Exploring Collatz-Like conjectures: Unveiling Consecutive and Alternate Twin Number Pairs in Sequences following those Conjectures.

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**Abstract.** The realm of mathematics is adorned with intriguing unsolved problems that have perplexed mathematicians for centuries. Among these, the Collatz Conjecture and the Prime Twin Conjecture stand as captivating enigmas, continuing to elude complete understanding. These conjectures have undoubtedly stimulated rigorous academic discourse, resulting in numerous papers, discussions, and research efforts across the mathematical community. This discussion aims to extend the computations of Collatz Conjecture modifying the basic equations involved. The modification thus lead to establishment of new conjectures discovered and unveiling the new twin number patterns occurred. The discussion will briefly lead to the conclusion as numbers sequences have many Hailstone Sequences.

**Keywords**: Collatz Conjecture, Twin Prime Conjecture, Conjectures, Hailstone Sequence.

# 1. Introduction

Mathematics considered to be inevitably important for all sciences holds a lot of mysteries in it. Among these, the Collatz Conjecture [1] and the Prime Twin Conjecture are relevant here, which are yet considered as unsolved question in number theory. This discussion aims to shed light on conjectures similar to them. Collatz like conjectures and the occurrence of consecutive twins and alternate twins amongst the numbers following them are discussed here.

The Collatz Conjecture, also known as the 3x+1 problem [2], revolves around an elementary yet confounding algorithm. Proposed by German mathematician Lothar Collatz in 1937, it posits that given any positive integer n, the following recursive sequence will eventually reach the value 1:

Step 1: If n is even, divide it by 2.

Step 2: If n is odd, multiply it by 3 and add 1. …………………………………. (1)

Iterating this process for new n coming out of above two computations, unleashes a mesmerizing journey of numbers dancing between odd and even values, seemingly wandering towards the elusive destination of 1.

On a parallel course, the Prime Twin Conjecture [3, 4] showing redundant occurrence of twin prime numbers has been a mystery to solve whether the pattern continues till infinity or not. The certainty of occurrence of such twin prime pairs is itself an intriguing mathematical phenomenon.

The field of number theory has been a source of fascination and mystery for mathematicians and enthusiasts. Be it Robert May’s population dynamics equation [5] which plotted as a bifurcation diagram and was studied by Takashi Tsuchiya and Daisuke Yamagishi [6] further lead to discovery of Feigenbaum Constant [7]. Which was later proved by the author in the negative region calling it as decay rate, proving the universality of Feigenbaum Constant in the negative region of growth rate also which is referred there as decay rate may help in understanding the decompositions process of biological materials of even decaying of radioactive waste [8]. The simple variation in the logistic map parameter “r” many times also referred as “μ” for example, Rajvaidya et al. used the logistic map with map parameter “μ” in the coupled map lattices to study the effect of delay[9,10], feedback[11] and asymmetry[10]. The study of numbers has often led to the discovery of intriguing patterns and conjectures that challenge our understanding of mathematical phenomena. The next section describes the extensions Collatz Equation and conjecture with occurrence of new twin number patterns.

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# 2. The Collatz like conjectures

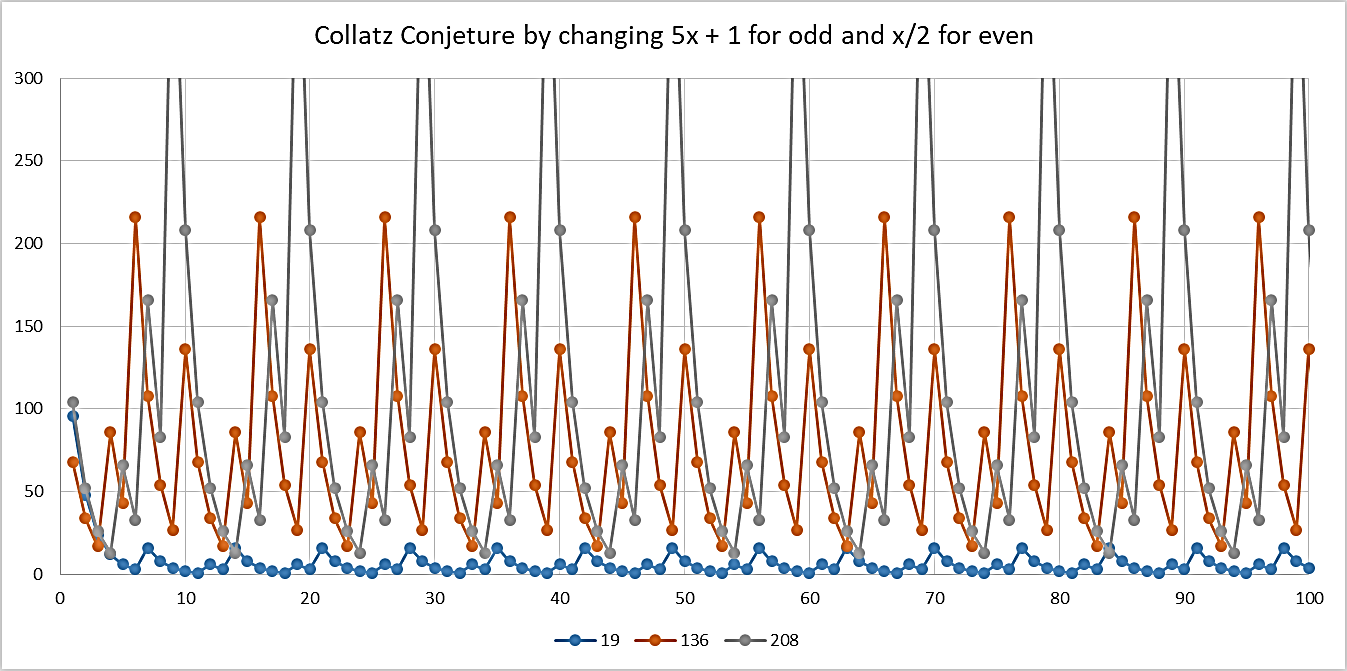
# The Collatz Conjecture, also known as the 3x+1 problem here is modified and studied as 5x+1, 7x+1, 9x+1 and 11x+1 problems. The simple equation of step 2 of Collatz process mention in equation (1) is revisited but with a different number equation which is intended to generate either even or odd number. The intension of 3x+1 problem was also the same, generating even and odd numbers with equal probabilities. Replacing 3 by 5 or 7 or even higher odd, does the same job.

# The main motto here was to study any other conjectures that resembles Hailstone sequence like Collatz. The equation for even number is kept same as x/2, while the odd number equation is tested for Collatz like effect starting form 5x+1 to 11x+1. There are some remarkable results obtained.

# In order to prove the occurrence of any conjecture, this equation 5x+1 studied for consecutive values and then observed that there exists three Hailstone sequences to end with for three different sets of numbers. Unlike Collatz, Not all numbers end at any self-repeating pattern but there are always few which end in three different repeating patterns.

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| TABLE 2.1. Collatz like Conjecture being studied for 5x+1 problem for the numbers showed repetitive occurrence of three different Hailstone Sequences as (16-8-4-2-1-6-3), (216-108-54-27-136-68-34-17-86-43) and (416-208-104-52-26-13-66-33-166-83) | | | | | | | | | |
| Sr No | **x** | **odd/even** | **19** | **x** | **odd/even** | **136** | **x** | **odd/even** | **208** |
| 1 | 19 | TRUE | 96 | 136 | FALSE | 68 | 208 | FALSE | 104 |
| 2 | 96 | FALSE | 48 | 68 | FALSE | 34 | 104 | FALSE | 52 |
| 3 | 48 | FALSE | 24 | 34 | FALSE | 17 | 52 | FALSE | 26 |
| 4 | 24 | FALSE | 12 | 17 | TRUE | 86 | 26 | FALSE | 13 |
| 5 | 12 | FALSE | 6 | 86 | FALSE | 43 | 13 | TRUE | 66 |
| 6 | 6 | FALSE | 3 | 43 | TRUE | 216 | 66 | FALSE | 33 |
| 7 | 3 | TRUE | 16 | 216 | FALSE | 108 | 33 | TRUE | 166 |
| 8 | 16 | FALSE | 8 | 108 | FALSE | 54 | 166 | FALSE | 83 |
| 9 | 8 | FALSE | 4 | 54 | FALSE | 27 | 83 | TRUE | 416 |
| 10 | 4 | FALSE | 2 | 27 | TRUE | 136 | 416 | FALSE | 208 |
| 11 | 2 | FALSE | 1 | 136 | FALSE | 68 | 208 | FALSE | 104 |
| 12 | 1 | TRUE | 6 | 68 | FALSE | 34 | 104 | FALSE | 52 |
| 13 | 6 | FALSE | 3 | 34 | FALSE | 17 | 52 | FALSE | 26 |
| 14 | 3 | TRUE | 16 | 17 | TRUE | 86 | 26 | FALSE | 13 |
| 15 | 16 | FALSE | 8 | 86 | FALSE | 43 | 13 | TRUE | 66 |
| 16 | 8 | FALSE | 4 | 43 | TRUE | 216 | 66 | FALSE | 33 |
| 17 | 4 | FALSE | 2 | 216 | FALSE | 108 | 33 | TRUE | 166 |
| 18 | 2 | FALSE | 1 | 108 | FALSE | 54 | 166 | FALSE | 83 |
| 19 | 1 | TRUE | 6 | 54 | FALSE | 27 | 83 | TRUE | 416 |
| 20 | 6 | FALSE | 3 | 27 | TRUE | 136 | 416 | FALSE | 208 |

# Here some numbers showed repetitive occurrence of three different Hailstone Sequences as (16-8-4-2-1-6-3), (216-108-54-27-136-68-34-17-86-43) and (416-208-104-52-26-13-66-33-166-83). The graphical representation can show the pattern.



**figure 2.1.** Collatz like Conjecture graphically plotted for three numbers 19, 136, and 208. All three showed some repetitive pattern at the end.

# Although it is not like Collatz in which all ended in a repetitive pattern but these numbers keep on occurring in the number sets and may range up to infinity which is to be proved. Few such numbers are shown below.

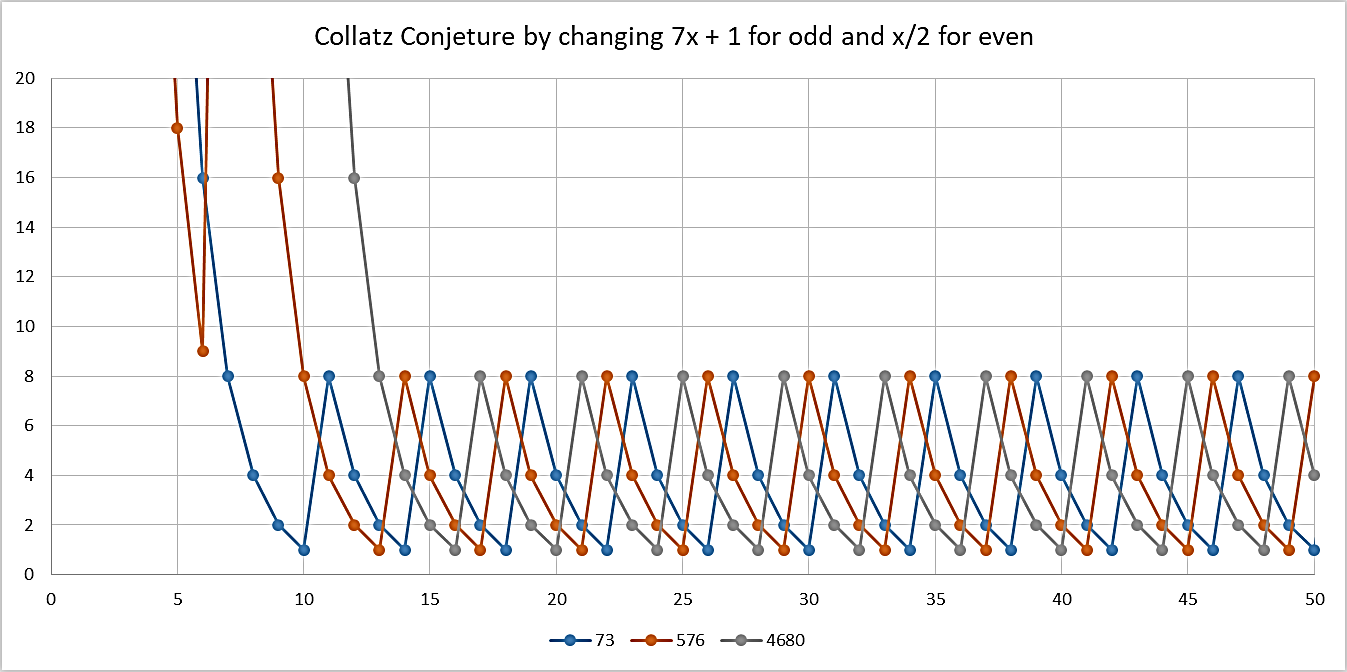
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| TABLE 2.2. Collatz like Conjecture being studied for 5x+1 problem for the numbers showed repetitive occurrence of three different Hailstone Sequences as (16-8-4-2-1-6-3), (216-108-54-27-136-68-34-17-86-43) and (416-208-104-52-26-13-66-33-166-83) | | | | | | | | | |
| *Numbers ending in the Hailstone Sequence (16-8-4-2-1-6-3) between 1 to 500* | | | | | | | | | |
| *1* | **8** | **12** | **15** | **16** | **19** | **24** | **30** | **32** | **38** |
| *48* | **51** | **60** | **64** | **65** | **76** | **96** | **97** | **102** | **120** |
| *128* | **130** | **48** | **51** | **60** | **64** | **65** | **76** | **96** | **97** |
| *102* | **120** | **128** | **130** | **137** | **152** | **155** | **163** | **175** | **192** |
| *194* | **204** | **219** | **243** | **256** | **260** | **274** | **304** | **307** | **310** |
| *326* | **343** | **350** | **384** | **388** | **397** | **408** | **417** | **429** | **438** |
| *480* | **486** | **491** | **…** |  |  |  |  |  |  |
| *Numbers ending in the Hailstone Sequence (216-108-54-27-136-68-34-17-86-43) between 1 to 500* | | | | | | | | | |
| *17* | **27** | **34** | **43** | **54** | **68** | **86** | **108** | **136** | **172** |
| *216* | **272** | **275** | **344** | **432** | **435** | **…** |  |  |  |
| *Numbers ending in the Hailstone Sequence (416-208-104-52-26-13-66-33-166-83) between 1 to 500* | | | | | | | | | |
| *5* | **10** | **13** | **20** | **26** | **33** | **40** | **52** | **66** | **80** |
| *83* | **104** | **132** | **160** | **166** | **181** | **185** | **208** | **211** | **245** |
| *264* | **320** | **332** | **362** | **370** | **416** | **422** | **453** | **457** | **463** |
| *490* | **…** |  |  |  |  |  |  |  |  |

However the interesting study is for number sequence coming after 7x+1 equation which is given in details next.

# 3. The 7x+1 Problem and Consecutive and Alternate Twin Patterns

# The Collatz like Conjecture 7x+1 problem is studied for occurrence of any repetitive pattern and plotted for a large data set ranging from 1 to 1010 numbers. The remarkable property is in this conjecture also few numbers showed repetitive pattern ending not in three but only one conjecture as (8-4-2-1).

|  |  |  |  |  |  |  |  |  |  |
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| TABLE 3.1. Collatz like Conjecture being studied for 7x+1 problem for the numbers showed repetitive occurrence of only one Hailstone Sequences as (8-4-2-1). | | | | | | | | | |
| Sr No | **x** | **odd/even** | **73** | **x** | **odd/even** | **576** | **x** | **odd/even** | **4680** |
| 1 | 73 | TRUE | 512 | 576 | FALSE | 288 | 4680 | FALSE | 2340 |
| 2 | 512 | FALSE | 256 | 288 | FALSE | 144 | 2340 | FALSE | 1170 |
| 3 | 256 | FALSE | 128 | 144 | FALSE | 72 | 1170 | FALSE | 585 |
| 4 | 128 | FALSE | 64 | 72 | FALSE | 36 | 585 | TRUE | 4096 |
| 5 | 64 | FALSE | 32 | 36 | FALSE | 18 | 4096 | FALSE | 2048 |
| 6 | 32 | FALSE | 16 | 18 | FALSE | 9 | 2048 | FALSE | 1024 |
| 7 | 16 | FALSE | 8 | 9 | TRUE | 64 | 1024 | FALSE | 512 |
| 8 | 8 | FALSE | 4 | 64 | FALSE | 32 | 512 | FALSE | 256 |
| 9 | 4 | FALSE | 2 | 32 | FALSE | 16 | 256 | FALSE | 128 |
| 10 | 2 | FALSE | 1 | 16 | FALSE | 8 | 128 | FALSE | 64 |
| 11 | 1 | TRUE | 8 | 8 | FALSE | 4 | 64 | FALSE | 32 |
| 12 | 8 | FALSE | 4 | 4 | FALSE | 2 | 32 | FALSE | 16 |
| 13 | 4 | FALSE | 2 | 2 | FALSE | 1 | 16 | FALSE | 8 |
| 14 | 2 | FALSE | 1 | 1 | TRUE | 8 | 8 | FALSE | 4 |
| 15 | 1 | TRUE | 8 | 8 | FALSE | 4 | 4 | FALSE | 2 |
| 16 | 8 | FALSE | 4 | 4 | FALSE | 2 | 2 | FALSE | 1 |



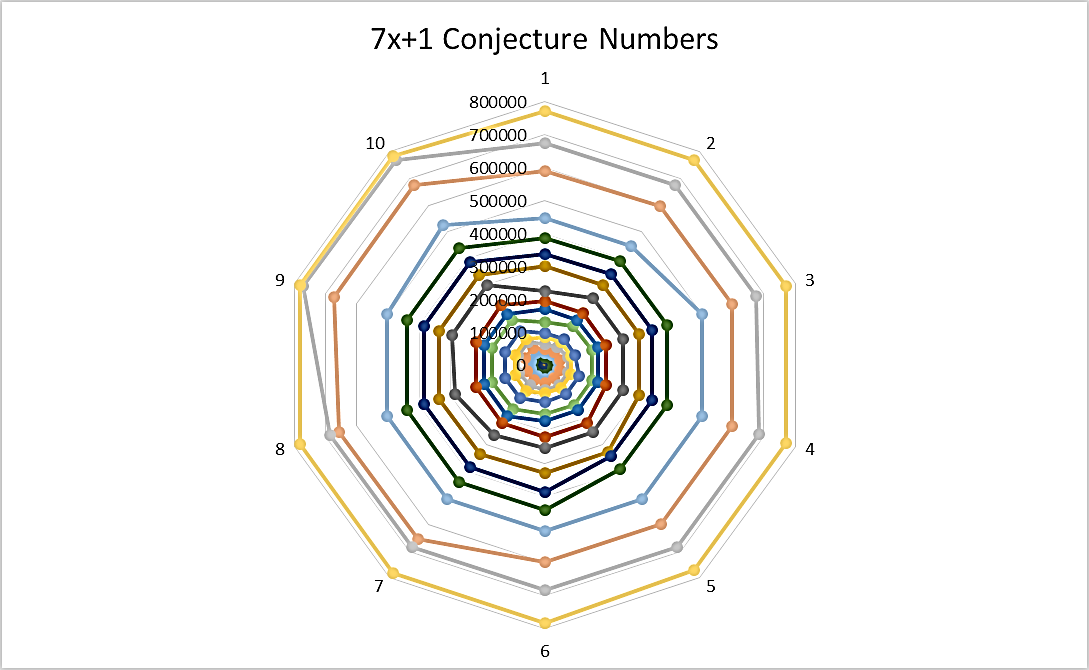
**figure 3.1.** Collatz like Conjecture graphically plotted for three numbers 73, 576, and 4680. All three showed same repetitive pattern (8-4-2-1) at the end.

Since this problem showed promising results of having a same conjecture being attained by many numbers. This problem is studied for a huge set of numbers. The exhaustive computations were done to find out how many and which numbers follow this conjecture. And the numbers showing this pattern are turned out to be remarkable. As they showed appearance in a particular pattern.

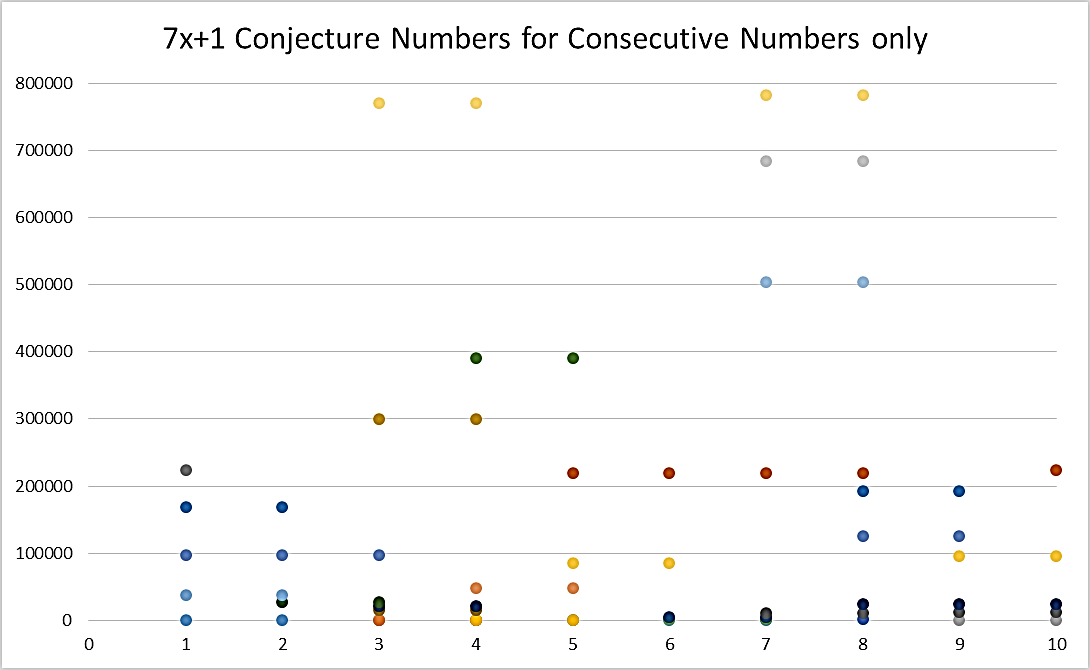
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| TABLE 3.2. Collatz like Conjecture: 7x+1 problem numbers between 1 and 108 showing Hailstone Sequences as (8-4-2-1). The pairs are highlighted for consecutive by gray cells and alternate by yellow cells. | | | | | | | | | |
| 1 | 2 | 4 | 5 | 8 | 9 | 10 | 16 | 18 | 20 |
| 32 | 36 | 40 | 41 | 64 | 73 | 80 | 82 | 128 | 144 |
| 146 | 160 | 164 | 167 | 256 | 288 | 292 | 320 | 328 | 329 |
| 334 | 512 | 576 | 584 | 585 | 640 | 656 | 658 | 668 | 1024 |
| 1152 | 1168 | 1170 | 1280 | 1312 | 1316 | 1336 | 1337 | 1965 | 2048 |
| 2304 | 2336 | 2340 | 2560 | 2624 | 2632 | 2633 | 2672 | 2674 | 3009 |
| 3439 | 3930 | 4096 | 4608 | 4672 | 4680 | 4681 | 5120 | 5248 | 5264 |
| 5266 | 5344 | 5348 | 6018 | 6878 | 7860 | 8192 | 9216 | 9344 | 9360 |
| 9362 | 10240 | 10496 | 10528 | 10532 | 10688 | 10696 | 10697 | 12036 | 12037 |
| 12225 | 13756 | 15720 | 15721 | 16384 | 18432 | 18688 | 18720 | 18724 | 20480 |
| 20992 | 21056 | 21064 | 21065 | 21376 | 21392 | 21394 | 24072 | 24073 | 24074 |
| 24450 | 27512 | 27513 | 27943 | 31440 | 31442 | 32768 | 36864 | 37376 | 37440 |
| 37448 | 37449 | 40960 | 41984 | 42112 | 42128 | 42130 | 42752 | 42784 | 42788 |
| 48144 | 48146 | 48148 | 48900 | 48901 | 55024 | 55026 | 55886 | 62880 | 62884 |
| 62887 | 65536 | 73728 | 74752 | 74880 | 74896 | 74898 | 81920 | 83968 | 84224 |
| 84256 | 84260 | 85504 | 85568 | 85576 | 85577 | 96288 | 96292 | 96296 | 96297 |
| 97800 | 97801 | 97802 | 110048 | 110052 | 111772 | 125760 | 125768 | 125769 | 125774 |
| 131072 | 147456 | 149504 | 149760 | 149792 | 149796 | 163840 | 167936 | 168448 | 168512 |
| 168520 | 168521 | 171008 | 171136 | 171152 | 171154 | 192576 | 192584 | 192585 | 192592 |
| 192594 | 195600 | 195602 | 195604 | 220096 | 220097 | 220104 | 220105 | 220109 | 223544 |
| 223545 | 251520 | 251536 | 251538 | 251548 | 251553 | 262144 | 287489 | 294912 | 299008 |
| 299520 | 299584 | 299592 | 299593 | 327680 | 328559 | 335872 | 336896 | 337024 | 337040 |
| 337042 | 342016 | 342272 | 342304 | 342308 | 385152 | 385168 | 385170 | 385184 | 385188 |
| 385191 | 391200 | 391204 | 391208 | 391209 | 440192 | 440194 | 440208 | 440210 | 440218 |
| 447088 | 447090 | 503040 | 503072 | 503076 | 503079 | 503096 | 503097 | 503106 | 524288 |
| 589824 | 598016 | 599040 | 599168 | 599184 | 599186 | 655360 | 657118 | 671744 | 673792 |
| 674048 | 674080 | 674084 | 684032 | 684544 | 684608 | 684616 | 684617 | 770304 | 770336 |
| 770340 | 770368 | 770376 | 770377 | 770382 | 782400 | 782408 | 782409 | 782416 | 782418 |
| 880384 | 880388 | 880389 | 880416 | 880420 | 880436 | 894176 | 894180 | …. | 1006080 |
| 1006144 | 1006152 | 1006153 | 1006158 | 1006192 | 1006294 | 1006212 | 1006213 | … | … |

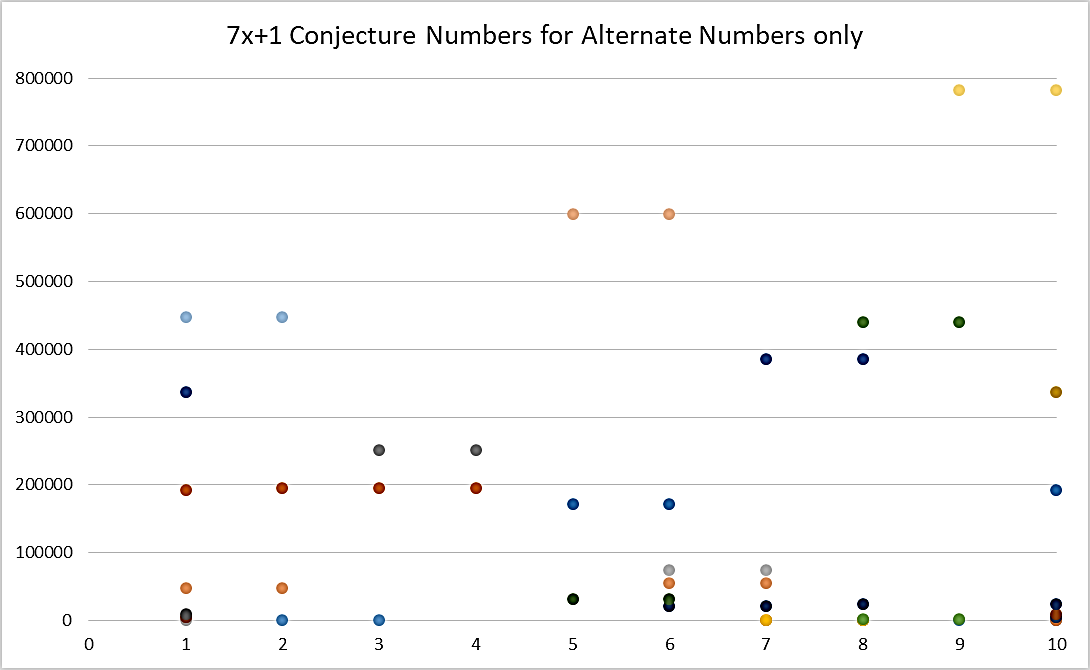
# 4. Results and Discussion and Conclusions

The intriguing nature of mathematics has led to another conjecture 8-4-2-1. Although unlike Collatz, all natural integers do not follow it but there are several which follow and also follow in a pattern which we called consecutive twin pattern and alternate twin pattern here. Plotting these conjecture numbers make a cob web structure as shown below which reminds about Fractals in nature. It can be a mathematical fractal of conjecture numbers appearing at all orders.



**figure 4.1.** Collatz like Conjecture 7x+1 problem graphically plotted for numbers ranging from 1 to 1000000 showing nested cobweb loops

Also if the Twin patterns are plotted separately for consecutive twins and alternate twins then the plots shows below structure on a scattered plots.

**figure 4.2.** Collatz like Conjecture 7x+1 problem scattered plotted for Consecutive Twin numbers only, occurring in between 1 to 1000000

**figure 4.2.** Collatz like Conjecture 7x+1 problem scattered plotted for Alternate Twin numbers only, occurring in between 1 to 1000000

In conclusion, the Collatz Conjecture and the Prime Twin Conjecture stand as testament to the enduring allure as well as tenacious intrigue of unsolved problems in mathematics. These conjectures, imbued with simplicity with shrouded in complexity, have left an indelible mark on the mathematical landscape. Their enigmatic nature continues to inspire meticulous investigations, computational explorations, and theoretical conjectures, underscoring the perpetual quest for understanding within the mathematical community. As these conjectures persist in eluding definitive resolution, they echo the ceaseless pursuit of knowledge and the unyielding fascination with the mysteries that permeate the realm of mathematics. But this is also the fact that they are not the only conjectures ever could exists. Mathematicians have tried their hands on in creating and identifying many such equations leading to intriguing results. This might be one way of solving the original problem.

The foundational principles that underpin the Collatz conjecture – the iterative transformation of integers based on specific rules – serve as a guiding framework for understanding the nature of these new conjectures. By delving into the similarities and divergences between the Collatz sequence and the newly uncovered patterns, researchers are gaining deeper insights into the underlying mechanisms governing the evolution of integers through iterative processes. Moreover, the examination of twin number patterns within these conjectures has revealed a captivating interplay with the inherent properties of the integers involved, shedding new light on the multifaceted nature of number theory.

Startled by the Collatz conjecture, Paul Erdos said "Mathematics is not ripe enough for such problems”. It is true to the same dimension as Ocean is not explored completely or all galaxies are not yet discovered. However, the discovery of these new conjectures and their associated twin number patterns holds profound significance for the field of number theory. The observation of recurring structures within the sequences generated by these conjectures has the potential to offer valuable insights into the inherent properties and tendencies of integers. The numbers are continuous and the identification of twin number patterns within these conjectures extends the exploration of twin primes, adding a new dimension to the ongoing pursuit of understanding the distribution and properties of prime numbers. The implications of these findings reverberate across the mathematical landscape, providing impetus for further investigations into the interconnections between conjectures, number sequences, and the prevalence of twin number patterns within large sets of integers.

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