**Combined Vibrations of a Two-Layer Cutted Conical Shell with a Viscoelastic Filler**

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**Abstract**. The reason why conical structure is used in this particular engineering is the same. The conclusion drawn from the article is that methods also an algorithm have been developed to solve the problem of natural vibrations of a viscoelastic, two-layer, truncated conical shell., the faces of which are freely supported or hinged. The movement equations for a conical shell made of two layers that are viscoelastic and contain a filler are created using Lagrange's second-order equations. The problem is approached by solving a particular mathematical equation called the characteristic equation, which is a type of frequency-transcendental equation, and this equation is solved using the Muller method. Based on experimental data, it was established that, the real and imaginary parts of the first and second natural frequencies reduce monotonically with decreasing thickness of the conical shell whereas the real part of the third and fourth frequencies reduce more slowly. Therefore, the related imaginary parts monotonically grow. It is discovered that the increment of number of faces causes the increment in the real part and imaginary part of the natural frequencies.

**Keywords:** own oscillations, cutted conical shell, filler, transcendent equation, damping, frequency

**INTRODUCTION**

In engineering practice, conical shaped structures are used various branches of mechanical engineering, aviation, and shipbuilding as well as rocket technology. To increase the overall stiffness of the structure, the thin-walled shell is reinforced with fillers. The slight addition of mass in this filler results in a huge increment in mechanical strength inspired by this filler at this small thickness.

In some studies [1, 2, 3], people found ways to calculate the natural shaking patterns of thin, cone-shaped elastic shells. It's still very important in real-world situations to look into and reduce the strong shaking effects that happen in layered shell structures. As the works [4, 5, 6] are dealing with theoretical and experimental investigation of vibrations of the circular conical structures, advanced computational methods to check the behavior of resonance under varied physical and geometric circumstances have not been developed. The research [7, 8, 9, 10], in its turn, provides analytical also experimental methods of determining the resonant frequencies also patterns of vibration in sealed cone-like sectors. An alternative analytical technique commonly used in shell analysis involves transforming the stability equations for conical shells into equivalent formulas for cylindrical shells with circular geometry. In research, they are usually used without momentum theory of shells or semi-momentless theory of shells. Approximate methods are also used.

An overview of existing research shows that the available constructions of such shells, determined by specific geometric and rheological characteristics, are practically impossible, therefore, the research is mainly theoretical.

This study looks at the issue of vibrations in a conical shell that has been cut off and has two layers.

**FORMULATION OF THE PROBLEM, SOLUTION METHODS, AND BASIC RELATIONSHIPS**

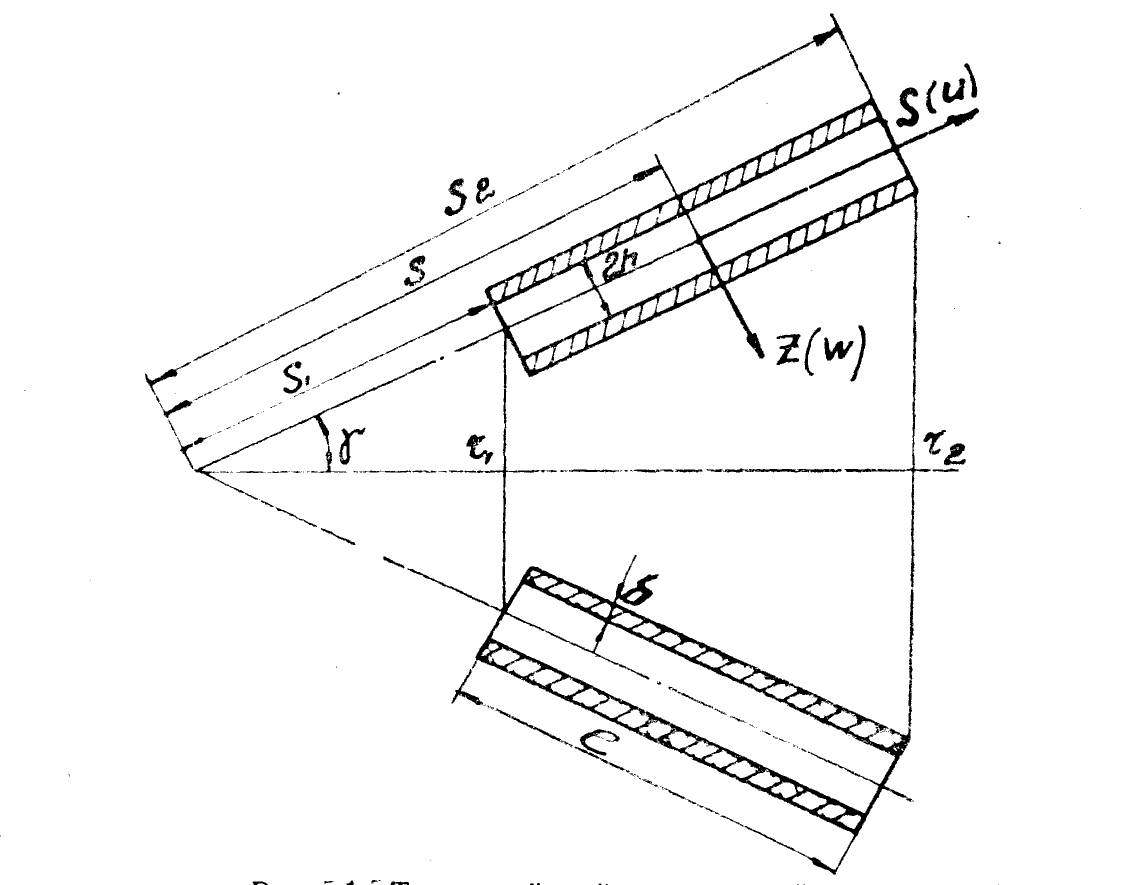
Let us examine a slender, two-layer truncated conical shell that is simply supported at its edges by non-deformable restraints.

A closed circular conical shell has a conicity angle, thickness (respectively along the longitudinal and circular directions) (Fig. 1). The shell structure is symmetrical with respect to the middle surface of the aggregate, i.e., the thicknesses of the supporting layers are equal. The packet thickness is constant. The stiffness of a two-layered truncated conical shell with a viscoelastic aggregate has the form:

(1)

where, - is the instantaneous cylindrical stiffness, -is the relaxation kernel, is the voluntary time function, is a deformation and, accordingly, a stresses respectively, the Poisson ratio *v (t)* that is time dependent and related to viscoelastic materials makes an impact in the deformation and stresses developed in the filler. Figure 1 has the other notations.

A coordinate system is regarded according to which the origin of that system is located at the mid-surface of the shell. This axes “” also “” of the cylindrical coordinate system, coinciding with the principal directions of curvature, are shown in Figure 1. The radial axis runs in a direction at right angles to the surface, pointing inward toward the curved, inner part of the shell. In the layers supporting the structure we make the correct Kirchhoff Love assumption assuming the normal straight. For the filler material, we assume it is a straight line. We consider the filler to be light and unable to be compressed. We will examine vibrations caused by the shell’s curved middle surface, especially those that match the so-called oblique-symmetric vibration patterns.



**FIGURE 1.** Truncated three-layered conical shell

The theoretical determination of the potential energy is done under this classical theory of elasticity, formula:

(2)

(These indices upper also lower carrier layers are represented by 1,2 (respectively). According to the given assumptions regarding the action of the aggregate solely in the sense of displacement the potential energy:

(3)

where,

(4)

(5)

and

(6)

Substituting (5) into (4), taking into account (1) and (2), we perform the integration of within the range of from to for the upper supporting layer and from to -for the lower supporting layer, respectively. Then, substituting into (6) taking into account (3), we perform the integration for the aggregate. In variables:

(7)

by performing the operations specified in (6), we obtain a system of five integro-differential equations describing the oscillation of a three-layer shell in a vacuum:

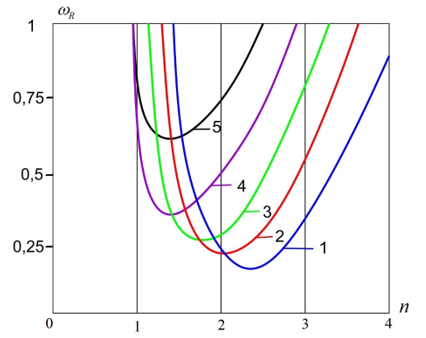
(8)

We will take the approximate expressions for displacements, as done in work [8], in the following form:

**RESULTS AND ANALYSIS**

The shape of displacements is such that the displacements in the ends is, i.e., free supported. In the case, the boundary conditions are approximately satisfied.

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**FIGURE 2.** The connection between the real component of the dimensionless natural frequency is 𝜔 𝑅 ωR and the mode number 𝑛. n) of a conical shell. The curves which are marked 1, 2, 3, 4, 5 are pertaining to various forms of the structure   
(or to various parameters of the form)

Imaginary part of a complex frequency of a truncated conical shell where n is a parameter of variation with different angles of conicities:

1= 0; 2. =0.20; 3. = 0.40; 4. = 0.60; 5. =0.80.

To find the frequency equation, the final expression is combined with the important variables and the result is set to zero. When this is done by adding up along the length of the curve, it creates a group of simple math equations. Making the biggest number in this group equal to zero gives the special equation that shows the frequency [9, 10]:

(10)

If the relaxation kernel of a two-layer truncated conical shell is considered null, the system behaves as a purely elastic structure. Under these conditions, the problem reduces to determining the natural frequencies of free vibrations in an elastic shell.

For modeling the viscoelastic behavior, the two-parameter Koltunov–Rzhanitsyn kernel is employed. Now the result is a complex-coefficient system of differential equations obtained by transformation of the integro-differential formulation

The following parameters are entered as initial data: Radii of the base, thicknesses and heights of the shells, the half-peak angle of the cone, the Young modulus of elasticity, the Poisson ratio, the parameters of the relaxation kernel, the geometric and the physical-mechanical parameters of the reinforcing ribs.

In Figure 2, it can be noted that the addition of the four longitudinal hard edges spurs the increment of this real also imaginary components of the natural frequencies.

The case φ = 0 corresponds to the hardened shell, and the values φ = 0.20; 0.40; 0.60 and 0.80 are attributed to different angles of conicity.

If we look at how the ring edges change, we can see that the highest points of the movement get smaller when the edges become larger.

**CONCLUSIONS**

A method has been created to study how rib-reinforced viscoelastic conical shells vibrate freely.

The set of finite element method (FEМ), freezing method and Muller, Laplace and Gauss methods constitute its set of method of solution.

Numerical bulk indicates that both the real and the imaginary parts of the first and the second natural frequencies will become lower when the thickness of the viscoelastic shell decreases. However, the actual frequencies of the third and the fourth frequencies decline at a moderate rate and the imaginary frequencies of the frequencies increment gradually.

Consideration of the rheological properties of the shell material enables you to compensate the values of the natural frequency within range 10%, which makes that analysis of vibration more accurate and general.

**FUTURE SCOPE**

Layer and Material Optimization: Determine how one can use new materials (lightweight and high-strength materials) to develop the design of the two-layered vanes of the conical shell. The vibration control can be achieved more effectively (the range of possibilities is extended) with the optimization of the properties of the viscoelastic seals (e.g., the viscosity, the idleness). Complications of Geometries and Practice: Experiments on volumes and dimensions of the truncated shapes of the conical shells of diverse forms. This would allow them to have even more applications in the aerospace, automobile and the construction businesses e.g. creating vibration resistant building, or specializing in acoustic insulation. Dynamic Characteristic Analyzing Styles More specifically: a more specific representation of visco elastic seal dynamic characteristics with respect to temperature, frequency and external loadings. This will lead to enhanced precision in the prediction of the vibration and its countering. Computer Simulations and Artificial Intelligence: Rapidly calculate the dynamic features of vibrations in simulation time or as accurately as possible by using computer simulations (e.g. finite element method) and artificial intelligence-based algorithms, and perform dynamic characteristics of vibrations in simulation time or as accurate as possible by using computer simulations (e.g. finite element method) and artificial intelligence-based algorithms, and describe the dynamic features of vibrations. This will be time saving in the designing and less cost. The elaboration of experimental researches: Experimental vibration of individual shells filled with ductility under multiple complex conditions (e.g. in high temperature, in condition of overloading). This will increase the reality of credibility. Sustainability and environment: Developing viscoelastic material and shells which can be recycled and are environmental friendly .. This will enable minimization of the environmental pollution and sustainability of the sector. Acoustic and vibration control:

Attenuation of noise and vibration using such structures (e.g. in aircraft, ships or industrial equipment). There is a possibility that in the future new solutions can be discovered in the process of acoustic engineering. The given areas will give the research in the field of the double-layered conical shells and viscoelastic filling in the further development.

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