**Formulation and Pareto Optimal Solutions of a Multi-Objective Decision-Making Optimization Model with Applications in Healthcare**

Anuradha Sahoo1, a), A. P. Maharana1, b)

*Department of Mathematics,*

*Siksha ‘O’ Anusandhan (Deemed to be University),*

*Bhubaneswar, 751030, Odisha, India.*

*akankshyamaharana05@gmail.com*

*\*Corresponding Author: anuradha25anu@gmail.com*

**Abstract:** Efficient decision-making in healthcare is critical for ensuring timely as well as cost-effective treatment, particularly when dealing with limited resources and diverse patient needs. This paper presents a general formulation of a decision-making multi-objective linear optimization model targeted at improving healthcare service delivery. The model simultaneously minimizes patient waiting time and the overall treatment cost while adhering to constraints on total available treatment time and a minimum budget. Further, a restriction that minimum number of each patient must be treated according to hospital policy. A main feature of the model is enabling specific resource allocation without sacrificing system efficiency. The proposed model shows its potential. It can improve operational outcomes and patient satisfaction in constrained environments. The resulting multi-objective model is solved by the weighted sum technique and then LINGO software is used to obtain optimal solutions.

**Keywords:** Decision-making Problem; Multi-objective Optimization; Weighted Sum Method; Pareto Optimal Solution.

1. **INTRODUCTION**

Multi-objective optimization deals with problems that have more than one goal to achieve at the same time. Instead of finding a single best answer, it tries to balance between different objectives, like minimizing cost and minimizing waiting time together. It helps decision-makers choose the most suitable solution based on their priorities. Hospitals are frequently struggling to treat a large number of patients with limited funds, time, or resources. Effective resource management is crucial because some patients require urgent care (critical), while others can wait (non-critical). A mathematical model is presented in this paper for helping hospitals in making better decisions. The model points out: shortening the wait time for patients, reducing the expense of treatment, within the constraints of time, money and minimum number of patient’s treatment with respect to granting essential patients’ priority. The model helps in balancing these objectives and improving healthcare services through the use of a technique known as multi-objective linear optimization. To demonstrate how the concept might be applied in real-world situations, an example is also provided.

Zhang et al. [1] developed a multi-objective optimization framework to improve hospital efficiency by addressing Length of Stay (LOS) and physician assignment, enhancing patient satisfaction. Using simulation models and decision guidelines, the study provides tailored improvement strategies for hospitals of different sizes. Bastian et al. [2] applied a genetic algorithm-based multi-objective optimization (MOO) approach to optimize the location of new healthcare facilities in Hong Kong. By balancing accessibility, coverage, and cost, the study provides a set of Pareto solutions to help urban planners make informed decisions.

Kazimieras Zavadskas et al. [3] proposed on multi-objective and multi-attribute decision-making methods in both certain and uncertain environments. These approaches are applied to real-world problems in areas like construction, transportation, and sustainable development. Sheikhalishahi et al. [4] explored multi-criteria decision-making (MCDM) techniques in healthcare to address complex, multi-dimensional challenges. A review of 140 articles (2013–2022) highlights key application areas, popular methods like the analytic hierarchy process, and trends in research, aiding healthcare professionals in effective decision-making.

Habib et al. [5] introduced a multi-objective nurse scheduling model that incorporates teamwork and decision-making styles to enhance job satisfaction. Using goal programming, the model optimizes team reliability, cost, and scheduling constraints, demonstrating its effectiveness through a real case study.

Torkayesh et al. [6] explained that Healthcare Waste Management (HWM) involves complex decisions with economic, environmental, and social impacts. This study proposes a multi-objective model using Improved Multi-Choice Goal Programming to reduce costs and risks while boosting job creation. Ghaderian et al. [7] explored how building energy use is a major global concern, and simulation-based optimization helps improve performance but is computationally intensive. This study combines surrogate models with a multi-objective evolutionary algorithm to optimize energy use and thermal comfort, achieving significant energy savings in a real case study. Verma et al. [8] introduced the use of NSGA-II in solving various combinatorial optimization problems, categorizing its applications into conventional, modified, and hybrid versions. It also analyses performance metrics, benchmarking methods, and includes a brief bibliometric analysis. Y li et al. [9] addressed the home health care routing and scheduling problem under uncertain travel and service times, using a fuzzy multi-objective model to balance service priority and cost. A discrete multi-objective grey wolf optimizer (DMOGWO) is proposed, showing strong performance compared to other optimization methods.

Tanantong et al. [10] highlighted that long wait times in public hospitals, especially in front-end departments like triage and medical records, reduce patient satisfaction due to inefficient resource management. Research at Thammasat University Hospital used simulation and fuzzy multi-objective optimization to balance operating costs and patient satisfaction. The study provided decision-making guidelines to improve hospital resource management and reduce bottlenecks. Chakraborty et al. [11] explored the use of biological immune system mechanisms in intelligent technology, applying multi-objective optimization to VR image segmentation for improved stability and accuracy. A study introduced a feature extraction method and a parallel search strategy to enhance segmentation performance. Results showed higher prediction accuracy and robustness compared to previous methods, making decision-making more reliable. Chemkomnerd et al. [12] explored multi-objective evolutionary algorithms (MOEA) for feature selection in disease prediction, improving classification accuracy in medical datasets. Research shows that the multi objective particle swarm optimization (MOPSO) method outperforms other MOEAs like NSGA-II and MOEA/D in optimizing accuracy with fewer features. Shen et al. [13] presented a bi-level optimization model for safe and efficient medical waste transport under uncertainty, considering costs, risks, and time constraints. Using simulated annealing and an improved genetic algorithm, the model outperforms standard methods and highlights the importance of travel and loading reliability.

We were inspired to study on this subject with the goal to help hospitals in better resource allocation decisions. We can enhance patient care and overall hospital efficiency by developing a model that prioritises reducing waiting times and expense while maintaining to time and financial constraints. In many hospitals patients have to wait a long time for care, because there are not sufficient staff members, money, or time. Urgent care is frequently required for essential patients, but it can be challenging to manage them with non-critical situations. We were inspired to study on this subject in order to assist hospitals in better resource allocation decisions. We can improve patient care and overall hospital efficiency by developing a model that focusses reducing waiting times and expenses while keeping to time, financial constraints and minimum number of each patient’s treatment

The structure of this paper is as follows: an introduction and a summary of the key references are provided in Section 1. In Section 2, the complete formulation of the problem is considered. In Section 3, the methodology is discussed. Section 4 presents a numerical example to demonstrate the proposed model. In section 5, we analysed the result. Finally, a brief conclusion is stated in Section 6.

1. **MODEL FORMULATION**

In this section, we propose a decision-making multi-objective model with application in healthcare. In healthcare decision-making, it is common to deal with more than one goal at the same time. In this model, we focus on two main objectives: minimizing the total waiting time and minimizing the total cost of treatment. Minimizing waiting time helps patients receive care faster, especially those in critical condition, while minimizing cost ensures that hospitals stay within budget and use their resources efficiently. However, these goals can sometimes conflict. For example, reducing waiting time may lead to higher costs.

Hospitals and other healthcare facilities can use this model to improve patient and resource management. Critical patients can receive treatment more quickly and possibly save lives if waiting periods are reduced. At the same time, hospitals operate more efficiently when treatment costs are kept reasonable and within budget. In real life, patients receive care more quickly and efficiently, more patients can be treated at hospitals without wasting money, better planning is possible for healthcare systems, particularly in times of crisis or high demand. All things considered, this model encourages better healthcare decision-making, which enhances patient outcomes and makes better use of limited resources.

Now we formulate a model corresponding to a real-life situation. Suppose a hospital in the USA wants to minimize/reduce the waiting time i.e. critical and non-critical patients wait to receive treatment, and also reduce the overall cost of treating critical and non-critical patients. To formulate the model, the following notations are used.

**Notations:**

: Number of critical patients treated.

: Number of non-critical patients treated.

: Waiting time per critical patient (Hours).

: Waiting time per non-critical patient (Hours).

: Cost per critical patient.

: Cost per non-critical patient.

: Time limit for critical patient.

: Time limit for non-critical patient.

: Total time limit for both critical and non-critical patients.

: Budget limit for critical patient.

: Budget limit for non-critical patient.

: Total budget limit for both critical and non-critical patient.

: Minimum number of critical patients treated.

: Minimum number of non-critical patients treated.

Now, the mathematical form for two objective functions to minimize the waiting time and cost for critical and non-critical patients are as follows.

**Objective Function:** Min, Min 

Also, the maximum time and minimum budget constraints are as follows.

**Constraints:** 



Again, according to the hospital policy at least  and  number of critical and non-critical patients must be treated respectively. So, the corresponding constraints are given as follows.





The objective of the decision-maker is to minimize the waiting times and overall treatment cost for both critical and non-critical patients, while considering limitations in time and budget. Also, minimum number of patients must be treated. So, the Decision-making Multi-objective Programming Problem (DMMP) model is given as follows.

**DMMP (1):** Min, Min 

Subject to 







In the following section, a complete technique is discussed to solve the DMMP (1) model and the corresponding optimal solutions are obtained.

**3. METHODOLOGY**

**3.1. Weighted Sum Method:**

A common technique for solving multi-objective optimization problems is the Weighted Sum Method. It can be difficult to find a single optimal solution which fulfils all of our goals equally when we have multiple objectives such as reducing healthcare costs and waiting times. We assign each objective a weight according to its significance when using the weighted sum method. By assigning different weights to each objective, the model can provide solutions that reflect the hospital’s priorities and help in better decision-making. These weights must sum up to one and are often values between 0 and 1. By multiplying each target by its weight and putting them all together, the goal is to create a single objective function.

In this paper, the weighted sum method is used to balance objectives like minimizing waiting time, minimizing cost, and staying within time and budget constraints in healthcare decision-making. By assigning weights, the DMMP (1) is converted to a Decision-making Single-objective Programming Problem (DMSP) model.

**DMSP (2):** Min 

Subject to 







Now, the above DMSP (2)can be solvedby using any linear programming techniques. Here, we used the LINGO software to obtain the optimal solutions.

**4. NUMERICAL EXAMPLE**

In this section, we introduce a numerical example to examine the applicability of the model under the consideration of the following parameters.

Suppose a hospital in the USA aims to optimize its treatment strategy by minimizing the total waiting time and overall treatment cost for two types of patients: critical and non-critical. Each critical patient contributes 2 hours to waiting time and costs $400 to treat, while each non-critical patient contributes 4 hours to waiting time and costs $250. Treatment of a critical patient requires 3 hours, and a non-critical patient requires 1 hour. Budget limit for critical and non-critical patient is 400$ and 250$ respectively. The hospital faces resource constraints: a maximum of 200 total treatment hours and a minimum of budget $15,000. The minimum number of each patient’s treatment is 5 and 10 respectively.

Let the DMMP (1) model corresponding to the above data be given by DMMP (3).

**DMMP (3):** 

Subject to 







Now, by using weighted sum technique to DMMP (3), we obtain the following model.

**DMSP (4):** Min 

Subject to 







The following Table 4.1 shows the optimal solutions for six different weights obtained by using LINGO software.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| 0 | 1 | 5 | 52 | 218 | 15000 | 15000 |
| 0.2 | 0.8 | 31 | 10 | 102 | 14900 | 12020.5 |
| 0.3 | 0.7 | 31 | 10 | 102 | 14900 | 10530 |
| 0.5 | 0.5 | 31 | 10 | 102 | 14900 | 7551.5 |
| 0.8 | 0.2 | 31 | 10 | 102 | 14900 | 3082 |
| 1 | 0 | 31 | 10 | 102 | 14900 | 102 |

Table 4.1

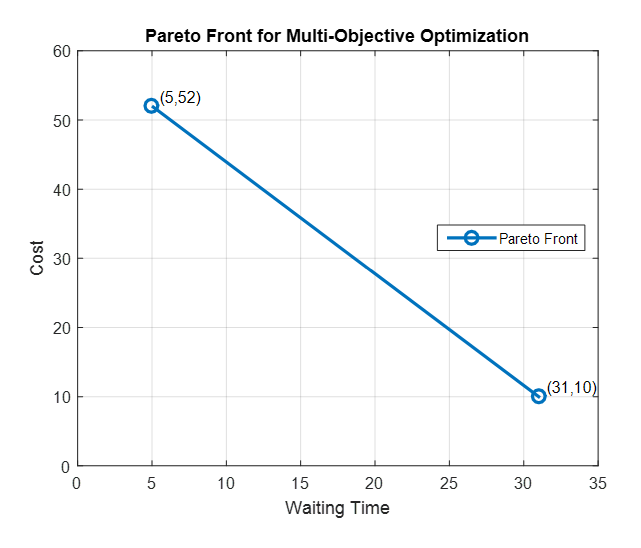
**5. RESULT ANALYSIS**

The model was tested using six different sets of weights for waiting time and cost: (0,1), (0.2, 0.8), (0.3, 0.7), (0.5, 0.5), (0.8, 0.2) and (1, 0). For the weight combination (0,1), where full priority was given to minimizing waiting time, the optimal solution obtained was 

However, for the other five weight combinations, the optimal solution remained the same with . This shows that when even a small importance is given to minimizing cost (other than zero), the model prioritizes a different selection. It also indicates that the model is sensitive to extreme weight changes and stable when there is a balance between cost and waiting time.

Thus, the model effectively adapts to different priorities and provides consistent solutions when reasonable importance is given to both objectives.

The Pareto graph is presented in Figure 1.



**Fig.1**

**6. CONCLUSION**

This study presents a multi-objective linear optimization model to support decision-making in healthcare, focusing on minimizing patient waiting time and treatment cost while staying within time and budget limits. The model also considers the priority of critical patients to ensure fair and effective treatment planning. By using the weighted sum method, we combined both objectives and tested the model with different weight combinations. The results showed that the same optimal solution was achieved in all cases, indicating that the solution is Pareto optimal and the model is stable and reliable. Even if decision-makers change how much they care about waiting time or cost, the hospital's best treatment plan stays the same.

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