**Solitary Waves in Anisotropic Magnetized Plasma with Trapped Electrons and Positrons**

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**Abstract:** The present research focuses on exploring the propagation of solitary waves (SWs) in a magnetized plasma with anisotropic ion pressure. This plasma consists of positive ions & negative ions (PINI), as well as trapped electrons and positrons (EP). The electrostatic potential is obtained using the reductive perturbation technique, resulting in a three-dimensional trapped Zakharov-Kuznetsov (TZK) type equation. We are specifically investigating the impact of EP trapping, as well as the concentrations of EP and negative ions. In addition, the effect of anisotropic pressure and a strong magnetic field on SWs for both PINI is briefly examined. The results of the present investigation could help explore nonlinear wave propagation in laboratory as well as space and astrophysical plasma systems where trapping exists in magnetized anisotropic plasmas.

**INTRODUCTION**

Over the last few decades, numerous researchers have investigated the nonlinear wave dynamics in anisotropic plasmas. In a collisionless medium, this anisotropic plasma behaviour is primarily due to strong magnetic fields. i.e. the various typical properties of the plasma have distinct values along the parallel and perpendicular according to the magnetic field [1-2]. Therefore, when analyzing the influence of ions pressure , it is essential to account for both the parallel and perpendicular components of ion pressure. This directional dependence, or anisotropy, in plasmas is effectively characterized by the Chew, Goldberger, and Low in 1956 and popularly known by Chew-Goldberger-Low (CGL) theory [3]. This concept is valid if there is no coupling between the parallel and perpendicular degrees of freedom [4]. To investigate anisotropies in plasma, separate momentum equations are required to determine the ion pressure components parallel  and perpendicular  to the magnetic field. When a strong correlation exists between these pressure components, the plasma tends to behave isotopically [5]. In space plasmas, anisotropy may develop due to plasma convection, which causes the magnetic field to compress or expand along its own direction. In recent years, numerous studies have explored pressure-anisotropic plasmas across various astrophysical settings and laboratory conditions. These investigations have primarily examined plasmas influenced by particle trapping, external magnetic fields, containing both PINIs, as well as dispersive and dissipative effects [6-11].

Particles in a plasma system are considered trapped when they remain confined within a certain region, following restricted paths during their oscillatory motion [12]. The idea of the trapping effect in plasma was first introduced by Bernstein et al. [13]. In a PINIs plasma containing trapped electrons, the nonlinear propagation of ion-acoustic solitary waves (IASWs) and ion-acoustic shock waves (IAShWs) is investigated by El-Monier S. Y. et al [14]. They noticed the influence of various plasma characteristics on the IASWs and IAShWs profiles by varying several parameters. In a dusty plasma, the profile of SWs and Shock Waves (ShWs) in a pair of trapped ions is studied by Adhikary N. C. et al [15]. The effect of positive to negative ion density ratios, dust thermal pressure and ion temperature on the profile of both types of waves is investigated. A three-dimensional ZK equation is derived in a study of ion acoustic waves (IAWs) in a quantized magnetized degenerate plasma with trapped electrons by Jahangir R. et al. [16]. Numerically analyses the characteristics of different parameters, and also discusses the stability of the ZK equation. Hamid N. et al [17] investigated the effect of electron trapping in an electron-ion (EI) dispersive and dissipative plasma. In an ion beam magnetized plasma containing relativistic degenerate EPs, the characteristics of SWs are investigated by Soltani H. et al [18].  Shah H. A. et al. [19] examine the impact of trapping in a degenerate quantum plasma that includes nondegenerate ions and degenerate electrons. They found SW structures when the electrons are fully degenerate, while in weakly nondegenerate plasma, both rarefactive and compressive SW structures appear with temperature changes. Zedan N. A. et al. [20] examined IASWs in a collisionless magnetized plasma that includes a degenerate pair of ions and trapped electrons within a quantizing magnetic field.

The nonlinear IASWs in a collisional plasma consisting of PINI with trapped electrons were theoretically studied by Boro et al. [21]. Their investigation revealed that solitons propagate more quickly as the trapped electron temperature ratio increases, but the amplitude of the wave decreases due to collisions with ions and neutral particles. Zerglaine et al. [22] investigated the pressure anisotropy effect in magneto-plasma, which is composed of superthermal electrons and static dust grains. They found that when pressure is in a perpendicular direction, both the amplitude and the breadth of rarefactive and compressive SWs increase. They also observed that the amplitude remained constant while the width decreased as the magnetic field's strength increased. Using the ZK equation, Adnan et al. [23] illustrate compressive and rarefactive solitons in an anisotropic plasma with nonthermal electrons. The nonthermal electrons and the anisotropy of the ion pressure were found to cause these solitons to move more quickly. The characteristics of solitary excitations in an superthermal anisotropic plasma were investigated by Adnan et al. [24] using Sagdeev's potential approach. They found that the SWs steeper and more localized. Furthermore, they draw the conclusion that the features of SWs are more significantly influenced by the ion  component. In a plasma that is highly magnetized, the solitary profile is studied by Mahmood et al. [25]. They observed that when ion  is stronger than  the amplitude and width of the SWs profile declines when  is stronger than  the SWs profile enhancing the width of the profile without affecting the amplitude. In a dense magnetized plasma, the effect of anisotropy as well as electron trapping is studied by Irfan M et al. [26]. They noted the effect of thermal electron and pressure anisotropy strongly influence the amplitude and width of IAWs. The effect of superthermal EP on pressure anisotropic ion magnetized plasma is studied by Khan S U et al. [27]. They investigated the effect of several parameters such as  of ion,  of ion, magnetic field and positron concentration on the characteristics of IASWs. Khalid M et al [28] investigated on both linear & nonlinear propagation of waves in a magnetized anisotropic plasma. The role of different physical plasma parameters on the propagation of SWs is noted along with stability of the soliton. Iqbal M J et al [29] studied the effect of trapping in an EPI along with quantizing magnetic field on the formation of SWs. The characteristics linear (nonlinear) waves in an EPI magnetized anisotropic plasma along with super thermal EPs are discussed by Adnan M et al [30]. Their observation mainly focuses on the disturbs of wave due to super thermality of EPs. In a magnetized ion pressure anisotropic plasma, the rigorous investigations on ion-acoustic periodic waves were carried out by Khalid M et al [31]. They noted that the influence of  and  on the conoidal SWs profiles.

The main purpose of the present objective is to analyze the impact of EP trapping on a four-component magnetized anisotropic PI plasma. To perform that, the RPT is used to derive the three-dimensional nonlinear ZK equation. The layout of the paper is organised as follows: Section 1 presents the introduction. The mathematical equations of fluid are laid out in Section 2, while Section 3 includes the derivations of the nonlinear equations by RPT. Sections 4 and 5 show the results, discussion, and conclusion, respectively.

**THE FLUID MODEL EQUATIONS**

The study focuses on the propagation of IASWs in a 3D multicomponent magnetized collisionless plasma. The plasma is composed of EPIs, trapped EPs. We made the assumption that the powerful magnetic field, aligned with the x-axis. We consider the set of basic fluid that describe the behavior of ions with pressure anisotropy in magnetized plasmas can be expressed as:

****  (1)

****  (2)

In case of magnetic plasma, the tensor of pressure of the ions is supposed to have different magnitudes along different directions relative to the surrounding magnetic field. Thus, the pressure tensor  can be expressed as [32,33,34]:

 (3)

Where () is the unit tensor (vector) along the magnetic field . The  &  components of pressure terms are denoted by [5]

 (4)

Here  and  represents the ion unperturbed  and  at equilibrium respectively.

Equation of poisons is provided by

****  (5)

The PINI fluids were described by the continuity and momentum equations, respectively, Eqs. (1) and (2). Poisson's Eq. (5) describes the changes in electrostatic potential as a result of fluctuations in charge over both space and time. The physical quantities is used to indicate the density ,  is used to indicate the mass,for the velocity of ions and  denotes the pressure due to motion of charge particles;  indicate the charge state of ions and the magnitude of the charge of an electron is represented by .

By normalizing all the variables of fundamental equations, one obtains the set of Eqs. (6)-(10) normalized set of governing equations:

****  (6)

****  (7)

****  (8)

****  (9)

 (10) The general quasi-neutrality is expressed as  Here , and  are used to represents the electron-to-positive, negative-to-positive ion and positron-to-positive density ratios respectively.

To normalize the basic plasma parameters the following normalization scale is used [5,7]

, , , ,,  and  and  .

The expressions form basic trapped EP after normalization expressed as

****  (11)

 (12)

Where  and. Here () representing the Temperature of free (trapped) electrons.

**LINEAR ANALYSIS**

To investigate the linear properties of IAWs of collisionless magnetized plasma assuming the first order perturbed physical quantities , we have linearized the above set of dimensionless normalized fluid Eqs. (6)-(12), the following linear dispersion relation was obtained:

 (13)







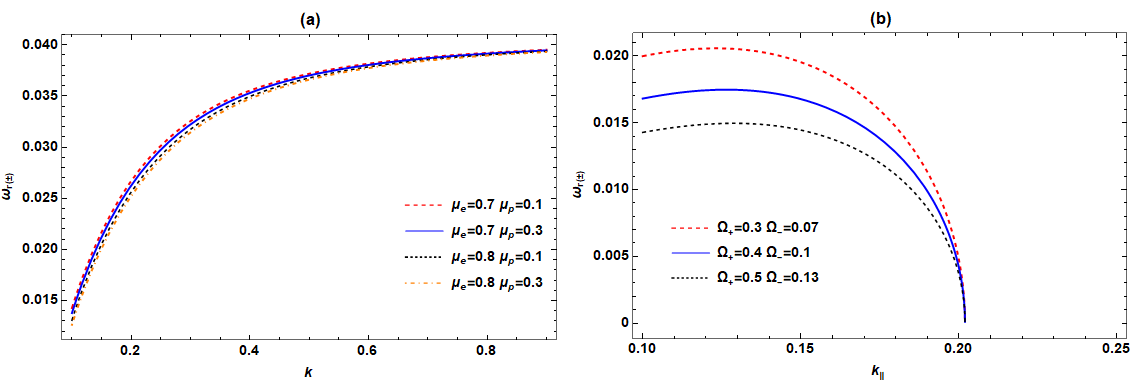


Such that the solutions of the dispersion relation in a viscous plasma specking complex and frequency  then reads , where the imaginary and real parts of  are

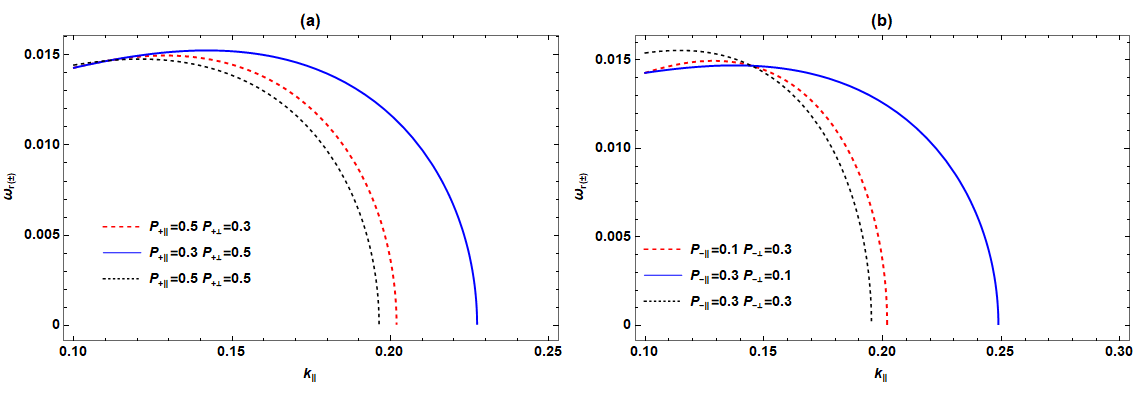
 and 

Where  is the wave vector defined as  with  and correspond to wavenumber in the parallel and perpendicular directions respectively and  is the angular frequency. Eq. (13) is quadratic in  with their possible roots.

It is clearly visible in Eq. (13) that the dispersion relation is multi variable function of strength of magnetic field , component of anisotropic pressure of positive ions  and component of anisotropic pressure of negative ions , density ratios ,  and , wave number in the parallel  & perpendicular  directions. Hence it is important to investigate the influence of these physical quantities on the linear characteristics of IAWs. The behaviour of real and imaginary frequency against the . From Fig.1(a), we can observe that the real angular frequency  against  for set of values of  and . It is noted that as the concentration of electron and positron increases the real angular frequency increases with  Fig.1(b) present the variation of real angular frequency against the wave vector  for set of values of  and . It can be noticed from the curve of Fig. 1(b) as the strength of magnetic field increases the real angular frequency declines to a certain value  Fig. 2 is plotted is plotted for set of values of  and  of PINI respectively in (a) and (b).

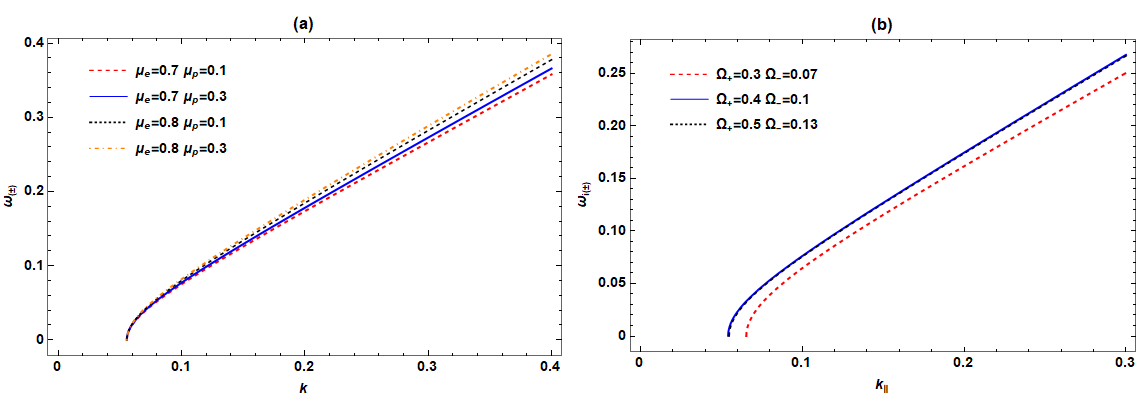


**Fig. 1**. Variation of real angular frequency  against the wave number  for set of values of  and  is depicted in (a). (b) showing the variation of  against  for  and 

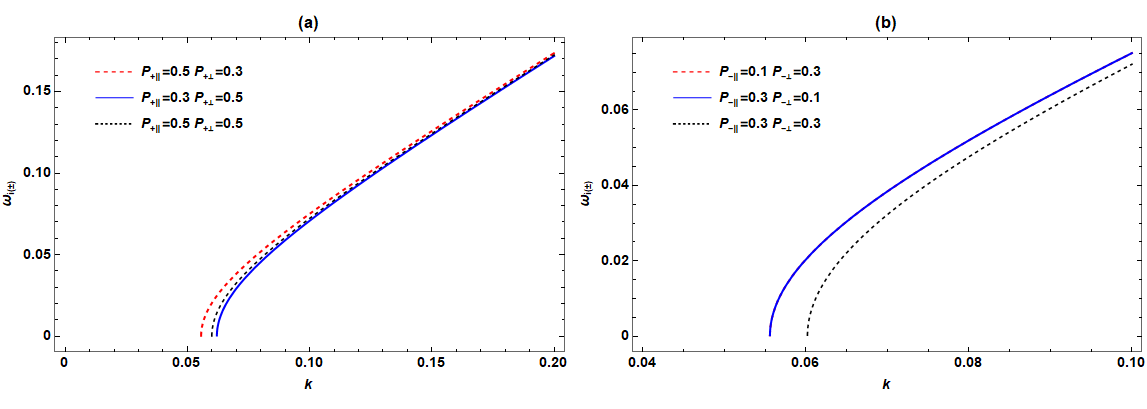
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**Fig. 2** Variation of real angular frequency  against  for set of values of   and  (b)  and .

In the Fig. 3 & 4 showing the imaginary part of Eq. (13) that the variation of imaginary angular frequency (damping rate). It is found that the imaginary angular frequency increases with increase in  and . Fig. 4 showing the variation of imaginary frequency, but this time we vary the  &  of ion in (a) and  &  of ion in (b) against . From Fig. 4(a), it can be noticed that as parallel pressure for positive ion increases the imaginary angular frequency, while it is decrease with perpendicular pressure for positive ion. However, from Fig. 4(b), it is clear that the imaginary angular frequency is declines with rise in pressure both parallel as well as perpendicular pressure.

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**Fig.3** Variation of imaginary angular frequency  against the wave number  for set of values of  and  is depicted in (a). (b) showing the variation of  against  for  and 



**Fig.4** Variation of real angular frequency  against  for set of values of   and  (b)  and 

**DERIVATION OF TZK EQUATION**

In order to determine the evolution equation of non-linear propagation of the SW from the last modified set of equations, i.e. from Eqs. (6)-(10), we adopt the well-known RPT. For this purpose, we use the stretched coordinates to represents independent space and time variables as: [16,26,28,30]

****  (14)

Where,  presents the ion excitation speed known as phase velocity. Other dependent perturbed variables such as  ,  and  are expanded in a dimensionless parameter which mesures the strength of nonlinearity as follows [35,36]:

 (15)

Using Eq. (15) along with space and time stretched coordinates Eq. (14) into set of normalized Eq. (6)-(10), then collecting the term of lowest co-efficient of  we obtain

****  (16)

****  (17)

Similarly, considering the lowest power co-efficient  from Eq. (10), we get

****  (18)

Substituting the above set of first order expressions (Eq. (15), (16) and (17)) in Eq. (18) we obtain a relation which gives the phase velocity as follows:

****  (19)

Where ****

****

and ****

The positive and negative sign in the Eq. (19) refers to fast and slow IA modes respectively. Hence, we compare the wave propagation of both acoustic mode in the paper.

In a similar manner collecting the coefficient order of , gives the expression for perturbation terms ,and  Now one can eliminate these physical quantities by the help of Eq. (16) & (16) and writing the equations in term of , finally we obtained the TZK equation as:

****  (20)

In the above Eq. (20),  is the complex fractional nonlinear term,  and  is the dispersion coefficients.

Here **** and 

Where **,**

****

****

To obtain the exact solution of Eq. (20), we describe a new transformation variable ,  is the transformed co-ordinate w.r.t velocity . The , and  are the directional cosines along the direction, andrespectively such that . After applying this transformation Eq. (20) becomes

 (21)

Where .

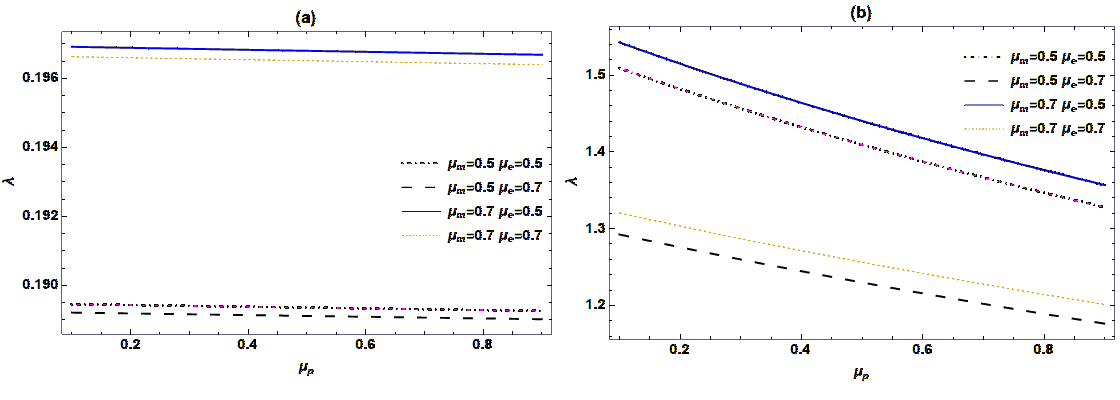
For the solution of TKdV nonlinear equation with fractional nonlinearity, the well know extended tanh method is utilized and the SW solution of Eq. (21) is obtained as follows [22].

 (22)

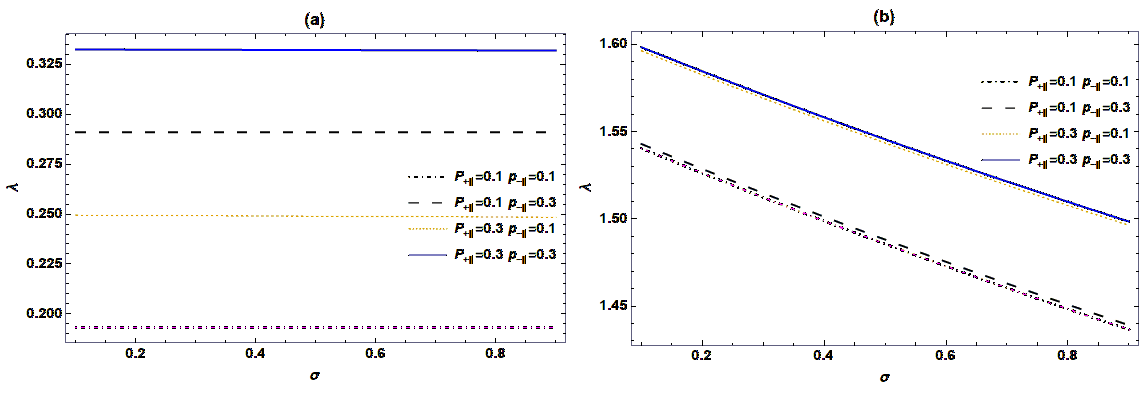
**RESULTS AND DISCUSSION**

In the present section, we want to analyse the dynamic effect of TZK Eq. (21) for the different plasma parameters on the characteristics of SW under the astrophysical situation. For the numerical evaluation, we considered all the plasma parameters in normalized form. So, the range of density ratio and the ambient magnetic field parameter is taken between 0.1 to 1 [12,15,23].

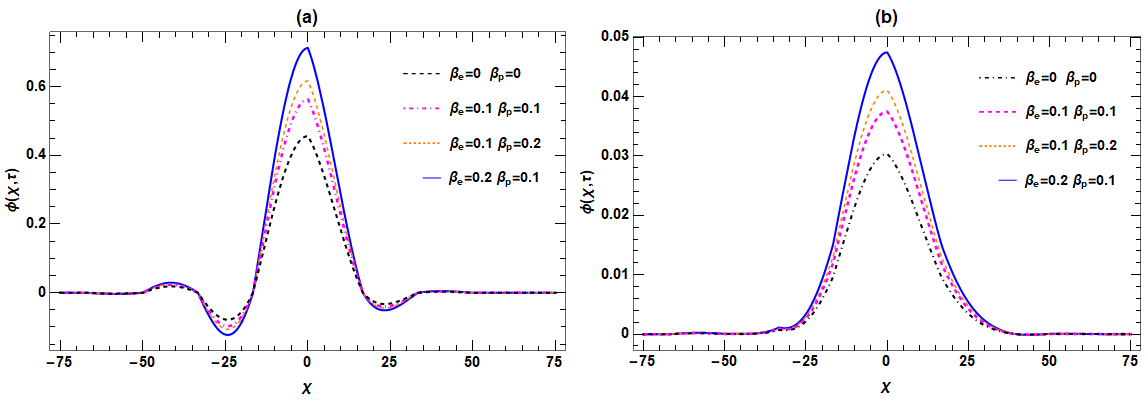
From Eq. (19) it is seen that  is a multi-variable function of density ratio , , , anisotropic pressure due to motion of charge particles of  & (positive & negative ions) and temperature ratio . The effect of the and  w.r.t the on the  of the nonlinear wave shown in Fig. 5. The variation of  basically in the subsonic and supersonic range for same set of  and  is depicted. From Fig.5(a) it is seen that the phase speed varies only in the subsonic range however, it varies in supersonic as shown in Fig.5(b). It is observed from the Fig.5(a)-(b), under different condition of  and  that the on-going plasma system supports both subsonic as well as supersonic plasma waves. As clear from the Fig.5 increasing  increases the phase velocity and decrease with . Again for it is noticed that the phase velocity independent on  for slow acoustic mode, but for fast acoustic mode  decreases the phase velocity.

**Fig. 5** Variations of  against  for set of values of  and  for (a) slow and (b) fast modes, with ,  and .

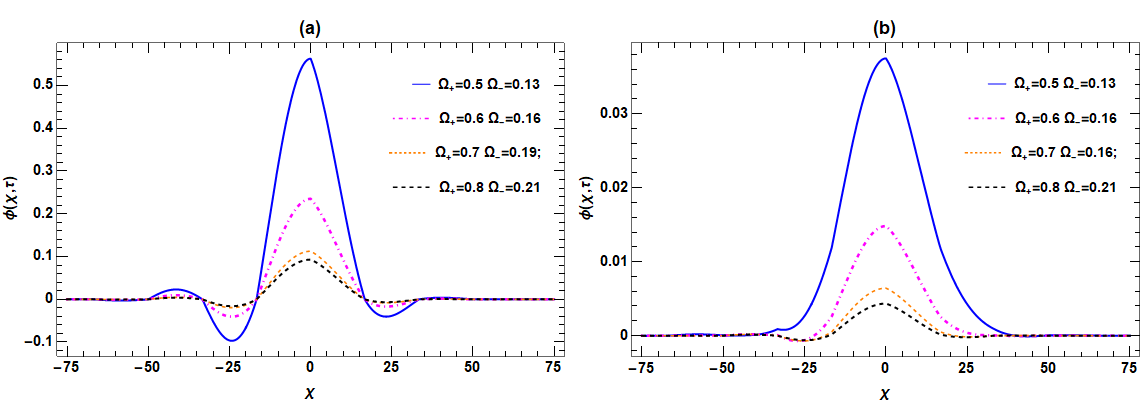
Furthermore, as the normalized phase velocity is function of ,  and  in Fig. 6 is showing the variation of  against the temperature ratio . It is clear Fig.6(a) and (b) that the normalized phase velocity is independent of temperature ratio when the velocity is in subsonic mode, however it helps in the reduction of the normalized phase velocity in supersonic mode. Apart from that it is noticed that the  increases with increase in  and . In subsonic mode the abrupt increase in the phase velocity is caused by parallel pressure due to negative ion, but in supersonic mode the it is due to the parallel pressure caused by positive ions.

 **Fig-6.** Variations of  against  for set of values of  and  for (a) slow and (b) fast modes, with ,  and .

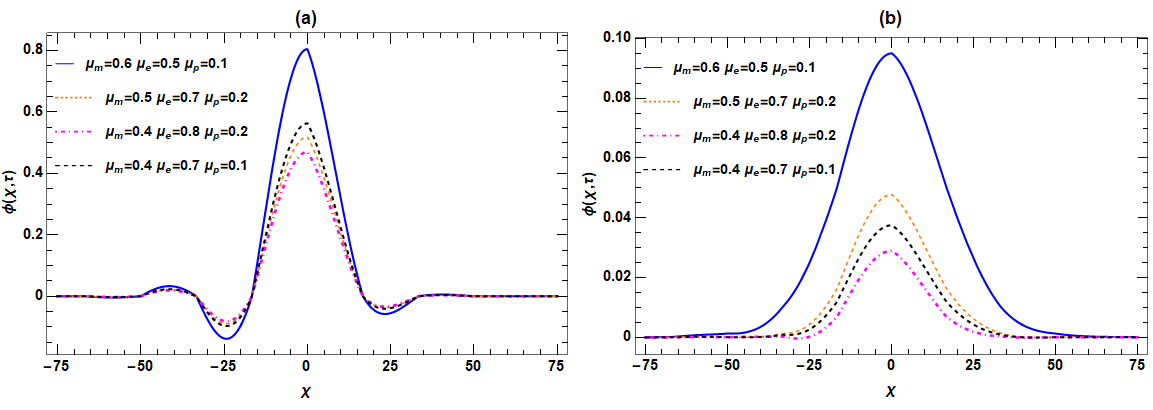
The trapping effect of electron  & positron  on characteristics of SWs profile is shown in Fig 7. Fig. 7(a) is described for slow mode  and Fig.7(b) described for fast mode  for same set of trapped parameters. Now from each curve of Fig.7 (a) and (b), it has been noted that the compressive SWs profile  reaches minimum for flat-topped distribution of electrons (black dashed curve) and it reaches maximum when electrons move towards thermal equilibrium (blue solid curve). The physical background behind these two graphs, as we increase the distribution of flat-trapped electrons provides the largest contribution of inertia to the plasma system. As a results, the nonlinearity of the system decreases and for this reason SWs potential significantly increases [13].

**Fig. 7** Variations of against  for set of values of  and  for (a) slow and (b) fast modes, where the other parameter such as , ,,  and  are fixed quantities.

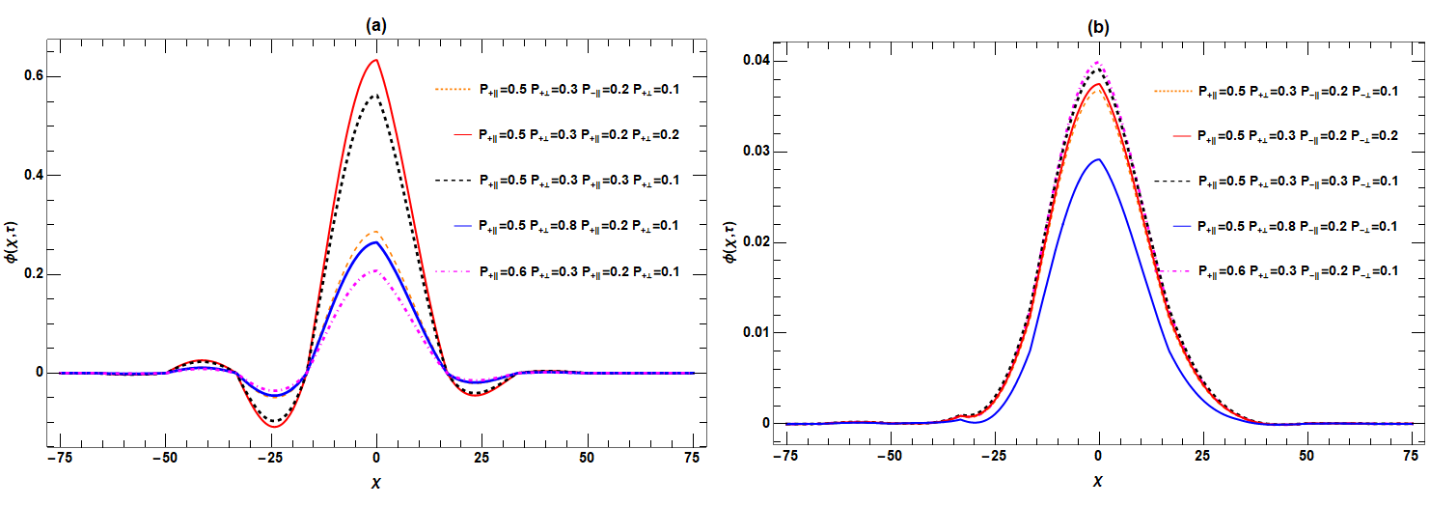
The role of the magnetic field of positive and negative on the profile of IASWs properties is depicted in Fig. 8 for (a) slow and (b) fast modes. Fig. 8(a)-(b) reveals clearly that the amplitude and steepness of the SWs decline in proportion to the magnetic parameter. However, it becomes more and more clear that the SWs development is sharply steep. By examining the dispersion coefficient, which is evaluated in proportion to the normalized cyclotron frequency, this study explores how the magnetic field affects the features of the IASWs profile [34].

**Fig. 8** Variations of against  for set of values of  and  for (a) slow and (b) fast modes, where the other parameter such as , ,,  and  are fixed quantities.

To express the effect of normalized density ratio parameter ,  and on the both (a) slow and (b) modes is shown in Fig. 9. From both modes, it is conclude that as the normalized density ratio of negative ion w.r.t positive increases the SWs profile abruptly increases, but declines with increase in  and .

**Fig.9** Variations of against  for set of values of ,  and  for (a) slow and (b) fast modes, where the other parameter such as  and  are fixed quantities.

A similar investigation of anisotropic pressure on SWs is depicted in Fig. 10 for both the modes (a) slow and (b) fast mode of velocity. From both 10(a) & (b), it can be noticed that when the parallel and perpendicular pressure due to negative ion increases the SWs profile is abruptly enhances for slow modes, however it increases slowly for fast acoustic modes. Similarly, it is evident that as the  is increases the SWs profile is declines abruptly for slow acoustic modes, however it has opposite role for fast acoustic modes. From both the figure, it can be concluded that when the phase velocity is low negative ion pressure plays a significant role, but when the normalized phase velocity is very high (subsonic)  plays a significant role.

**Fig.10** Variations of against  for set of values of ,,  and for (a) slow and (b) fast modes, where the other parameter such as  are fixed quantities.

**CONCLUSION**

In this paper a detailed analysis on the basic properties on the profile of arbitrary amplitude of IASWs were examined in the presence of trapped EPs in a magnetoplasma. The system contains both PINIs exhibiting pressure anisotropy. With the help of RPT, we derived the TZK equation. The effect of negative ion, EP to positive ion density ratio, positive to negative ion temperature ratio, magnetic field and pressure anisotropy of both PINIs are investigated. We hope, outcomes from present investigations may have implications in astrophysical environments like white dwarfs and neutron stars.

**REFERENCE**

[1] K. Singh and N. S. Saini, *Radio Sci.* **54**, 1192 (2019).

[2] R. A. Treumann and W. Baumjohann, *Advanced Space Plasma Physics* (Imperial College Press, London, 1997).

[3] G. F. Chew, M. L. Goldberger, and F. E. Low, *Proc. R. Soc. Lond. A* **236**, 112 (1956).

[4] G. K. Parks, *Physics of Space Plasmas: An Introduction* (CRC Press, Boca Raton, 2019).

[5] C. R. Choi, C. M. Ryu, D. Y. Lee, N. C. Lee, and Y. H. Kim, *Phys. Lett. A* **364**, 297 (2007).

[6] A. N. Dev, B. Boro, and N. C. Adhikary, *AIP Conf. Proc.* **2819**, 020002 (2023).

[7] B. Boro, A. N. Dev, R. Sarma, B. K. Saikia, and N. C. Adhikary, *Plasma Phys. Rep.* **47**, 557 (2021).

[8] A. N. Dev, M. K. Deka, R. K. Kalita, and J. Sarma, *Eur. Phys. J. Plus* **135**, 823 (2020).

[9] K. Habib, M. R. Hassan, M. S. Alam, and S. Sultana, *Plasma Phys. Control. Fusion* **66**, 065027 (2024).

[10] B. Pradhan, B. Boro, M. K. Deka, A. N. Dev, J. Manafian, and N. A. Alkader, *Results Phys.* **59**, 107617 (2024).

[11] S. Y. El-Monier and A. Atteya, *Sci. Rep.* **15**, 7524 (2025).

[12] H. Schamel, *Phys. Rep.* **140**, 161 (1986).

[13] I. B. Bernstein, J. M. Greene, and M. D. Kruskal, *Phys. Rev.* **108**, 546 (1957).

[14] S. Y. El-Monier, and A. Atteya, *Waves Random Complex Media* **32**, 299 (2022).

[15] N. C. Adhikary, A. P. Misra, M. K. Deka, and A. N. Dev, *Phys. Plasmas* **24**, 073704 (2017).

[16] R. Jahangir, and S. Ali, *Front. Phys.* **9**, 622820 (2021).

[17] N. Hamid, W. Masood, and H. Rizvi, *Contrib. Plasma Phys.* **59**, e201900015 (2019).

[18] H. Soltani, T. Mohsenpour, and F. Sohbatzadeh, *Contrib. Plasma Phys.* **59**, e201900038 (2019).

[19] H. A. Shah, M. N. S. Qureshi, and N. Tsintsadze, *Phys. Plasmas* **17**, 032312 (2010).

[20] N. A. Zedan, A. Atteya, W. F. El-Taibany, and S. K. El-Labany, *Waves Random Complex Media* **32**, 728 (2022).

[21] B. Boro, A. N. Dev, B. K. Saikia, and N. C. Adhikary, *Eur. Phys. J. Plus* **136**, 842 (2021).

[22] N. Zerglaine, K. Aoutou, and T. H. Zerguini, *Astrophys. Space Sci.* **366**, 72 (2021).

[23] M. Adnan, S. Mahmood, and A. Qamar, *Contrib. Plasma Phys.* **54**, 724 (2014).

[24] M. Adnan, A. Qamar, S. Mahmood, and I. Kourakis, *Phys. Plasmas* **24**, 032114 (2017).

[25] S. Mahmood, S. Hussain, W. Masood, and H. Saleem, *Phys. Scr.* **79**, 045501 (2009).

[26] M. Irfan, S. Ali, and A. M. Mirza, *Phys. Plasmas* **24**, 052306 (2017).

[27] S. U. Khan, M. Adnan, S. Mahmood, H. Ur-Rehman, and A. Qamar, *Braz. J. Phys.* **49**, 379 (2019).

[28] M. Khalid, A. Althobaiti, S. K. Elagan, S. A. Alkhateeb, E. A. Elghmaz, and S. A. El-Tantawy, *Symmetry* **13**, 2232 (2021).

[29] M. J. Iqbal, W. Masood, H. A. Shah, and N. L. Tsintsadze, *Phys. Plasmas* **24**, 012302 (2017).

[30] M. Adnan, G. Williams, A. Qamar, S. Mahmood, and I. Kourakis, *Eur. Phys. J. D* **68**, 301 (2014).

[31] M. Khalid, and A. U. Rahman, *Astrophys. Space Sci.* **364**, 28 (2019).

[32] R. E. Denton, B. J. Anderson, S. P. Gary, and S. A. Fuselier, *J. Geophys. Res. Space Phys.* **99**, 11225 (1994).

[33] R. Boström, G. Gustafsson, B. Holback, G. Holmgren, H. Koskinen, and P. Kintner, *Phys. Rev. Lett.* **61**, 82 (1988).

[34] J. A. Bittencourt, *Fundamentals of Plasma Physics* (Springer, New York, 2013).

[35] B. Pradhan, B. Boro, A. N. Dev, J. Manafian, and N. A. Alkader, *Nonlinear Dyn.* **112**, 17403 (2024).

[36] S. Sultana, and I. Kourakis, *Physics* **4**, 68 (2022).