Integrated Inventory Model for Single Producer and Multi Buyers to Determine the Optimal Production Quantity

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**Abstract.** An integrated inventory model for determining production quantity can yield optimal outcomes for stakeholders within the supply chain network. This study develops a supply chain network model involving one manufacturer and two buyers, aimed at minimizing inventory costs. The manufacturer alternately delivers products to each buyer, who then sells them in the market until they reach the final consumers. The annual demand for each buyer is assumed to be identical, as is their inventory cycle time. The optimal production quantity for the manufacturer in a single production cycle is determined using a straightforward algorithm. Numerical examples are employed to test the modified algorithm. The experimental results indicate a delivery frequency of nine times per production cycle, with an optimal production quantity of 1,565 units per cycle. The optimal quantity per delivery to each buyer is 174 units, resulting in a minimum total inventory costs in the one-manufacturer-two-buyers inventory model.

**Keywords:** Integrated inventory model, Single producer, Multi buyer, EPQ

# INTRODUCTION

Inventory management is a critical aspect of supply chain operations, as it directly impacts production efficiency, cost reduction, and customer satisfaction. In a competitive market environment, businesses strive to optimize their inventory levels to balance production and demand while minimizing holding and shortage costs. Integrated inventory models have gained attention as they offer a collaborative approach, enabling producers and buyers to align their strategies for mutual benefit. Specifically, models that involve one producer and multiple buyers are crucial for understanding the dynamics of multi-echelon supply chains, where coordination between parties can lead to significant cost savings and improved service levels [1]{Liu, 2022 #97}{Liu, 2022 #237}{Liu, 2022 #237}{Liu, 2022 #97}.The focus of this study is on an integrated inventory model involving one producer and two buyers, aiming to determine the optimal production quantity that meets demand while minimizing total supply chain costs.

Previous studies have extensively explored integrated inventory models, highlighting various approaches to optimize production and inventory decisions. For instance, Jauhari [2] developed an economic production quantity (EPQ) model that integrates the buyer-supplier relationship to minimize total costs across the supply chain. Similarly, Jauhari and Pujawan [3] introduced a joint economic lot size model that coordinates the production and ordering policies between a single producer and a single buyer. These foundational works laid the groundwork for further research into more complex models that involve multiple buyers and more intricate supply chain relationships.

In recent years, researchers have expanded on these earlier models by incorporating more realistic assumptions and constraints. Sarker and Wu [4] extended the joint economic lot size model to include scenarios where multiple buyers are involved, focusing on how coordination can reduce overall costs in the supply chain. Additionally, Handayani, et al [5] and Yuniarti, Masudin [6] explored an integrated production-inventory model for a vendor and multiple buyers, considering issues such as lead time, batch production, and quantity discounts. These studies underscore the importance of integrating production and inventory decisions in multi-echelon supply chains, yet they also reveal the complexity that arises when multiple buyers are involved.

Despite the advancements in integrated inventory modeling, a research gap remains in the context of models that specifically address the interaction between one producer and two buyers. Most existing models either focus on a single buyer or involve a larger network of buyers without addressing the unique challenges posed by a two-buyer scenario. This study seeks to fill this gap by developing an integrated inventory model that optimizes production quantity in a supply chain consisting of one producer and two buyers. The proposed model provides insights into how such a configuration can be managed effectively to minimize costs and enhance supply chain performance, contributing to the broader understanding of inventory management in multi-echelon supply chains.

# LITERATURE REVIEW

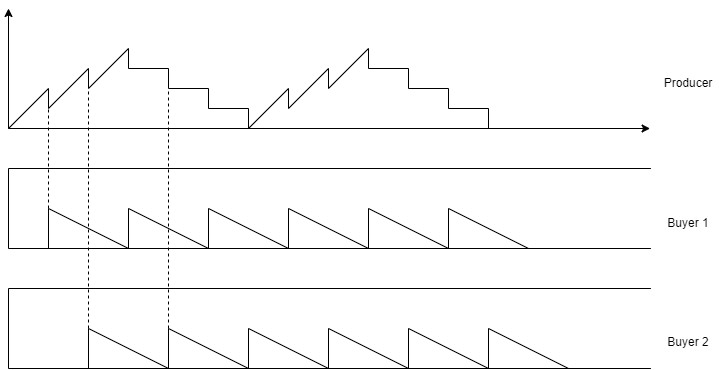
Raw material inventory management is a crucial element in running a successful business. Research conducted by Gao, Chen [7] emphasizes the importance of raw material inventory management strategies. They stress that business owners must understand how much raw material is needed in the production process and when is the most appropriate time to reorder. Furthermore, research by Taleizadeh [8] highlights the aspect of advance payment policies in inventory management. They formulated models that consider important aspects such as demand levels that evolve over time and time-related damage. In the real world, this becomes key in facing inventory management challenges.

Research by [9] highlights the role of ABC Analysis in inventory management. The conclusion from this research is that ABC Analysis is an essential tool for classifying inventory based on specific criteria [10]. Proper implementation of ABC Analysis helps business owners be more efficient in inventory planning and forecasting, minimizing the risk of raw material shortages, and improving the effectiveness of their supply chain. This conclusion underscores the importance of inventory management in a competitive and ever-changing business world.

The results of Emar, Al-Omari [11] research indicate that the implementation of inventory policies is crucial in managing slow-moving goods, such as Continuous Slow-Moving Products (CSP). Coordination among all employees involved in the management of slow-moving CSPs is emphasized as a key factor, with special attention to addressing the failures of some CSPs that can independently affect slow sales. On the other hand, research by Shilul Imarah and Roni [12] concluded that the ABC Analysis Method is effective in reducing the difficulties of conducting periodic stock checks and improving inventory data accuracy. The effective forecasting method for inventory needs planning is Exponential Smoothing [13], while Economic Order Quantity (EOQ) has been proven to enhance inventory purchasing planning at a higher level [14]. Therefore, implementing these methods can bring significant improvements in inventory management and supply planning. Previous research serves as a basis for guiding and inspiring the ongoing research, which uses an integration of the EOQ and EPQ methods that have been developed and unified into one formula in the context of one producer and two buyers to determine the optimal production quantity [15]. This research methodology distinguishes itself from previous studies by providing new contributions to inventory analysis for situations involving two buyers with one supplier/producer.

# METHODS

The supply chain network model between the producer and two buyers can be seen in **FIGURE 1**. After the single product is produced consistently by the producer, it is distributed to the two buyers. The marketing of the product is carried out in collaboration between the producer and the buyers until the product reaches the consumers. Each delivery is done alternately and sequentially; for instance, the first delivery goes to the first buyer, the second delivery to the second buyer, and so on. This supply chain network model can be seen in the figure below.



**FIGURE 1**. Supply Chain Inventory Model for One Manufacturer and Two Buyers

Figure 1 represents an inventory model for a supply chain involving one producer and two buyers. The top section of the figure illustrates the producer's inventory levels, which fluctuate due to production activities, leading to a step-like pattern. The two lower sections correspond to Buyer 1 and Buyer 2, showing their respective inventory levels. Both buyers experience a gradual depletion of inventory over time, represented by the downward sloping lines, which are periodically replenished by the producer. The timing of replenishment for each buyer is staggered, with Buyer 1 receiving inventory before Buyer 2, indicating a coordinated scheduling to ensure continuous supply while managing inventory levels efficiently across the supply chain.

## PRODUCER'S INVENTORY COSTS

This study focuses on optimizing production and inventory costs within a supply chain network through a detailed model. As illustrated in **FIGURE 2**, the model employs several equations to compute essential metrics, including production cycle time, total inventory costs, and order quantities [16].

A diagram of a graph

Description automatically generated

**FIGURE 2**. Producer Inventory Model

Total quantity required

(1)

Production cycle time :

(2)

Based on the research by Firmansyah [17], the formula to calculate with the model mentioned above is as following equation (3). While the inventory model for buyer is displayed in **FIGURE 3**.

(3)

A triangle with a letter d and a letter d

Description automatically generated with medium confidence

**FIGURE 3.** Inventory model for buyer

Buyer inventory costs :

(4)

(5)

Order cycle time:

(6)

Equation for calculating total inventory cost:

(7)

(8)

Production quantity equation of products per cycle derived from TIC:

(9)

Some notations used in research on supply chain network model development include:

Cr1: Cost per order for buyer 1 (Rp/Order)

Cr2: Cost per order for buyer 2 (Rp/Order)

Crp: Setup cost for producer (Rp/Setup)

Ch1: Holding cost for buyer 1 (Rp/Unit/Year)

Ch2: Holding cost for buyer 2 (Rp/Unit/Year)

Chp: Holding cost for producer (Rp/Unit/Year)

q: Quantity per shipment to buyer (Unit)

Q: Quantity of product per production cycle (Unit)

d: Demand at buyer (Unit/Year)

D: Demand at producer (Unit/Year)

P: Production rate of producer (Unit/Year)

T: Length of producer's production cycle (Year)

t1: Length of buyer's inventory period (Year)

n: Frequency of deliveries in one cycle

The model illustrated in Figure 2 addresses the optimization of production and inventory costs within a supply chain network. It utilizes several equations to determine key metrics such as production cycle time, total inventory costs, and order quantities. Equation (1) calculates the total quantity of products needed, while Equation (2) determines the production cycle time by dividing the total quantity by the demand rate. The total inventory cost (TIC) for both the producer and buyer is derived from Equations (3) through (5), incorporating setup costs, holding costs, and order quantities. The cumulative total inventory cost is given by Equation (8), which combines various cost components. The optimal order quantity for each production cycle is computed from the derivative of the TIC equation, as shown in Equation (9). This model is instrumental in managing costs and improving efficiency by analyzing the interactions between production, inventory, and order parameters within the supply chain.

# RESULTS AND DISCUSSION

This study involves one producer shipping products to two buyers with the same aggregate demand. As per the assumptions previously explained, the costs that are variables in determining the total cost are known and fixed. Here are some of the known variables:

|  |  |
| --- | --- |
| * Cr1: IDR 841,000/order | * Ch2: IDR 10,000 (ton/year) |
| * Cr2: IDR 446,000/order | * Chp: IDR 20,000 (ton/year) |
| * Crp: IDR 1,690,000/setup | * D : 936 (ton/year) |
| * Ch1: IDR 10,000 (ton/year) | * P : 1000 (ton/year) |

The results of the calculations using the above data can be seen in the **TABLE 1** below:

**TABLE 1**. Total Inventory Cost Calculation

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| N | Q (ton) | q (ton) | TIC TOTAL (IDR) | T | t1 |
| 1 | 325.34 | 325.34 | 13.430.185.83 | 0.347589369 | 0.70 |
| 2 | 511.85 | 255.93 | 10.892.182.79 | 0.546849033 | 0.55 |
| 3 | 681.19 | 227.06 | 9.954.486.44 | 0.72776928 | 0.49 |
| 4 | 841.45 | 210.36 | 9.491.551.76 | 0.898984641 | 0.45 |
| 5 | 995.27 | 199.05 | 9.236.100.32 | 1.063322042 | 0.43 |
| 6 | 1.143.89 | 190.65 | 9.090.141.01 | 1.22210853 | 0.41 |
| 7 | 1.288.04 | 184.01 | 9.008.939.42 | 1.376114113 | 0.39 |
| 8 | 1.428.20 | 178.52 | 8.969.089.84 | 1.525853653 | 0.38 |
| 9 | 1.564.72 | 173.86 | 8.957.138.32 | 1.671706717 | 0.37 |
| 10 | 1.697.88 | 169.79 | 8.964.797.90 | 1.813972639 | 0.36 |
| 11 | 1.827.91 | 166.17 | 8.986.686.08 | 1.952898739 | 0.36 |
| 12 | 1.955.02 | 162.92 | 9.019.156.85 | 2.088696099 | 0.35 |

Based on the calculations above, there were 12 trials with varying values of nnn, ranging from n=1n = 1n=1 to n=12n = 12n=12. For trials 1 to 9, the TIC values consistently decreased, until trial 10, where the TIC began to increase. The optimal delivery frequency can be considered as n=9n = 9n=9, with a total TIC of IDR 8,957,138 and a production quantity of 1,565 tons per cycle. In contrast, subsequent trials continued to show increasing TIC values, such as TIC n=10n = 10n=10 at IDR 8,964,797 with a production quantity of 1,697 tons, and TIC n=11n = 11n=11 at IDR 8,986,686 with a production quantity of 1,827 tons. Adding three more values of nnn resulted in further increases in TIC. Therefore, based on the results of this calculation, the optimal delivery frequency per cycle is 9 deliveries to buyers in rotation. With a delivery frequency of 9 times per cycle, the producer requires 1.6717 years for one cycle. To meet the buyer's needs, the producer must carry out production and delivery according to the buyer's known requirements. For the quantities, deliveries to both buyer 1 and buyer 2 are the same at 174 tons, conducted 9 times each cycle in rotation. Meeting the buyer's needs is determined by the varying requirements of each buyer, calculated by comparing the percentage needs of each buyer.

# CONCLUSIONS

This study employs the EOQ and EPQ methods to optimize inventory control. The EOQ method is used to determine the optimal quantity that results in minimum inventory costs for each production cycle, while the EPQ method is applied in the context of economic production to achieve greater efficiency. Thus, these methods provide in-depth insights into potential risks that may arise in the company. The study models a supply chain network involving one producer and two buyers. The inventory models for both parties are integrated to achieve the optimal total inventory cost, which is the main objective of this research. The goal is to determine the delivery frequency to each buyer within a production cycle to avoid inventory delivery delays and achieve the most efficient total inventory cost in the supply chain model. The study facilitates effective coordination between the two buyers, allowing the producer to calculate the optimal production and delivery quantities for each buyer.

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