**Theory of Developing a Mathematical Model for Calculating the Zone of Determination of a Mobile Unit**

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**Abstract.** This article explores and discusses the development of a mathematical model for automation sensors with current potential information readout, and also provides a method for determining the maximum shunting zone of a train unit when moving away from the rail line sensors. Based on the developed method and algorithm of mathematical calculation, it is possible to do research and also determine the optimal parameters of sensors with a current potential receiver. Based on the developments, it is possible to take technical measures to increase the length of the train release of the rail line.

**Key words:** track circuits, jointless track circuits, shunt, four-terminal network, transmission resistance

**INTRODUCTION**

The wide development of industry and economic development requires an increase in the transportation process using advanced technologies. Especially with the development of high-speed traffic, it is necessary to develop new systems in the railway industry. The current state of railway automation and telemechanic is characterized by the processes of intensive development and implementation of devices implemented using modern achievements of microelectronics, microprocessor technology [1], the development of a new theory of signal transmission and processing, the development and improvement of the synthesis and theory of track circuits [2]. Therefore, in connection with the development of high-speed traffic, there is a need for an important and responsible approach to the problems solved by these systems, and a number of specific requirements are imposed on them [3]. In addition, when constructing devices of the "Modern System for the Development of Railway Automation and Telemechanic", various external factors are taken into account that affect the implementation of the interval control system of the main production and technological algorithm, and its productivity [4], as well as the requirement to ensure the efficiency and safety of the transportation process [5, 6, 7, 8, 9, 10, 11]. The main elements of almost all railway automation systems, including microelectronic ones, are new-generation track circuits, which perform important functions of monitoring the condition of track sections during the transportation process, reliable transmission of information to the locomotive and, in some cases, transmission of information between automatic [2] blocking and signaling units. Track circuits operate under difficult conditions of traction current interference and environmental influences [11, 12, 13, 14, 15].

Due to the growing needs of industrial and economic growth, there must be improvements in the transportation, especially in the railway sector as high-speed traffic was developed. This has brought intensive development and exploitation of modern automation of the railway and telemechanic, which makes use of microelectronics and micro processing.

An essential part of such automation systems is the track circuit, which controls the condition of sections of the track and sends the information. These circuits though are used in environments where there is interference caused by traction current and other external interferences. One major problem with jointless track circuits is the signal of one sensor bleeding into neighboring one, potentially causing the wrong sensor to be shunted to when a train is nearby, disabling an otherwise free track, 7which the paper proposes to solve with a special sensor with readout of current potential.

**The Problem of the "Danger Zone"**

When the track receiver has a potential connection the track receiver is excited not when the train passes the sensor and has gone a short way, but only when the train has traveled some distance away. This length is called the danger zone of shunting (ldzsh). Measurement of this zone in the most accurate way is very important when considering the efficiency and safety of the train movements, particularly high-speed rail.

**Proposed Solution**

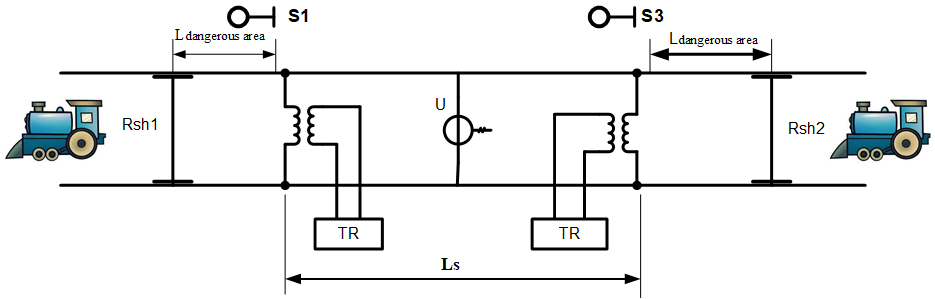
We would like to suggest studying this danger zone with a mathematical model that will enable accurate calculations. They take the track circuit and model it as an equivalent electrical circuit and this leads to a set of differential equations which characterize the voltage and current along the rail line as a difference equation. These formulas take into consideration the energy that has accumulated in the rail line and consider the coefficients as time dependent and this nature defines the transient behavior of the tracks in the circuit masses.

Decision on the length of the dangerous shunting zone under different conditions can be made using a computation algorithm presented in the paper based on this mathematical anchor. This enables specifications of the boundaries of the bypass zone which is a necessary condition to create high-speed and safe transport systems.

**METHODOLOGY**

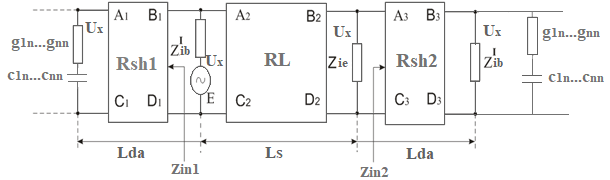
When conducting a study of the control sensor, it is necessary to take into account many factors affecting the operation of the control sensor, but one of them is very significant, namely, in the absence of insulating joints, the neighboring sensor also receives a signal from the neighboring sensor along the rail line, therefore, when the moving unit approaches to the train, the sensor is shunted to the traffic light, which leads to an unreasonable stop of the train on a free track circuit. To reduce the influence of the shunt effect, we considered a new type of sensors with current potential information readout.

Therefore, with a potential connection of the track receiver to the track circuit fig. 1 in normal mode, the track receiver TR is excited not at the moment the train releases the supply end of the sensor, but when the train moves away for a certain distance *l*dzsh, which is called the danger zone of shunting.

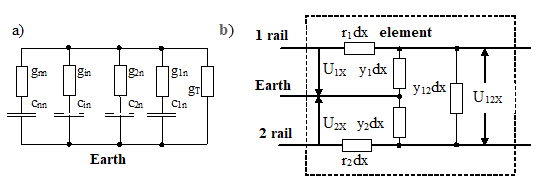


**FIGURE 1.** Diagram of a sensor with a current potential receiver

To determine this zone, let's represent the circuit Figure 1 as an equivalent circuit Figure 2.



**FIGURE 2**. Equivalent circuit



**Figure 3.** Insulation resistance equivalent circuit

Taking into account the fact that the ballast on each of the track circuits can have different values, we denote the wave propagation coefficients γ and wave resistances Zw, respectively, through various states of the ballast. After changing the polarity of the supply voltage for the 70th min. only those capacitors сin have time to recharge, the circuits of which have a small time constant τin [4, 5]. The remaining capacitors are either partially or slightly discharged. At the moment of turning off the external power source, the magnitude and polarity of the voltage on the rails is determined by the voltage of the capacitors, in series with which the conductance’s gin are connected, which have a large value.

These capacitors begin to discharge to the gТ element and other parallel branches, the capacitors of which are charged to a lower voltage value or have a charge of reverse polarity. Over time, the voltage between the rail threads increasingly begins to depend on the voltage on the capacitors, in series with which elements with a lower conductivity are connected. The discharge of these capacitors proceeds slowly, and the voltage on the rails changes smoothly.

The equivalent circuit of the dx element of a symmetrical rail line is shown in fig. 3b. Complex conductivities y1 and y2 contain: *g1n, g2n,…gin,…gnn* - specific conductivities of the grounding branches of rail threads (not shown in the figure), *с1n, с2n,…сin,…сnn* - specific capacitances of the grounding branches of rail threads , gT – grounding conductivity of rail lines in steady state, complex conductivity y12 contain *g1р, g2р,…giр,…gnр* - specific conductivities of the branches between rail lines, *с1р, с2р,…сiр,…сnр* - specific capacitances of the branches between rail threads, gt – specific conductivity between rail threads in steady state, *r*1 and *r*2 – specific resistance of rail threads, *r1* = *r2 = r*.

The complex specific conductivity of the grounding of a rail thread can be represented in the following form:

where *x* – voltage between the first track and ground, *, ,…* is capacitor voltage, , ,………. is time constants.

If you start the countdown at the very beginning of the transient process, when   
 *,* that

(1)

Equation (1) is reduced to the form:

,

where

,

Similarly, the complex conductivity of the second rail thread and the specific complex conductivity of the upper layer of the ballast and sleepers are expressed:

,

(2)

where

;

;

.

Based on Kirchhoff's law, we can write:

; (3)

; (4)

; (5)

(6)

The solution of the resulting system of equations when representing as a function of the coordinate is rather complicated, therefore, we replace the voltage values ​​on the capacitors *с1n, с2n,…сin,…сnn* with the voltage Umid middled over the differentiable section.

Equations (4 and 6) are reduced to the form:

(7)

(8)

where

;

Differentiating equations (3 and 5) and then substituting in them instead of and their values from equations (7 and 8), we obtain:

(9)

(10)

If we solve equation (9) with respect to U2X, then find the second derivative and, based on them, replace equation (10), we get:

(11)

The general solution of equation (11) will be the sum of the particular solution of this equation and the general solution of the corresponding equation without the right side

. (12)

A particular solution of the differential equation (12) is the expression

. (13)

Taking the second derivative of equation (13) and substituting into equation (12), we obtain:

. (14)

Expression (14) is a biquadratic equation, the roots of which are determined by the formula:

Denote:

(15)

(16)

The general solution of differential equation (14) will be

A particular solution of differential equation (11) is the following expression:

(17)

Adding the particular solution of equation (11) with the general solution of equation (12) and performing transformations, we obtain:

(18)

The values A1, A2, A3 and A4 are integration constants and are found for each particular track circuit based on the boundary conditions.

Differentiating equation (3), replacing the value in it with its value from equation (7) and then solving it with respect to , we get:

Making a replacement based on equation (18), we find:

(19)

Substituting into equations (3 and 5) instead of and their values from equations (18 – 19) and solving the resulting system of two equations for *I1X*and *I2X* , we find:

(20)

(21)

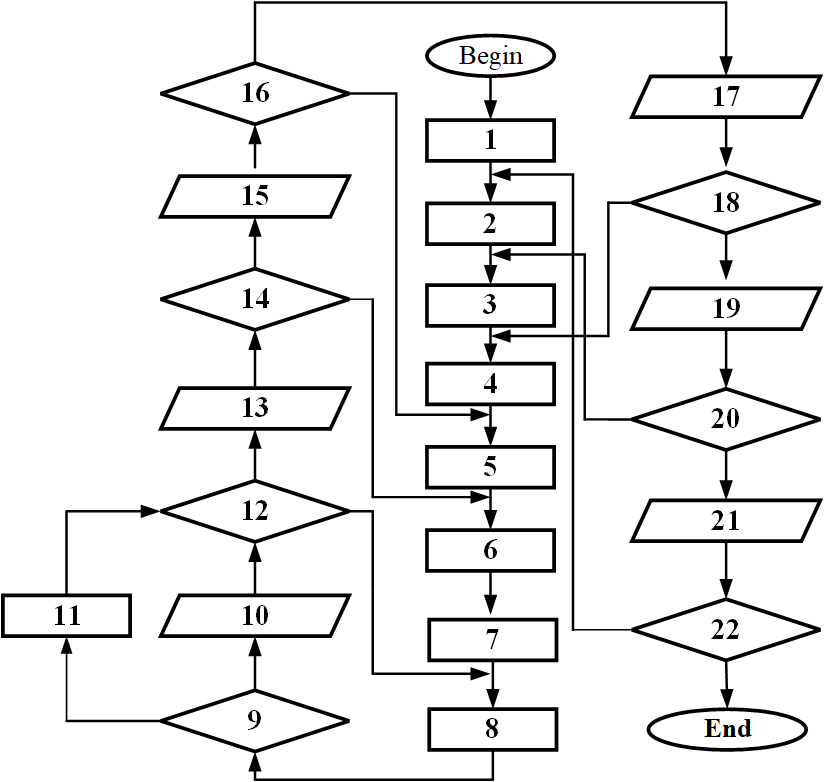
where

;

;

**RESULT**

Equations (18 - 21) express the pattern of voltage and current changes along the rail line depending on its parameters and the distance *x* from the end of the rail circuit to which the load is connected. The produced mathematical expression made it possible to create an algorithm of work and a block diagram was created.



**FIGURe 4.** Algorithm for determining the length of the additional bypass zone

The block diagram of the algorithm for determining the length of the dangerous shunting zone is shown in fig. 4 and contains the following blocks:

1 is block for input of initial values of rail resistance Z, ZIib and their arguments fz, fib;

2 is input block of the initial value of the length of the rail circuit *l*;

3 is block for input of initial values of insulation resistance of adjacent track circuits ri1, ri3;

4 is block for input of initial values of insulation resistance of the studied track circuit ri;

5 is block for input of initial values of determined length of additional shunting zone *l*sha ;

6 is block for calculating transmission resistance in normal mode ;

7 is block for calculating the wave resistance of the track circuit transmission ;

8 is comparison block ;

9 is input block of the initial value of the length of the rail circuit *l*;

10 is block for input of initial values of insulation resistance of adjacent track circuits ri1, ri3;

11 is block for input of initial values of insulation resistance of the studied track circuit rи;

12 is block for input of initial values of determined length of additional shunting zone *l*sha;

13 is block for calculating transmission resistance in normal mode ;

14 is block for calculating transmission resistance in the presence of a shunt on an adjacent track circuit ;

15 is block ;

16 is parameter change block *l*нsha = *l*shna +Δ *l*sham ;

17 is output of the found value *l*дшн at the given values: frequency of the signal current; insulation resistance; track chain length; input resistance of the ends of the track circuit;

18 is condition check block *l*shab <= *l*;

19 is block for changing the parameter ri = ri  + Δri;

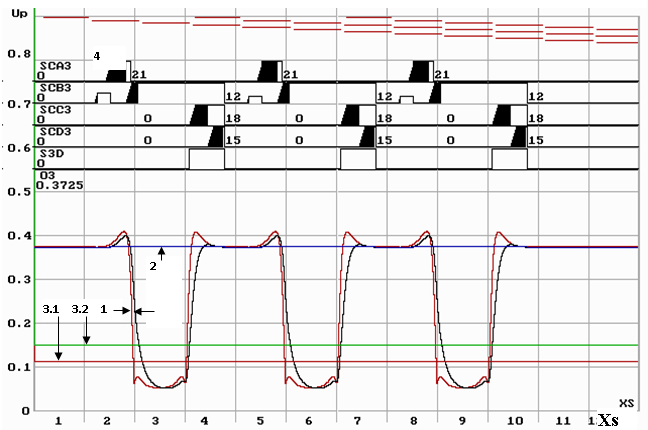
20 is condition check block ;

21 is a block for changing the parameters of the insulation resistance of adjacent track circuits;

22 is condition check block for ri1 ,ri3;

23 is end-of-calculation block.

After the calculations were made and a graph of the influence of the dangerous zones of the approach and departure of the mobile unit is shown



**FIGURE 5**. Shunting schedules for the arrival and departure of a train unit

**CONCLUSION**

The resulting equations differ from the known ones in that they have additional terms that take into account the accumulated energy in the rail line. In addition, the coefficients of the equations, which coincide in form with those derived earlier, and additional terms are functions of time. This dependence characterizes transient processes in track circuits. This allows you to clearly take into account, regardless of the factors that arise at any time of the day, take into account the danger zone when shunting a rail line by a train unit. The conducted researches of this question show the development of electric rail circuits of tone frequency on having iso-joints. The question of definition of the additional zone of bypass, and now there is no clear definition of the boundaries of this zone, it will be possible to clearly define, respectively, and in the future conduct researches to ensure high speeds of movement and safety of transportation.

**FUTURE SCOPE**

The study of the tone-frequency jointless rail circuit which makes use of transient processes as well as time-dependent parameters leaves a number of intriguing perspectives in potential prospects to look into the future:

The future research can be put on mathematically modeling the transient behaviors in track circuits, incorporating multifaceted dynamic interactions between rolling stock and the infrastructure. Next generation of signaling systems will be designed and tested using more precise simulation tools.

The detection of accurate bypass (or shunting) zones is one of the most important applications as well. Improving the current standards of energy buildup and perception of signals within insulated joint zones allows formalizing the algorithm of danger zones identification and making a significant breakthrough in the overall enhancements of operation safety in that context.

By using models included in real-time monitoring systems, predictive diagnostics can be achieved and failures in track circuits can be minimized as well as the downtimes. Intelligent maintenance strategies will have time-dependent behavior analysis to assist in them.

Intelligent and adaptive control systems are the way ahead as far as railway signaling is concerned. The models will be ready to be latterly assembled into AI-based platforms, digital twins, and automated decision support systems where the real-time controls of signals are essential, especially in multidimensional or a high-velocity setting.

With modernization and expansion of rail networks in the form of high-speed corridors and in cities with a significant density of metros, the enhanced perception and competencies of jointless circuits and transient behavior will be essential to provide a comprehension necessary to ensure stability, reliability, and safety when exposed to challenging operational conditions.

Future studies can also help in defining some common guidelines in the design and implementation of tone-frequency signaling systems. This will also be able to support interoperability between various rail networks and bring integrations in railways globally.

Future research into the storage, loss and transportation of energy in the rail line would help towards creating energy efficient signaling infrastructure, which falls in line with the green and sustainable transport philosophy.

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