**A Comparative Study of Picard’s Method and Differential Transform Method to Solve Ordinary Simultaneous Differential Equations of First and Second Order**

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**Abstract**: In this work, we derived Picard’s successive approximation technique and proposed the Differential Transform Method (DTM) for solving systems of differential equations. Both methods produced similar outcomes, indicating their efficiency and applicability.

**INTRODUCTION**

**The differential transformation method is a numerical method which applies a Taylor expansion. The method develops an analytical representation expressed in a polynomial. Zhou [14], first introduced and employed the concept of the differential transform method as a means of solving linear and nonlinear initial value problems when analysing electric circuits. Demirovski et al. called this method by Chen and Liu to solve problems with two boundaries [1].The two-dimensional differential transform method was used when tackling partial difference equations by Jang, Chen and Liu among other researchers [9]. Yu and Chen concentrated on the differentiation transformation operation in the optimisation of rectangular fins with varying thermal parameters [5]. Unlike conventional high-order Taylor series approach, where the solution is obtained through lengthy symbolic manipulation, a differential transform approach is an iterative procedure that serves to obtain Taylor series solutions. This method is effective and does not need numerous computer time when used on nonlinear or parameter-varying systems. It finds a solution of analysis in the form of a polynomial. Nevertheless, it is unlike the Taylor series method because one does not have to calculate any high order derivatives. Differential transform method operates as an iterative procedure of transformed equations of original functions in order to solve the differential equations.**

**A promising semi-analytical method called the Differential Transform Method (DTM) can be applied to solve differential equations and also to simultaneous systems of first-order ordinary differential equations (ODEs). It builds algebraic recurrence relations out of differential equations, and so it is relatively easy to program and quite fast in most applications. DTM is a variant of the Taylor series expansion; however, in contrast to the complete Taylor series, the coefficients are computed recursively, as a transformation rule, and do not need the symbolic derivatives.**

**A classical numerical way to find approximate solutions of ordinary differential equations (ODEs) is the method of Picard, also called Picard iterative method or Picard successive approximations. It is especially applied in situations where analytical solutions cannot be made or performed with difficulty. Although first formulated to solve single first-order ODE, Picard method can be generalized to systems of competing first order differential equations. This successive method is founded on the integral form of the differential equation and functions under continuity and Lipschitz continuity assumption.**

**The current paper will analyse three systems of ordinary differential equations by using the technique of the differential transformation. Under this method, one has either a closed-form series solution or an approximate solution. These computed values are then compared with the accurate solutions computed by Picard numerical method.**

**BASIC DEFINITIONS**

As in Refs. [6, 7, 2, 3, 8], the basic definition of the differential transformation are introduced as follows:

**Definition 1:** The one-dimensional differential transformation of the kth-order derivative of function  is defined as follows:

 (1)

Where  is analytic and differentiated continuously with respect to in the domain of interest and is the transformed function, which is called the T-function in brief""""".

"""**Definition 2**""**:** ""The differential inverse transform of  is defined as follows""":

" "" (2)

"""When then (1) and (2) can be written as":

"**Definition 3:** The one-dimensional differential transform of function  is defined as follows:"

"  " (3)

"where  is the original function and is the transformed function, which is called the -function".

"**Definition 4:** The differential inverse transform of  is defined as follows":

 (4)

"Substituting (3) into (4) we have"

 " " (5)

"In real applications, the function by a finite series of (4) can be written as"

 (6)

and (4) implies that



""is neglected as it is small. Usually, the values of n are decided by a convergency of the series coefficients"""".

Some of the basic operations of differential transformation method.

**Theorem 1**: If then  (7)

**Theorem 2**: If  then  (8)

Where is constant.

**Theorem 3:** If then  (9)

**Theorem 4**: If  then

 (10)

**Theorem 5**: If  then  (11)

**Theorem 6:** If then  (12)

""**Theorem 7**: ""If  then""  ""

""**Theorem 8**"": ""If  then"" 

""**Theorem 9**"":"" If then"" ""

**NUMERICAL EXPERIMENTS**

**Example 1:**

Consider the following first order liner differential system:

 and (13)



With initial conditions  (14)

Taking the differential transform method to equations (13) and (14), we obtain

 (15)



**TABLE 1**

"Differential transformation values of Example 1", "for 

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 0 | 1 | 3/2 | -3/2 |
| 1 | 2 | -3/4 | 3 |
| 2 | 3 | 4 | -13/4 |
| 3 | 4 | -87/200 | 55/16 |
| 4 | 5 | 151/80 | -51/10 |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |

Rearranging the equations (15)



 (16)

With initial conditions"" (17)

The numerical results for the differential transform method are presented in Table 1.

From (16) the approximate solution when  is given by ""

 and (18)

 (19)

Substituting the values of  and  into equations (18) and (19), we obtain 6





Using Picard’s method, the solution of this example is the form

 and



The numerical solutions are presented in the figure 1.



**FIGURE 1.** Numerical solutions of differential transform method and exact solutions when and .

**Example 2:**

Consider the following first order linear differential system:

 and

 (20)

With initial conditions . (21)

Taking the differential transform method to equations (20) and (21), we obtain



 (22)

Rearranging the equations (22)



 (23)

With initial conditions  (24)

The numerical results for the differential transform method are presented in Table 2.

**TABLE 2**

Differential transformation values of Example 2, for 

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 0 | 1 | 8 | 1 |
| 1 | 2 | 0 | -3 |
| 2 | 3 | 4/3 | 1/2 |
| 3 | 4 | 0 | -1/3 |
| 4 | 5 | 0 | 1/24 |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |

From (6) the approximate solution when  is given by

 and (25)

 (26)

Substituting the values of and  into equations (25) and (26), we obtain





Using Picard’s method, the solution of this example is the form





The numerical solutions are presented in the figure 2.



**FIGURE 2.** Numerical solutions of differential transform method and exact solutions when  and .

**EXAMPLE 3:**

Consider the following non-homogenous differential equations

 and

 (27)

With initial conditions . (28)

Taking the differential transform method to equations (27) and (28), we obtain

 (29)  (30)

Rearranging the equations (29) and (30)



 (31)

With initial conditions . (32)

The numerical results for the differential transform method are presented in Table 2.

From (6) the approximate solution when  is given by

 and (33)

 (34)

**TABLE 3**

Differential transformation values of Example 1, for 

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 0 | 1 | 0 | 0 |
| 1 | 2 | 0 | 1/2 |
| 2 | 3 | 1/6 | 0 |
| 3 | 4 | 0 | -1/24 |
| 4 | 5 | -1/120 | 0 |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Substituting the values of and  into equations (33) and (34), we obtain





Using Picard’s method, the solution of this example is of the form





The numerical solutions are presented in the figure 3.



**FIGURE 3.** Numerical solutions of differential transform method and exact solutions when  and .

**CONCLUSION AND DISCUSSION**

Researchers have used the differential transform technique on one dimensional linear and non-linear ordinary differential equations systems. Solutions take a form of Taylor series with all calculations being simple. The findings are rather dependable. The promise of this approach has been confirmed with numerical examples and this means that future research towards this direction will be worth undertaking.

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