A Study of the Dust Charging Process in Vasyliunas-Cairns Plasmas in Presence of Secondary Electron Emission

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**Abstract.** In this paper, the dust charging process is investigated in the Vasyliunas-Cairns plasmas containing suprathermal and nonthermal particles simultaneously. A simple model for dusty plasmas is used, including inertialess primary electrons, secondary electrons, ions, and immobile dust grains. Electrons and ions are assumed to follow the Vasyliunas-Cairns velocity distribution. Expressions of electron current, ion current and secondary electron emission currents are derived considering negatively and positively charged dust grains. Numerical analysis shows that the suprathermal and nonthermal particle population has a significant effect on equilibrium dust grain potential.

# INTRODUCTION

The phenomenon of charging an object immersed in a plasma environment has implications in various fields such as astrophysics, technological plasma applications, fusion-related studies, and laboratory dusty plasmas [1-7]. In this context, a dust grain acquires charge by accumulating plasma electrons and ions on its surface. The balance between electron and ion fluxes in the stationary state determines the floating potential of the object, with electrons being more mobile, resulting in a negative floating potential roughly equal to the electron temperature. This negative potential prevents most electrons from overcoming the barrier between the object and the surrounding plasma, enabling the balance of ion and electron fluxes [8].

Dust particles in a plasma environment acquire electric charges, which can vary widely from zero to potentially hundreds of thousands of electron charges. The charge accumulation is influenced by factors such as the size of the particle and the conditions of the plasma. Dust particles gain charge by attracting electrons and ions from the surrounding plasma, and in some cases, they may also emit electrons themselves. In plasmas where emission processes are negligible, the equilibrium charge tends to be negative because electrons flux towards the uncharged surface more readily than ions. However, in environments where electron emission is significant, the equilibrium charge of the dust particle tends to be positive [9].

The importance of calculating the charge on a particle is the initial step in any theory regarding dusty plasmas. The electron and ion plasma charging current densities, as well as the secondary electron current density, had previously been calculated using the Maxwellian distribution [10-12]. Jingyu Gong and Jiulin Du discussed the dust charging process with nonextensive power law distribution and the effect of secondary electron emission on this charging process [13]. The effect of secondary electron emission on dust acoustic and dust ion acoustic wave propagation has been investigated considering the Maxwellian and Lorentzian Kappa distribution of electrons and ions [14-16].

This article discusses the widely accepted 'orbit-limited' theory of charge collection before going on to describe many phenomena that can have a substantial impact on particle transport. These effects include a decrease in charge due to high dust density, positive charging produced by electron emission, and charge fluctuations.

In space plasma systems, plasma species are not in thermal equilibrium [17-20]. Cairns et al. [18] introduced the Cairns distribution function for electron species, which is a type of nonthermal distribution function observed with the help of Freja satellite and Viking spacecraft data. Various studies have utilized the Cairns distribution to analyze phenomena such as ion-acoustic waves, oblique modulation of ion-acoustic waves, expansion of laser-produced plasma, electrostatic waves, Landau damping of dust-acoustic waves, and derivation of dispersion relations for ordinary modes [21-27]. Researchers have employed this distribution function as a theoretical model to understand non-Maxwellian or nonthermal space plasmas effectively [28-31].

Currently, it is widely acknowledged that space and astrophysical plasmas often contain high-velocity particles and are characterized by non-Maxwellian velocity distributions. One common model used to describe this distribution is the Lorentzian kappa velocity distribution, which features a suprathermal tail representing a population of high-energy particles [20, 32,33]. The suprathermal population can have a significant impact on the behavior of waves and instabilities in space plasmas [34-38].

Scientists have later adopted a combination of the Vasyliunas distribution function and the Cairns distribution function, creating a more generalized distribution function known as the Vasyliunas-Cairns distribution function [39-42]. This combined model incorporates aspects of both suprathermality and nonthermality and has been utilized in various studies by researchers [25, 43-46]. Shahzad et al. [47] used the 3D Vasyliunas-Cairns distribution function for non-thermal electrons to examine the dispersion relation and damping rate of ion-acoustic waves, finding differences compared to Maxwellian distributed electron plasmas. This velocity distribution draws the significant attention of physicists to study dusty plasma. The effect of polarization force on dust acoustic kinetic Alfvén solitary waves in a magnetized dusty plasma with Vasyliunas-Cairns distributed electrons was studied [48]. In another study, the nonlinear properties of dust-ion-acoustic shock waves in a plasma with charge-fluctuating dust, inertial ions, and Kappa-Cairns distributed electrons were investigated [49]. Rehman et al. [50] analyzed the propagation of electron plasma waves in hot, un-magnetized Vasyliunas-Cairns distributed plasmas using the Poison-Vlasov model.

For th species of plasma, the three-dimensional Vasylinas-Cairns distribution function could be introduced as [39, 41]



where,  and . Here is the suprathermal index and is the nonthermal index. The function must satisfy  and [41]. In the velocity distribution function,  is the equilibrium number density and is the thermal velocity containing the temperature and mass  of the corresponding th species of particle. From this Vasyliunas-Cairns distribution function, one can obtain the Maxwellian velocity distribution function for,  and Kappa or Vasyliunas velocity distribution function for .

This article provides a thorough examination of the relationship between the nonthermal coefficient and the total current. Secondary electron emission, together with ion and electron attachment, affects dust charge variations. The flux of positive and negative charges to the dust grains is the basis for the calculation of these variations. The potential of the dust grains might be either positive or negative in a balanced state. This dust potential in equilibrium is significantly influenced by the nonthermal coefficient.

# THE BASIC MODEL: ORBIT-LIMITED THEORY

Electrostatic probe theories in plasmas serve as the foundation for the majority of dusty plasma charging ideas. Ion and electron currents are anticipated by these theories. The currents are termed 'orbit-limited' when the particle radius, is very small compared to the Debye length and collisional mean free path between neutral gas atoms and either electrons or ions. In that case, the currents are determined based on the assumption that the ions and electrons are collected by the dust grain surface [9, 51, 52]. Dust grains acquire charge through the attachment of electrons and ions, as well as through secondary electron emission, which contributes positively to their charge. The flux of electrons and ions is affected by the surface potential of the dust grains, leading to variations in their charge. This phenomenon, where the flux of electrons and ions influences dust charge, is referred to as charging current [53].

Dust grains collect electrons and ions from the surrounding plasma, leading to variations in their charge. As electrons have a higher thermal speed than ions, dust grains tend to acquire more electrons, resulting in a negative surface potential. This potential affects the currents of primary electrons and ions, with electrons experiencing repulsion and ions experiencing attraction when the equilibrium dust charge is negative. Using the orbit-limited motion (OLM) approach, one can calculate the charging currents for electrons and ions under different dust charge conditions, whether negative or positive [14]. When electrons or ions strike a dust grain, they can lose energy, exciting surface electrons, which may escape, making the grain positively charged. Secondary electron release occurs due to both electron and ion impacts, but ion contribution is usually negligible.

In dusty plasma, significant secondary electron emission occurs with high-density incident particles. This creates a positive current on dust particles. For the secondary electron emission, the secondary yield is defined as the ratio of the emitted electron flux to the primary electron flux. Secondary emission yield depends on impact energy and grain material. The expression of the secondary yield due to electron impact is [11, 12, 54, 55]



where is the maximum value of  and is impact energy. The quantities  and  are determined by the materials of dust grains. In a plasma, electrons create a negative current while ions generate a positive current on dust surfaces. Secondary electron current equals positive current to the dust. Lower secondary electron yield results in negatively charged grains; higher yield leads to positively charged ones [14].

In this paper, we will first derive electron currents, ion currents and secondary electron emission currents both in Vasyliunas-Cairns distributed plasmas considering both negative and positive dust grain surface potential. Then we will determine the equilibrium condition by plotting the total current as a function of the grain surface potential.

## Negative Surface Potential

Using the orbit-limited motion (OLM) technique, we can determine the charging current for electrons [11,12].



The normalized electron plasma current has been calculated for negatively charged dust grain using the distribution function from equation (1) in equation (3).



where.

When the nonthermal term is zero, both and reduced to one.

Here  , containing dust grain surface potential and electron temperature .

Similarly, we obtain the ion current expression substituting the ion velocity distribution (1) in the equation below [11, 12].



The normalized ion plasma current has been calculated using equation (5).



where and  and . This disappears when nonthermality disappears.

The expression of secondary emission current is obtained using the yield function from equation (2) in the equation below [54].



Using the Vasyliunas-Cairns distribution function from equation (1) into equation (7) we obtain the normalized secondary electron emission currents



where = and .

The dust charge is variable due to the variation of the current flow to the dust surface. Thus, the variable dust charge  satisfies the grain charging equation.



The net current to the dust grain is zero in the equilibrium condition. The normalized form of the net current in the case of negative dust grains respectively is



## Positive Surface Potential

Using the similar approach used in the previous subsection, we can calculate the charging current for electrons and ions when the equilibrium dust charge is positive. The normalized electron plasma current has been calculated for positively charged dust grain using equation (3).



where .

Similarly, the normalised form of ion current for positively charged dust grain is obtained using equation (5).



where 

The secondary electron emission current is calculated using equation (7) for positive dust grain.



whereand .

In equilibrium, the net current to the dust grain is zero. The normalized form of net current for positive dust grains is



# NUMERICAL ESTIMATION AND DISCUSSION

The study investigates the impact of suprathermal and nonthermal particles on dust charging in Vasyliunas-Cairns plasmas. Analytical derivations of charging currents are conducted for both negative and positive dust grains in equilibrium. Space plasmas like the solar wind, interplanetary space, and cometary tails are considered [20, 32, 33]. Kappa index values between 2 and 6 are chosen to align with astrophysical plasma observations [32] and considered to be in the range 0 to 1. For numerical estimation, we have considered the plasma parameters [13, 14, 28], ,,. Our primary emphasis lies in examining the influence of suprathermal and nonthermal particle populations on the dust charging process, particularly with regard to secondary electron emission.

To calculate the secondary emission current, we calculated the core integral in the secondary emission current and numerically. We have plotted the total current as a function of dust surface potential. The numerical calculation of the total current is done using Equation (10) for and Equation (14) for . In equilibrium condition, we plot this total current in Figure (1) for different values of the nonlinear index. We have considered the other parameters, ,  and. Figure 1 shows that decreases with increasing potential and becomes zero at some negative value of . This negative value increases with increasing. So equilibrium current balance condition indicates that the dust surface potential will be negative and increasing nonlinearity makes the dust surface potential more negative. In Figure 2 we have considered  unlike in Figure 1. This makes the equilibrium dust surface potential positive and increasing the nonlinear coefficient increases this equilibrium potential. Figure 3 is plotted considering higher kappa indices and it shows Maxwellian-like three points of equilibrium at . Here we have obtained a specific scenario characterized by parameters  and . It shows the existence of three equilibrium points within the system: two of these points are negative, while one is positive. This configuration is observed in Maxwellian plasmas [11], where equilibrium states can exhibit both stability and instability. Figure 4 shows that, in the absence of a secondary current, we do not have any equilibrium point in our considered range of potential for a non-zero nonlinear coefficient. However, in the absence of a nonlinear coefficient we obtain an equilibrium point at negative potential. On the other hand, figure 5 shows that if we consider higher kappa indices, we obtain equilibrium points in the negative potential zone and this negative value increases with increasing nonlinear coefficient. So we may conclude that lower has a high probability of obtaining negative dust whereas high may give positive dust. The equilibrium dust potential is primarily determined by the secondary yield: a low secondary yield results in a negative potential, while a high secondary yield leads to a positive potential. In a balanced state, the potential of the dust grains can be either negative or positive. The nonthermal coefficient significantly impacts this equilibrium dust potential. When the equilibrium potential is negative, an increase in the nonthermal coefficient further lowers the potential. On the other hand, when the equilibrium potential is positive, a higher nonthermal coefficient raises it.



**FIGURE 1.** Plot of total current as a function of potential considering and .



**FIGURE 2.** Plot of total current as a function of potential considering and.



**FIGURE 3.** Plot of total current as a function of potential considering and .



**FIGURE 4.** Plot of total current as a function of potential considering and.



**FIGURE 5.** Plot of total current as a function of potential considering and .

# CONCLUSION

An analytical theory on the effect of secondary electron emission on the dust charging process in nonthermal dusty plasmas is presented in this article. This dusty plasma model is considered the presence of both nonthermal and suprathermal particles. The coexistence of nonthermal and suprathermal particles is outlined by Vasyliunas-Cairns velocity distribution. Electron and ion impact currents along with secondary electron currents have been derived considering both negative and positive dust grain potentials. A detailed discussion on the effect of the nonthermal coefficient on secondary electron emission current as well as total current is presented. The electron and ion attachment and secondary electron emission make the dust charge variable. This dust charge variation is calculated using negative and positive charge flux to the dust grains. In the balanced condition potential of dust grains may be either negative or positive. However, this equilibrium dust potential mainly depends on secondary yield. Low secondary yield means negative potential and high secondary yield means positive potential. The value of the nonthermal coefficient has a great impact on the equilibrium dust potential. When the equilibrium dust potential is negative, increasing the value of the nonthermal coefficient makes the equilibrium dust potential more negative. On the other hand, this nonthermal coefficient makes equilibrium dust potential more positive. In the particular situation of the study considering and, we obtain three equilibrium points, two negative and one positive. It is quite similar to Maxwellian plasmas [11]. Our results show the similarities between the investigated scenario and Maxwellian plasmas and advance our understanding of equilibrium behaviors in plasma systems. Investigating these equilibria further could provide important information on the system's dynamical responses and stability characteristics. So this study is quite effective in the case of non-Maxwellian plasmas.

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