**Mathematical Modeling of the Influence of Spatial Inclination of Reinforced Concrete Poles on the Geometry of the Overhead Contact System in Railway Electrification**

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**Abstract.** The article examines the relationship between the structural parameters of reinforced concrete poles in the overhead contact system and the reliability of the catenary operation. Based on an analysis of the contact system’s configuration and the characteristics of supporting structures, approaches are proposed for assessing the technical condition of reinforced concrete poles, taking into account deviations from the vertical and the direction of inclination. A mathematical model is presented for calculating the deviations in contact wire stagger and height under various pole tilt angles. The results contribute to improving diagnostic accuracy and preventing emergency situations in railway power supply systems.

**Keywords:** Catenary system, reinforced concrete poles, stagger, inclination angle, deviation, diagnostics.

**INTRODUCTION**

The overhead contact system of electrified railways plays a key role in ensuring reliable traction power supply. Reinforced concrete poles, as an integral part of the supporting structure, must maintain high stability and preserve their geometry throughout the entire service life. Any tilt of these poles leads to displacement of the contact wire and distortion of the catenary clearance, which may result in arcing, overheating, mechanical damage to pantographs, and even accidents.

In this context, the contact wire suspension system acts as a critical component of the entire traction power infrastructure. The efficiency and safety of power supply depend significantly on the condition of supporting structures—primarily the reinforced concrete poles. Pole inclinations caused by climatic, vibrational, or soil-related factors can distort the spatial geometry of the catenary, disrupt current collection, and accelerate wear of both contact wires and pantographs. These disturbances may lead to arc discharges, wire burnout, and unscheduled train stops.

The reliability of the overhead contact system directly depends on the geometric stability of its supporting structures. Deviations of the poles from their standard vertical alignment result in violations of contact wire geometry, increasing the risk of pantograph detachment. Even short-term separation can generate high-temperature electric arcs, which in turn can lead to wire burnout and emergency train stops. The arc, forming within milliseconds, produces high-frequency electromagnetic fields and its formation is influenced by various factors, including air pressure, temperature, and the distance between the pantograph and contact wire [1, 9].

Reinforced concrete pole degradation is a complex physical and chemical process that occurs over time and is influenced by both the nature of the materials used and environmental exposure. Centrifuged concrete, commonly applied in catenary poles, exhibits significant heterogeneity across the wall thickness, forming multiple layers with varying textures and structural properties. During operation, poles are exposed to cyclic temperature changes, chemically aggressive environments, and microbiological activity, all of which accelerate deterioration. In addition to these factors, the influence of the electromagnetic field generated by the overhead contact system contributes to the development of transverse cracks, primarily in the upper zone of the pole [6, 7, 8, 12, 13, 14, 15]. One of the earliest and most reliable indicators of structural degradation is a change in the pole inclination angle, which directly affects the geometry of the overhead system and can be used as a diagnostic parameter. This highlights the necessity of continuous condition monitoring and the development of advanced diagnostic tools to detect dangerous inclinations and prevent pole failure, thereby ensuring the safety and continuity of railway operations [10-11].

Under high load conditions, as identified through traction power calculations, the technical state of both the contact system and its supporting structures becomes a key factor influencing energy reliability and operational safety. Therefore, the development of structural-parameter-based assessment methods for pole condition and advanced diagnostic tools for detecting deviations from vertical alignment is a highly relevant engineering challenge.

Special attention must also be paid to mathematical modeling that quantifies the impact of pole inclination on contact system parameters and enables early detection of critical zones. Such models could serve as the foundation for real-time diagnostic tools, supporting preventive maintenance, reducing operational costs, and improving safety, especially on high-speed lines.

**STRUCTURAL-PARAMETRIC DESCRIPTION OF REINFORCED CONCRETE POLE SYSTEMS AND THEIR RELATION TO THE RELIABLE OPERATION OF THE OVERHEAD CONTACT SYSTEM**

The overhead contact system is a spatial structure consisting of reinforced concrete poles, cross-arms, insulator strings, messenger wires, and contact wires. The poles fix the position of the wire above the railway track, and their spatial orientation determines key parameters such as stagger (horizontal displacement of the contact wire relative to the track axis) and suspension height. Any change in the inclination angle of a pole distorts these parameters, especially in cases of diagonal tilting, which results in simultaneous longitudinal and transverse deformations[2, 3, 4, 5, 6, 7, 8].

With the development of electrified railways, the overhead contact system - a core component of the traction power supply infrastructure - plays a critical role in ensuring sufficient power delivery for railway loads. As is well known, the contact system comprises support structures, suspension components, the catenary system itself, and the compensating unit [1, 10, 11].

Like any complex system, the overhead contact system consists of both primary and auxiliary elements, each of which can be considered a subsystem in its own right. A sequential logic block diagram (Fig. 1) presents the general structure of the railway overhead contact system under normal operating conditions.



**FIGURE 1.** Structure of the Railway Overhead Contact System

The failure frequency of individual components, under which the overhead contact system operates, is not taken into account here. Normal operation assumes proper functioning of all elements in the block diagram, as they are logically connected in a sequential manner. The first block in the sequential structure of the overhead contact system (OCS) is the support structures, which include the support pole and the foundation. A failure in this block will sequentially lead to the failure of all subsequent blocks and, ultimately, to a failure of the entire system. Therefore, enhanced monitoring of the condition of all blocks—starting from the support structures-is essential.

System integrity means that each element contributes to the achievement of the system’s functional goal. One of the main indicators of OCS integrity is the accuracy of the contact wire's geometric positioning (Fig. 3). A classification of the key OCS parameters can be developed based on three geometric dimensions (Fig. 2):

**Length**: parameters of the OCS distributed along the entire electrified track;

**Width**: parameters perpendicular to the railway axis;

**Height**: parameters related to the vertical position of the main elements of the contact system [2, 3, 4, 5].

Based on this classification, a formalized model has been developed for determining the coordinates of contact wire points at the support node.

|  |  |
| --- | --- |
|  | (1) |

where ***l***is the span length. In this study, all spans are assumed to be intermediate, with equal spacing between support poles; ***a*** is the contact wire stagger.

A positive sign is assigned when the stagger is directed toward the increase of the clearance gauge, and a negative sign when it is directed toward the decrease. According to regulatory documents, on straight track sections the stagger is taken as a = 0.3 m, and on curves as a = 0.4 m. The permissible values depend on the effective length of the pantograph strip; – denotes the constructional height of the contact wires.

Coordinates of the contact wire point under the first supporting node

|  |  |
| --- | --- |
|  | (2) |

coordinates of the contact wire point under the second supporting node

|  |  |
| --- | --- |
|  | (3) |

In the particular case

|  |  |
| --- | --- |
|  | (4) |

The allowable deviation of the contact wire is determined by:

|  |  |
| --- | --- |
|  | (5) |

where – allowable stagger deviation.

Limit deviation of the wires from the pantograph axis due to wind action:

|  |  |
| --- | --- |
|  | (6) |



**FIGURE 2.** Classification of key parameters of the overhead contact system

*-* working zone of the pantograph. For a collector strip length of 1600 mm, the pantograph’s working zone is – 1200 mm, For a collector strip length of 1950 mm, the pantograph’s working zone is – 1550 mm [1]; D - the pantograph displacement range is 200 mm and 225 mm, respectively.

Depending on the type of pantograph, ranges from 400 mm to 550 mm. At the same time, the *permissible* value must not exceed the maximum allowable deviation of the contact wire:

|  |  |
| --- | --- |
|  | (7) |

where- extreme position of the tensioned wire under wind load[16]. – tolerance due to pole deflection and cantilever sway 0,02-0,04 m; .

The values of the contact wire stagger were determined based on measurements from the Overhead Contact System Test Car (OCSTC), and the probability density function of the normal distribution for the stagger magnitude was constructed for both straight and curved track sections in accordance with the dependencies given below.

Static frequency of stagger range repetition:

|  |  |
| --- | --- |
|  | (8) |

where, – Repetition frequency of the i-th stagger range.



**FIGURE 3.** **Spatial arrangement of overhead contact system wires: *a*** – side view**, *b*** – top view

The mathematical expectation of the mean stagger value for the *i*-th range is determined by the following formula:

|  |  |
| --- | --- |
|  | (9) |

The mathematical expectation of the mean stagger value for the *i*-th range is calculated using the following formula:

|  |  |
| --- | --- |
| . | (10) |

The theoretical frequency is determined based on the Poisson distribution law:

|  |  |
| --- | --- |
|  | (11) |
|  | (12) |

The statistical and theoretical frequency distributions of the contact wire stagger range, measured by the OCSTC, for both straight and curved track sections are presented in Tables 1 and 2.

**TABLE 1.** Statistical and Theoretical distribution of the contact wire stagger range frequency measured by the OCSTC on a Curved Track Section

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Stagger range OCS *a*i, mm | Repetition frequency of the range, *fi* | Statistical frequency of stagger *pi* | Mathematical expectation *mi* | Theoretical frequency *Pi* |
| 240-250 | 4 | 0,01 | 2,45 | 0,0041 |
| 250-260 | 8 | 0,02 | 5,1 | 0,0189 |
| 260-270 | 18 | 0,045 | 11,925 | 0,0602 |
| 270-280 | 49 | 0,1225 | 33,6875 | 0,1328 |
| 280-290 | 85 | 0,2125 | 60,5625 | 0,2062 |
| 290-300 | 85 | 0,2125 | 62,6875 | 0,2280 |
| 300-310 | 84 | 0,21 | 64,05 | 0,1815 |
| 310-320 | 40 | 0,1 | 31,5 | 0,1051 |
| 320-330 | 23 | 0,0575 | 18,6875 | 0,0446 |
| 330-340 | 4 | 0,01 | 3,35 | 0,0141 |
| Total | 400 | 1 | 294 | 0,9954 |

**TABLE 2.** Statistical and Theoretical distribution of the contact wire stagger range frequency measured by the OCSTC on the straight section

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Stagger range OCS *a*i, mm | Repetition frequency of the range, *fi* | Statistical frequency of stagger *pi* | Mathematical expectation *mi* | Theoretical frequency *Pi* |
| 360-370 | 1 | 0,0033 | 1,2167 | 0,0058 |
| 370-380 | 3 | 0,01 | 3,75 | 0,0194 |
| 380-390 | 9 | 0,03 | 11,55 | 0,0499 |
| 390-400 | 33 | 0,11 | 43,45 | 0,0995 |
| 400-410 | 53 | 0,1767 | 71,55 | 0,1552 |
| 410-420 | 59 | 0,1967 | 81,61 | 0,19 |
| 420-430 | 58 | 0,1933 | 82,16 | 0,1838 |
| 430-440 | 36 | 0,12 | 52,2 | 0,1413 |
| 440-450 | 28 | 0,0933 | 41,53 | 0,0867 |
| 450-460 | 14 | 0,0467 | 21,23 | 0,0427 |
| 460-470 | 6 | 0,02 | 9,3 | 0,0169 |
| Total | 300 | 1 | 419,56 | 0,9954 |

On the straight section, 400 stagger measurements were recorded (Fig. 4a). The statistical mean stagger value was 294.0 mm, the statistical variance was 295.5 mm², the standard deviation was 17.2 mm, and the coefficient of variation was 0.0585.

|  |  |
| --- | --- |
| *а* | *b* |

**FIGURE 4.** Histogram of statistical and theoretical distributions of stagger measurements from the OCSTC on straight (a) and curved (b) track sections

On the curved section, 300 contact wire stagger measurements were recorded (Fig. 4b). The statistical mean value of the stagger was 419.56 mm, the statistical variance was 368.15 mm, the standard deviation was 19.18 mm, and the coefficient of variation was 0.0457.

**MATHEMATICAL MODEL OF THE INFLUENCE OF POLE INCLINATION ON THE GEOMETRY OF THE OVERHEAD CONTACT SYSTEM**

The mathematical model describes the spatial deviation of a support considering two angles: β — the inclination angle from the vertical, and γ — the direction of inclination in the horizontal plane relative to the track axis. These parameters determine the displacement of the contact wire attachment point, expressed through projections onto the horizontal and vertical axes, corresponding to stagger deviation (Δa) and suspension height deviation (Δh), respectively.

Δ*a*= H⋅sin(β)⋅cos(γ)

Δh= H⋅sin(β)⋅sin(γ),

where, **H** — Height of wire attachment on the support.

Radius of rotation (deflection) of the suspension cable fixation point R2, and the fixation point of the contact wire R1 (Fig. 5) are determined by the following formulas:

|  |  |
| --- | --- |
|  | (13) |
|  | (14) |
|  | (15) |

where, – Radius of pole rotation relative to the track axis; – Radius of pole rotation relative to the axis of the catenary wire; – Radius of pole rotation relative to the axis of the contact wire.



**FIGURE 5.** Determination of changes in the geometric parameters of the overhead contact system due to pole inclination perpendicular to the track

– Radius of the support column at the height of the conditional foundation top level (CFTL)

|  |  |
| --- | --- |
|  | (16) |

– Radius of the pole at the rail top level (RTL)

|  |  |
| --- | --- |
|  | (17) |

- Height from the RTL to the center of pole rotation.

To develop the mathematical model, it is assumed that the center of pole rotation is located at the level of the CFTL:

|  |  |
| --- | --- |
|  | (18) |

The angle of the pole rotation radius relative to the track axis is defined as:

|  |  |
| --- | --- |
|  | (19) |

The angle of the pole rotation radius relative to the catenary wire axis is defined as:

|  |  |
| --- | --- |
|  | (20) |

The angle of the pole rotation radius relative to the contact wire axis is determined by the following formula:

|  |  |
| --- | --- |
|  | (21) |

When the pole is inclined at an angle β, the height of the catenary wire and the contact wire is determined accordingly as follows:

|  |  |
| --- | --- |
|  | (22) |
|  | (23) |

The "+" sign indicates the direction toward the field side, and the "−" sign indicates the direction toward the track.

|  |  |
| --- | --- |
|  | (24) |
|  | (25) |

By subtracting equation (24) from equation (23), we determine the new value of the stagger and the magnitude of its change.:

|  |  |
| --- | --- |
|  | (26) |
|  | (27) |

Now let us consider the changes in geometric parameters when the pole is inclined diagonally. In this case, the stagger deviation consists of two components (Fig. 6):



**FIGURE 6:** Determination of changes in the geometric parameters of the overhead contact system due to diagonal pole inclination

|  |  |
| --- | --- |
|  | (28) |

where the deviation is caused by inclination along the track: Deviation perpendicular to the track: where – Diagonal deviation, it is determined by:

(29)

– Pole inclination direction angle.

The formula for determining the stagger deviation ∆a for a general case is as follows:

|  |  |
| --- | --- |
| + | (30) |

where – Length of the additional clamp

. (31)

Based on the developed mathematical model (28), the dependence of stagger deviation on the inclination angle was constructed for eight spatial directions (Fig. 7).



**FIGURE 7:** Dependence of ∆a(β) on the pole inclination direction γ

Based on the developed mathematical model, it is possible to determine the deviations in contact wire height, clearance, and stagger depending on the inclination angle of poles in any direction. The following values of the inclination direction angle γ are assumed:

γ = 0° — the pole tilts outward, away from the track (toward the field);

γ = 90° — the pole tilts along the track in the direction of train movement;

γ = 180° — the pole tilts inward, toward the track;

γ = 270° — the pole tilts along the track, opposite to the direction of movement;

γ = 45° and γ = 315° — the pole tilts diagonally outward (toward the field);

γ = 135° and γ = 225° — the pole tilts diagonally inward (toward the track).

Maximum displacements occur under diagonal tilting, where both the stagger and suspension height deviate simultaneously, creating the most hazardous conditions for current collection at high speeds. Model analysis has shown that the contact suspension is sensitive to pole inclination: at an angle of β = 2°, the stagger deviation may reach 15 mm, and at β = 3°, it may exceed 100 mm, which goes beyond the permissible limits established by technical standards.

According to regulations, the maximum permissible stagger deviation is ±300 mm on straight sections and ±400 mm on curves, while the allowable deviation in suspension height is ±50 mm. Exceeding these limits can result in pantograph impacts, electrical arcing, and accelerated wear of the contact wire.

For high-speed railways such as Afrosiyob, such deviations are especially critical as they directly affect operational safety.

**CONCLUSION**

The analysis showed that deviations of reinforced concrete poles from the vertical have a significant impact on the geometry of the overhead contact system. The developed mathematical model enables a quantitative assessment of the influence of pole inclination on system parameters and helps to identify potentially hazardous conditions. Incorporating the direction of inclination (γ) into regular diagnostics opens the possibility for a shift toward proactive maintenance, reducing operational costs and increasing safety levels. The model can be implemented as a software module for real-time evaluation of hazardous inclination directions, enabling timely identification of critical sections, forecasting the remaining service life of the contact suspension, and setting priorities for maintenance activities.

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