits species buildup near the surface and promotes a more uniform solute distribution throughout the flow. Figure 7 illustrates that how the thermal radiation parameter (Rd) influences the temperature (Fig. 7a) and concentration (Fig. 7b) profiles. In Fig. 7(a) shows that increasing thermal radiation parameter Rd results in a noticeable rise in temperature. This is because higher thermal radiation enhances radiative heat flux, resulting in greater heat absorption by the fluid. Consequently, the concentration gradient region becomes thinner, indicating a faster rate of species diffusion. Figure 7(b) demonstrates a decline in concentration with increasing Rd values. The coupling between thermal and solutal transport processes is responsible for the observed effect. When the temperature rises due to increased radiation, molecular diffusion becomes more pronounced, leading to enhanced mass transfer farther from the surface.

Figure 8 illustrates how the thermophoresis parameter (Nt) influences thermal and solutal distributions. In Figure 8(a), higher values of Nt promote thermophoretic motion of nanoparticles away from the heated surface, contributing to heat retention and thickening the thermal boundary layer. As shown in Figure 8(b), the concentration distribution also increases with higher Nt​, since more nanoparticles migrate away from the wall and accumulate within the fluid domain. This leads to an extended solutal boundary layer, indicating that thermophoresis significantly enhances both thermal and concentration transport in the flow. Figure 9 presents the influence of parameter Q related to heat generation on thermal and solutal behavior. As depicted in Figure 9(a), an increase in Q noticeably elevates temperature levels due to added heat generation, which increases energy in the adjacent flow region and, in turn, broadens the heat-affected zone. Similarly, Figure 9(b) shows that higher values of Q elevate concentration, which can be attributed to the coupling between thermal and solutal diffusion—where elevated temperatures from heat generation promote nanoparticle movement, enhancing concentration levels throughout the fluid domain.

Figure 10 illustrates how the Schmidt number (Sc) affects the distributions of temperature and concentration. From Figure 10(a), as Sc increases, the temperature profile slightly diminishes, indicating that momentum diffusivity becomes more dominant over thermal diffusivity. In Figure 10(b), a higher Schmidt number results in a notable decrease in concentration, as it indicates lower mass diffusivity and consequently a thinner concentration boundary layer. In Figure 11, the influence exerted by the chemical reaction coefficient (σ), where Figure 11(a) shows a marginal drop in temperature due to the weak coupling effect. Meanwhile, in Figure 11(b), increasing σ causes a notable decline in concentration, as the chemical reaction consumes nanoparticles, thereby weakening the solutal boundary layer. Figures 12 to 14 present the variations in skin friction and Nusselt number under the influence of different parameters. In Fig. 12, subfigure (a) demonstrates that the skin friction coefficient declines as the magnetic parameter (M) increases across various Weissenberg numbers (We), while subfigure (b) exhibits a similar downward trend with increasing permeability factor (K), indicating a thinning of momentum transfer layer. Fig. 13 analyses Nusselt number (Nu) variations with thermal radiation parameter Rd: (a) shows Nu increases with Rd and higher We, while (b) shows Nu decreases with increasing permeability K. Fig. 14 illustrates in (a) that Nu increases with magnetic parameter (M), particularly at higher values of Rd, while (b) shows that higher thermophoresis parameter (Nt) reduces Nu, especially when the Brownian motion parameter (Nb) is low. These trends emphasize the interplay between thermal and flow control parameters on heat transfer rates. Figure 15 depicts how the Sherwood number ( changes because of thermal radiation (Rd), magnetic field strength (M), Brownian diffusion (Nb) and the thermophoretic parameter (Nt). Subfigure (a) shows thatincreases (becomes less negative) with rising Rd for all values of M, indicating enhanced mass transfer as radiation increases. In subfigure (b),  also increases significantly with higher Nb, especially at lower Nt values, demonstrating that stronger Brownian motion boosts mass transfer despite the opposing effect of thermophoresis.

Table 2 presents the changes in the local skin friction coefficient () corresponding to variations in the Weissenberg number (We), magnetic parameter (M), and the permeability of the porous medium (K). A rise in the Weissenberg number clearly leads to greater skin friction, suggesting that enhanced fluid elasticity contributes to increased wall resistance. For instance, as We increases from 0.00 to 0.03 (Sc. Nos. 2–4), , rises from 3.9247 to 5.4227, indicating increased shear stress in the vicinity of the wall. Conversely, a rise in the magnetic field coefficient (M) (Sc. Nos. 5–8) leads to a decrease in , the Lorentz forces generated by the magnetic field counteract fluid motion, thus reducing wall shear. Similarly, the increase in permeability (K) (Sc. Nos. 9–12) results in a notable decline in , due to reduced flow resistance in a more porous medium that facilitates fluid penetration through the surface. These trends align well with the dynamic characteristics of magnetized nanofluids exhibiting non-Newtonian properties within porous media, thereby reinforcing the reliability of the current numerical model.

|  |  |
| --- | --- |
|  |  |
| (a) | (b) |
|  | |
| (c) | |
| **FIGURE 6** The Suction parameter () effect on velocity, temperature, and concentration profiles. | |

**Table 2.** Computed values of with different values of We, M and K

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **S.No.** | **We** | **M** | **K** |  |
| 1 | 0.01 | 0.1 | 0.1 | 4.2639 |
| 2 | 0.00 |  |  | 3.9247 |
| 3 | 0.02 |  |  | 4.7682 |
| 4 | 0.03 |  |  | 5.4227 |
| 5 |  | 0.0 |  | 6.1311 |
| 6 |  | 0.1 |  | 4.2639 |
| 7 |  | 0.2 |  | 3.6131 |
| 8 |  | 0.3 |  | 3.1441 |
| 9 |  |  | 0.0 | 6.1311 |
| 10 |  |  | 0.1 | 4.2639 |
| 11 |  |  | 0.3 | 3.1441 |
| 12 |  |  | 0.5 | 2.4418 |

|  |  |
| --- | --- |
|  |  |
| (a) | (b) |
| **FIGURE 7** The thermal radiation (Rd) effect on temperature and concentration profiles. | |

|  |  |
| --- | --- |
|  |  |
| (a) | (b) |
| **FIGURE 8** The thermophoresis (Nt) effect on temperature and concentration profiles. | |

|  |  |
| --- | --- |
|  |  |
| (a) | (b) |
| **FIGURE 9** The heat generation (Q) effect on temperature and concentration profiles. | |

Table 3 presents the influence of various governing parameters on the local Nusselt number , which reflects the rate of heat transfer from the surface to the nanofluid. It is evident that increasing the Weissenberg number (We) (Sc. Nos. 2–4) slightly enhances , indicating that greater fluid elasticity supports better thermal diffusion near the wall. The magnetic parameter (M) (Sc. Nos. 5–8) shows a decreasing trend in , as the Lorentz force suppresses convective transport and thickens the thermal boundary layer. Likewise, increasing permeability (K) (Sc. Nos. 9–12) reduces , attributed to the reduced resistance and weaker thermal gradients near the wall. A significant positive impact is observed with thermal radiation parameter Rd (Sc. Nos. 13–16), where greater Rd enhances , owing to radiative heat addition improving energy transport. Interestingly, heat generation/absorption (Sc. Nos. 17–20) shows that heat absorption (lower Q) strongly increases , while generation (higher Q) reduces it, consistent with energy accumulation in the system. Thermophoresis parameter Nt (Sc. Nos. 25–28) increases notably, implying stronger thermal transport due to nanoparticle migration. Electrical conductivity σ (Sc. Nos. 29–32) also enhances heat transfer moderately due to Joule heating effects. The Eckert number E (Sc. Nos. 33–36) slightly lowers, due to viscous dissipation reducing the temperature gradient. Lastly, an increase in Schmidt number Sc (Sc. Nos. 31–34) leads to a decline in , reflecting weakened mass diffusivity, which indirectly affects thermal transport in this coupled system. Collectively, these results demonstrate how thermal behavior is influenced by both fluid rheology and multiphysical interactions near the surface.

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| --- | --- |
|  |  |
| (a) | (b) |
| **FIGURE 10** The Schmidt number (Sc) effect on temperature and concentration profiles. | |

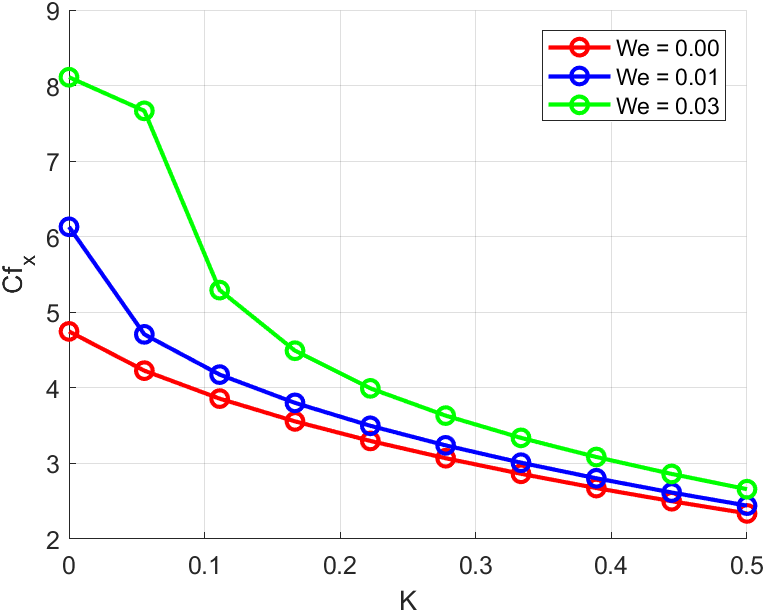
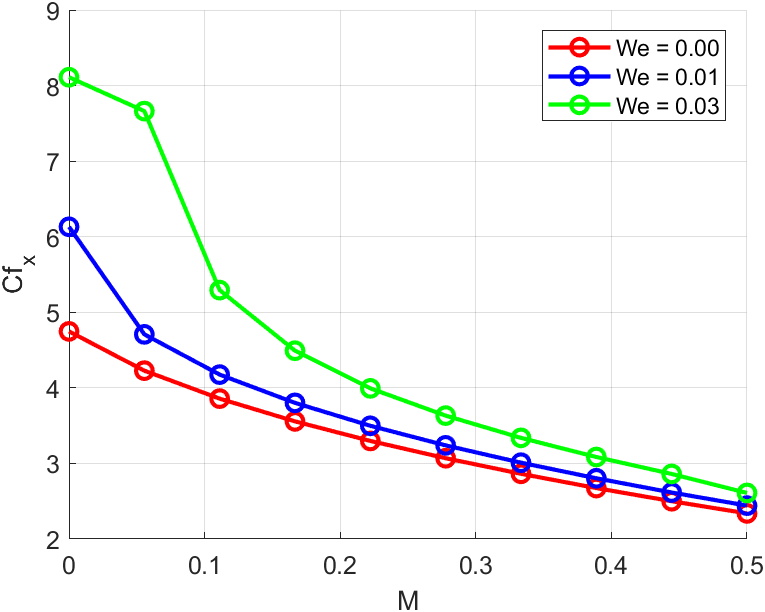
**TABLE 3.** Computed values of with different values of We, M, K, Rd, Nt, Q, , E and Sc

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **S.No.** | **We** | **M** | **K** | **Rd** | **Nt** | **Q** |  | **E** | **Sc** |  |
| 1 | 0.01 | 0.1 | 0.1 | 0.3 | 0.5 | 0.5 | 0.1 | 0.1 | 3 | 8.7773 |
| 2 | 0.00 |  |  |  |  |  |  |  |  | 8.7439 |
| 3 | 0.02 |  |  |  |  |  |  |  |  | 8.8269 |
| 4 | 0.03 |  |  |  |  |  |  |  |  | 8.8902 |
| 5 |  | 0.0 |  |  |  |  |  |  |  | 8.9928 |
| 6 |  | 0.1 |  |  |  |  |  |  |  | 8.7773 |
| 7 |  | 0.2 |  |  |  |  |  |  |  | 8.6786 |
| 8 |  | 0.3 |  |  |  |  |  |  |  | 8.5936 |
| 9 |  |  | 0.0 |  |  |  |  |  |  | 8.9928 |
| 10 |  |  | 0.1 |  |  |  |  |  |  | 8.7773 |
| 11 |  |  | 0.3 |  |  |  |  |  |  | 8.5936 |
| 12 |  |  | 0.5 |  |  |  |  |  |  | 8.4383 |
| 13 |  |  |  | 0.0 |  |  |  |  |  | 8.5965 |
| 14 |  |  |  | 1.0 |  |  |  |  |  | 8.9492 |
| 15 |  |  |  | 2.0 |  |  |  |  |  | 9.1090 |
| 16 |  |  |  | 3.0 |  |  |  |  |  | 9.3164 |
| 17 |  |  |  |  | 0.1 |  |  |  |  | 12.7473 |
| 18 |  |  |  |  | 0.2 |  |  |  |  | 11.5484 |
| 19 |  |  |  |  | 0.3 |  |  |  |  | 10.4938 |
| 20 |  |  |  |  | 0.4 |  |  |  |  | 9.5743 |
| 25 |  |  |  |  |  | 0.0 |  |  |  | 7.7461 |
| 26 |  |  |  |  |  | 0.5 |  |  |  | 8.7773 |
| 27 |  |  |  |  |  | 1.0 |  |  |  | 9.5517 |
| 28 |  |  |  |  |  | 2.0 |  |  |  | 10.7851 |
| 29 |  |  |  |  |  |  | 0.0 |  |  | 8.7480 |
| 30 |  |  |  |  |  |  | 0.5 |  |  | 8.8631 |
| 31 |  |  |  |  |  |  | 1.0 |  |  | 8.9313 |
| 32 |  |  |  |  |  |  | 2.0 |  |  | 9.0089 |
| 33 |  |  |  |  |  |  |  | 0.1 |  | 8.7773 |
| 34 |  |  |  |  |  |  |  | 0.5 |  | 8.7697 |
| 35 |  |  |  |  |  |  |  | 1.0 |  | 8.7628 |
| 30 |  |  |  |  |  |  |  | 3.0 |  | 8.7511 |
| 31 |  |  |  |  |  |  |  |  | 0.5 | 12.5724 |
| 32 |  |  |  |  |  |  |  |  | 1.0 | 11.2087 |
| 33 |  |  |  |  |  |  |  |  | 2.0 | 9.6293 |
| 34 |  |  |  |  |  |  |  |  | 3.0 | 8.7773 |

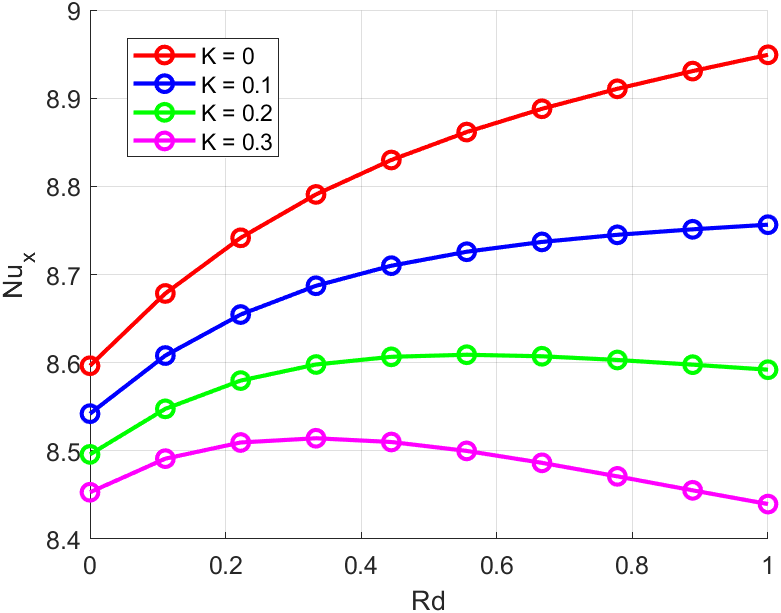
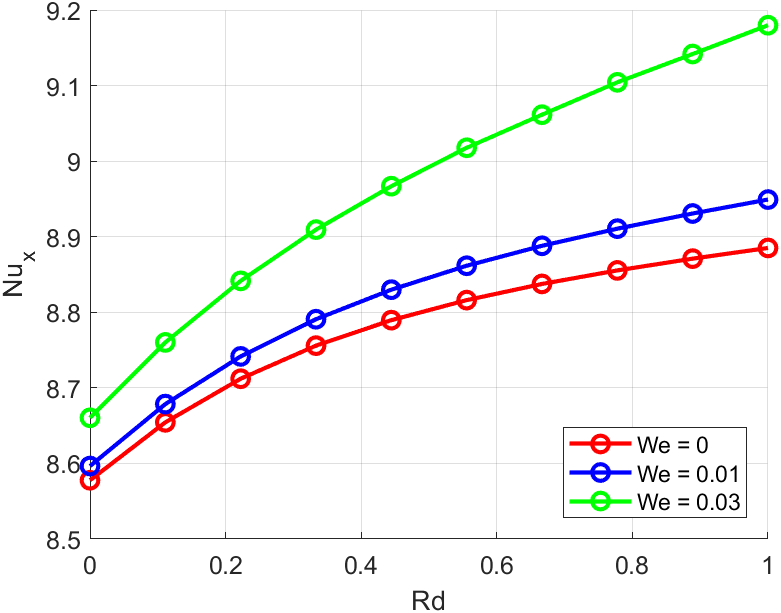
**TABLE 4.** Computed values of with different values of We, M, K, Rd, Nt, Nb, Q, , E and Sc

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **S.No.** | **We** | **M** | **K** | **Rd** | **Nt** | **Nb** | **Q** |  | **E** | **Sc** |  |
| 1 | 0.01 | 0.1 | 0.1 | 0.3 | 0.5 | 0.8 | 0.5 | 0.1 | 0.1 | 3 | -3.9184 |
| 2 | 0.00 |  |  |  |  |  |  |  |  |  | -3.9035 |
| 3 | 0.02 |  |  |  |  |  |  |  |  |  | -3.9406 |
| 4 | 0.03 |  |  |  |  |  |  |  |  |  | -3.9689 |
| 5 |  | 0.0 |  |  |  |  |  |  |  |  | -4.0146 |
| 6 |  | 0.1 |  |  |  |  |  |  |  |  | -3.9184 |
| 7 |  | 0.2 |  |  |  |  |  |  |  |  | -3.8744 |
| 8 |  | 0.3 |  |  |  |  |  |  |  |  | -3.8364 |
| 9 |  |  | 0.0 |  |  |  |  |  |  |  | -4.0146 |
| 10 |  |  | 0.1 |  |  |  |  |  |  |  | -3.9184 |
| 11 |  |  | 0.3 |  |  |  |  |  |  |  | -3.8364 |
| 12 |  |  | 0.5 |  |  |  |  |  |  |  | -3.7671 |
| 13 |  |  |  | 0.0 |  |  |  |  |  |  | -5.3728 |
| 14 |  |  |  | 1.0 |  |  |  |  |  |  | -2.3971 |
| 15 |  |  |  | 2.0 |  |  |  |  |  |  | -1.5527 |
| 16 |  |  |  | 3.0 |  |  |  |  |  |  | -1.1646 |
| 17 |  |  |  |  | 0.1 |  |  |  |  |  | -1.1381 |
| 18 |  |  |  |  | 0.2 |  |  |  |  |  | -2.0622 |
| 19 |  |  |  |  | 0.3 |  |  |  |  |  | -2.8108 |
| 20 |  |  |  |  | 0.4 |  |  |  |  |  | -3.4194 |
| 21 |  |  |  |  |  | 0.1 |  |  |  |  | -31.3474 |
| 22 |  |  |  |  |  | 0.2 |  |  |  |  | -15.6737 |
| 23 |  |  |  |  |  | 0.3 |  |  |  |  | -10.4491 |
| 24 |  |  |  |  |  | 0.4 |  |  |  |  | -7.8369 |
| 25 |  |  |  |  |  |  | 0.0 |  |  |  | -3.4581 |
| 26 |  |  |  |  |  |  | 0.5 |  |  |  | -3.9184 |
| 27 |  |  |  |  |  |  | 1.0 |  |  |  | -4.2642 |
| 28 |  |  |  |  |  |  | 2.0 |  |  |  | -4.8148 |
| 29 |  |  |  |  |  |  |  | 0.0 |  |  | -3.9054 |
| 30 |  |  |  |  |  |  |  | 0.5 |  |  | -3.9567 |
| 31 |  |  |  |  |  |  |  | 1.0 |  |  | -3.9872 |
| 32 |  |  |  |  |  |  |  | 2.0 |  |  | -4.0218 |
| 33 |  |  |  |  |  |  |  |  | 0.1 |  | -3.9184 |
| 34 |  |  |  |  |  |  |  |  | 0.5 |  | -3.9150 |
| 35 |  |  |  |  |  |  |  |  | 1.0 |  | -3.9119 |
| 36 |  |  |  |  |  |  |  |  | 3.0 |  | -3.9067 |
| 37 |  |  |  |  |  |  |  |  |  | 0.5 | -5.6127 |
| 38 |  |  |  |  |  |  |  |  |  | 1.0 | -5.0039 |

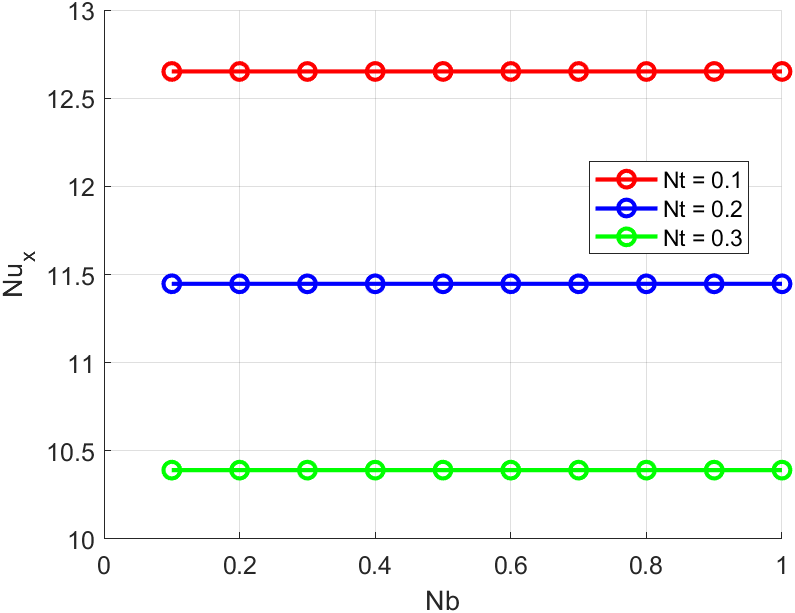
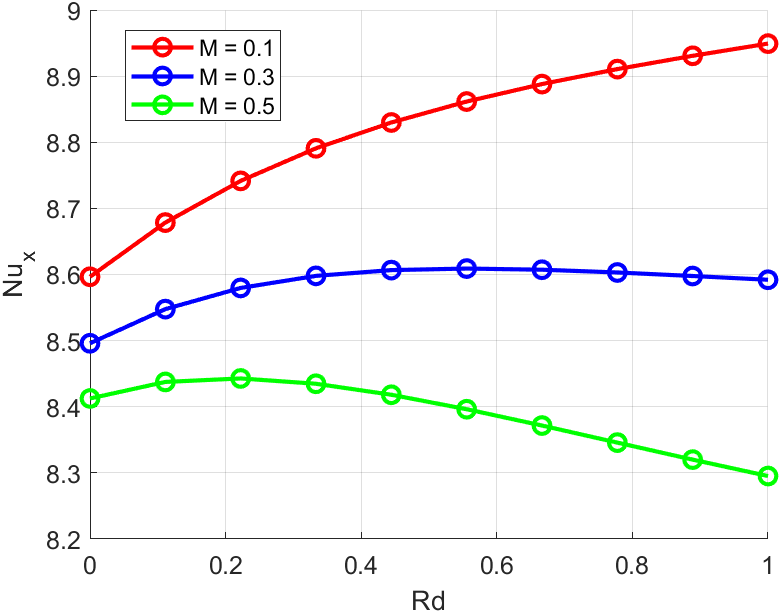
|  |  |
| --- | --- |
|  |  |
| (a) | (b) |
| **FIGURE 11** The Chemical Reaction parameter () effect on temperature and concentration profiles. | |



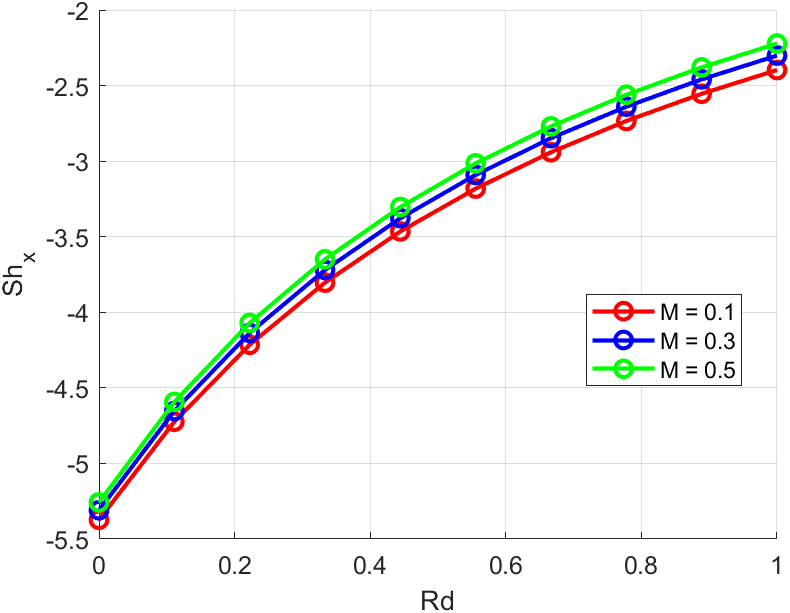
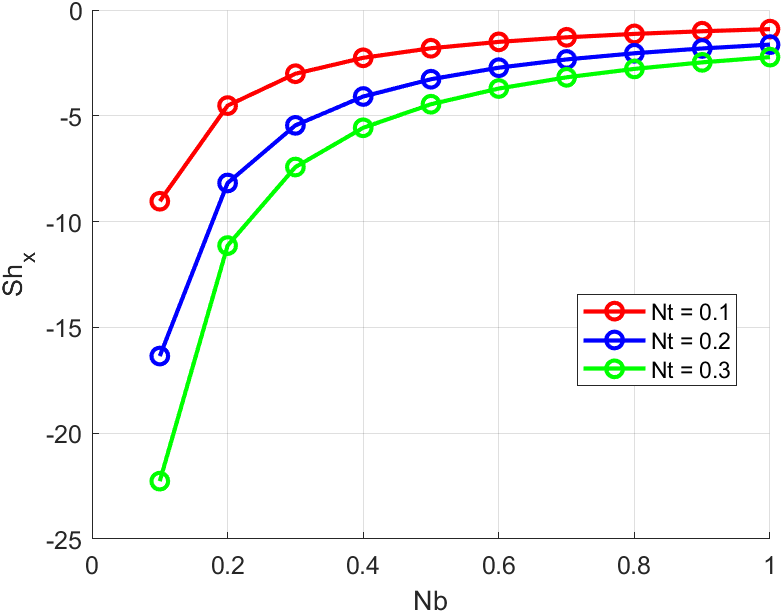
**FIGURE 12.** Variation in skin friction for (a) M against We and (b) K against We,



**FIGURE 13.** Variation in Nusselt number for (a) Rd against We and (b) Rd against K,



**FIGURE 14.** Variation in Nusselt number for (a) Rd against M and (b) Nb against Nt.

**FIGURE 15**. Variation in Sherwood number for (a) Rd against M and (b) Nb against Nt.

Table 4 encapsulates how essential physical factors affect the local Sherwood number , offering insights into the mass transfer dynamics within the nanofluid system. A progressive increase in the Weissenberg number (Sc. Nos. 2–4) marginally enhances the magnitude of , reflecting increased elasticity leading to stronger fluid stretching and better concentration gradient at the wall. The magnetic field strength M (Sc. Nos. 5–8) shows a clear decline in with increasing M, due to magnetic damping that suppresses convective effects and reduces concentration diffusion. Similarly, increasing porous medium permeability K (Sc. Nos. 9–12) results in a lower , indicating diminished mass transfer as resistance to flow is lessened. Thermal radiation Rd (Sc. Nos. 13–16) exhibits a strong inverse relation, where higher Rd causes to sharply drop, due to enhanced thermal energy overpowering mass diffusion. Thermophoresis Nt (Sc. Nos. 17–20) strongly reduces , as nanoparticle migration away from hot regions weakens the concentration gradient at the wall. Conversely, increasing Brownian motion Nb (Sc. Nos. 21–24) significantly reduces , especially at lower Nb, indicating the destabilizing role of nanoparticle diffusion on concentration transfer. Heat generation/absorption Q (Sc. Nos. 25–28) shows that heat absorption improves , while heat generation deteriorates it. Electrical conductivity σ (Sc. Nos. 29–32) and Eckert number E (Sc. Nos. 33–36) both show modest negative effects on , implying that increased Joule heating and viscous dissipation suppress concentration transport. Finally, a rising Schmidt number Sc (Sc. Nos. 37–40) significantly boosts , as reduced mass diffusivity steepens the concentration boundary layer. This comprehensive analysis confirms that mass transport is highly sensitive to thermal and nanoparticle interactions, fluid rheology, and magnetic or porous effects.

# CONCLUDING REMARKS

This research explores the dynamics of magnetized Williamson-type nanoparticle-laden fluid flow over a porous surface with extending/shrinking motion in the presence of internal heating, chemical reactions, radiative heat transfer, and nanoparticle effects. Through numerical simulations, the influence of different physical factors on flow speed, temperature distribution, and concentration gradients was systematically investigated. Furthermore, crucial engineering metrics such as frictional drag, heat transfer rate (Nusselt number), and mass transfer rate (Sherwood number) were thoroughly evaluated. Based on the simulation outputs and graphical results, the principal findings are summarized below:

1. Heat Generation Enhances Profiles:

An increase in internal heat generation (Q) significantly amplifies temperature and concentration gradients, signaling more vigorous thermal and material transport near the surface region.

1. Diffusion and Reaction Suppress Concentration:

An enhanced Schmidt number (Sc) and reaction rate parameter (σ) lead to a significant decrease in the concentration distribution, owing to reduced mass diffusivity and increased species consumption.

1. Magnetic Field and Porosity Reduce Skin Friction:

The surface shear stress decreases notably with higher values of the electromagnetic influence (M) and permeability factor (K), indicating less resistance of the liquid near the boundary.

1. Radiation Improves Heat and Mass Transfer:

Thermal radiation parameter (Rd) enhances both the Nusselt number and Sherwood number , indicating increased the rates of heat and mass transfer.

1. Brownian Motion and Thermophoresis Influence Mass Transfer:

The Brownian motion parameter (Nb) increases the Sherwood number, while a higher thermophoresis parameter (Nt) reduces it, highlighting their contrasting effects on nanoparticle diffusion.

# Limitation

Some constraints of this examination include that the considered numerical model is based on idealized conditions and may not accurately reflect real-world variability.

# Date Availability

The complete set of findings obtained or examined in the course of the present research has been included in the released paper.

# Declaration of Competing Interest

The authors state that there are no financial or personal affiliations that may have affected the findings discussed in this paper.

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