**Nonlinear Oscillations of a Viscoelastic Wing**

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**Abstract**. The problem of the aircraft structural strength and its reliability and the question of fatigue in materials and specific components of the architecture continue to be of focal importance to the designers. Of late, even though there have been variations in the dimensions in airplanes, there is also an increase in the flying speed, thus making these problems all the more relevant. These problems can be effectively solved provided the consequences of aircraft vibrations and deformations during the action of the external variable loads are accurately accounted. This proposed methodology would suffice to solve the problem and the computational algorithm would be developed to determine the critical velocity of bending-twisting flutter of a large, elongated viscoelastic wing. The transcendental equation has been solved using Muller method. The algorithm developed enables one to construct own wing shapes, and taking into consideration the viscoelastic features of the material, it has become apparent that by taking into consideration the viscous properties of the material, the critical velocity increases by as much as 15 per cent.

**Keywords:** critical velocity, wing, aerodynamic load, flutter, self-form, stationarity hypothesis

**INTRODUCTION**

Structural integrity and reliability of the aircraft with parts, and fatigue in the material, and life of a particular part of the aircraft structure is always one of the issues of utmost importance to engineers [1, 2, 3, 4]. These problems have acquired additional significance in the recent past as there has been an increase in the mass and size of aircraft and additionally the velocity of flight. To solve these issues, it is very important that the impact of oscillations and deformations of aircrafts under the influence of external variable forces be accurately assessed. When comparing the structural strength, cost-effectiveness, and producibility of the main components of the aircraft (LA), and regarding the major rheological and hereditary-deformable properties of these materials at all temperatures, it is possible to suppose that the LA is not only the elastic being but a hereditary-deformable solid body. The considerations of these material properties during the calculation of the stress-strain state of the aircraft (LA) enhance the extent of alignment of the theoretical model with the real working conditions of the applicant [5, 6]. The problems of aeroelasticity are classified in the works [7, 8] and those pertaining to the dynamic loads are artificially split into the problems of the other types of loads i.e., flatter, divergence, etc. [9, 10, 11, 12, 13].

The study aims at an exploration of the flight of aircraft flutter under the influence of loads due to atmospheric pressure within a gas. The issues relative to the deformation and oscillation of the hereditary-deformable elements of the aircraft structures are discussed in their inseparable organism of relation with these goals. Simultaneously, the design of the aircraft is a complex system of oscillatory deform-abable dynamics as hereditary.

**METHODS**

A console arrow-shaped hereditary-deformable wing of large elongation with a straight deformable axis normal to the root cross-section is considered. We will assume that the wing is rigidly fixed in the massive fuselage, which we will consider stationary. Thus, the analysis excludes the impact of the remaining parts of the aircraft (LA) and, in particular, the fuselage on the wing flutter. Considering that the wing material obeys the linear hereditary theory of viscoelasticity [14, 15], we present the main solving equations of the bending-twisting flatter in the direct formulation of the problem:

(1)

where

Partial derivatives (1) describe the weakly singular system of integro-differential equations (SID) of the problem of bending-twisting stabilization of the flatter of the hereditary-deformable wing of finite amplitude. In order to solve the SID system (1) three boundary conditions must be obtained at each end of the wing.

In the reinforcement, at the left end of the wing, the deflection, the first derivative of it with respect to the coordinate x and the rotation angle must be equal to zero:

(2)

At the right load-free end of the wing, the bending and torsional moments and the shear force will be equal to zero:

(3)

### The system (1) together with the boundary conditions (2), (3) represents the direct formulation of the problem of the bending-twisting stabilized flatter of the hereditary-deformable wing. Since a steady-state process is considered, the initial conditions for system (1) are not set.

Let's assume that at the flatter boundary, the motion is harmonic. Therefore, the functions determining the unknown deflection and the angle of twist can be represented as

(4)

where A, B are unknown constants; *ω* is the unknown flutter frequency.

If it is necessary to investigate the stability of the lower forms of oscillations, then as the first approximation for the functions and can be taken, for example, the shapes of the main tones of purely bending and purely twisting oscillations of a cantilever wing modeled by rods. In this case, the following will serve as the specified functions:

(5)

or

. (6)

If we use easily provable integral identities:

where

and substituting (5) into (3), then after performing the known Bubnov-Galerkin procedure, we obtain

(7)

The system (7) constitutes a homogeneous set of algebraic equations concerning the coefficients A and B. A necessary and sufficient condition for the presence of non-trivial solutions is that the complex determinant equals zero.

. (8)

Equations (8) completely coincide with the equations obtained in [7, 8].

We give an algorithm, which is thoroughly tried, of solving the system (15) numerically, by the Muller method.

Assuming , we calculate , where θ– corresponding is a chosen and non-zero constant. Let's construct an iterative circuit:

(9)

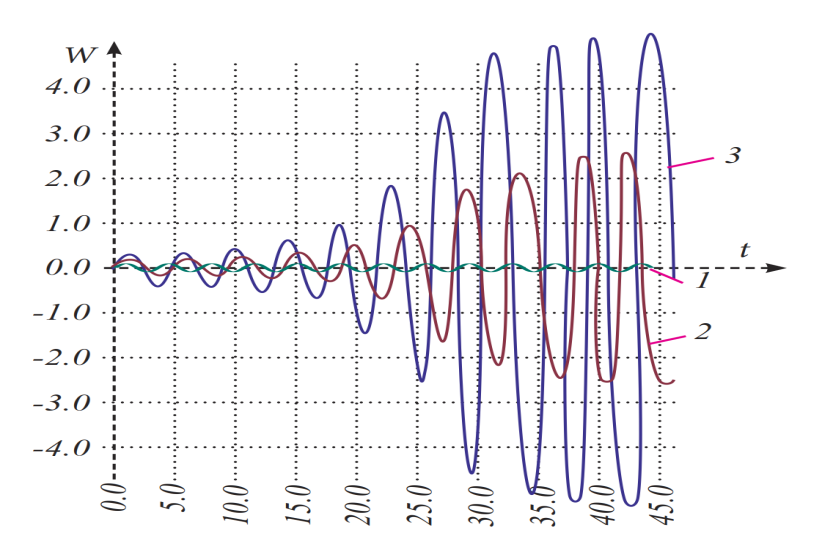
and we calculate

If the quantities (9) become smaller than the previously given small quantity , then the sequential approximation process is completed and the solution of system (9) is taken ; otherwise, the iteration is repeated. Having identified the velocity and frequency of oscillations during flatter it is possible to find the ratio A/B which defines the amplitudes of bending and torsional oscillations. The general algorithm of solving nonlinear problems of dynamics and dynamic aero-stability of hereditary-deformable systems of type (9) on computer was proposed, which allows to examine the flutter phenomena in certain conditions.

**RESULTS AND ANALYSIS**

The calculation was performed as in the case of perfect elastic linear and nonlinear (*ε=0*), as well as in viscoelastic linear and nonlinear settings, taking into account and without taking into account aerodynamic damping. The relaxation behavior is described using the three-parameter Koltunov–Rzhanitsyn relaxation kernel   
, The calculation results are presented in Figure 1. The arrival of the flutter phenomenon was investigated.

On figure 1. The dependence of displacements on time at different values of the relaxation nucleus amplitude for 1. А=0.10; 2. А=0.010; 3.А=0.001.



**FIGURE 1.** Dependence of displacements on time at different values of the relaxation nucleus amplitude:   
1.А=0.10; 2. А=0.010; 3.А=0.001

It can be seen that with increasing viscosity, the displacements decrease depending on the time.

**CONCLUSIONS**

The direct formulation of the problem of forced oscillations of hereditarily deformed aircraft in the gas flow of the flight mode up to sound with distributed parameters has been formulated;

a new method for finding the critical velocity and frequency of the problem of the bending-twisting steady-state flatter of a hereditary-deformable wing of great elongation has been developed.

**FUTURE SCOPE**

Nonlinear oscillations of viscoelastic wings are an area of study that is open to plenty of research possibilities and real-life involvement in the area of the aeroelasticity of structures, dynamics of structures and smart structures in the aerospace sector. Due to the evolution of aerospace systems to be lightweight, easily bendable, and adaptive, the complex viscoelastic phenomena are more demanding to be understood. The major future research scopes are provided below: Advanced Material Models Development In further studies it is possible to include more realistic constitutive models to describe the viscoelastic behavior, e.g. fractional-order constitutive models or hereditary models are also found to better reflect the time dependent and memory effects commonly observed in today composites and polymeric materials found in an aircraft wing. Coupled Aeroelastic-Acoustic- Structural Analysis There is a new developing field in the combination of nonlinear viscoelastic wing dynamics with acoustic and aerodynamic settings under the condition of real flight. This multiphysics coupling may be critical when it comes to making quieter and more stabilized flight system particularly in the UAV work and even high speed aircraft. System Identification and Experimental Validation Theoretical and numerical models can be verified through high-fidelity experimental (such as wind tunnel tests and real-time measurements of vibration) setup. The damping and stiffness parameters can also be obtained directly in the form of flight data by use of system identification methods. Passive and Active Vibrational Control Strategies As interest increases in adaptive structures, the integration of such smart materials (e.g. piezoelectrics, shape-memory alloys) into viscoelastic wings provides avenues towards the achievement of active damping. At the same time, the passive control means optimization (e.g., tuned mass dampers, viscoelastic layers) is also a feasible research topic. Bifurcation and nonlinear stability Analysis Possible optimisation of the study of the beginning of nonlinear phenomena, like limit cycle oscillations, subharmonic resonance and bifurcation behaviour, under different flow velocities and structural configurations would allow predicting and preventing the flutter and other aeroelastic instabilities.Morphing and Bio-inspired Wings Laser signal encoding and signal duplication There are a few possible applications to morphing and bio-inspired wings the first being to a laser signal encoding and signal duplication. It is also possible to use results of nonlinear viscoelastic analysis and relate them to morphing wing technologies, wherein the shape change is of primary concern and the flexibility is key. Future flight performance is significant to the nonlinear and viscoelastic performance of these types of adaptive structures. The Integration of Machine Learning and Numerical Simulation Use of numerical techniques such as reduced-order modeling, isogeometric analysis and deep learning has a lot of potential to improve accuracy of the simulations and even the execution of the simulations. It is also helpful to use supervised learning method in control and real-time estimation of wing vibrations.

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