**The inductive response of the coil to the concentration of ions in the solution**

Bui Dinh Tu1,a), Nguyen Anh Tuan2, Vuong Van Hiep2, Nguyen Hai Chau1, and Hoang Nam Nhat1,b)

1 *Faculty of Engineering and Nanotechnology, VNU-University of Engineering and Technology, 144 Xuan Thuy, Cau Giay, Ha Noi 100000, Viet Nam*

2 *Faculty of Physics, VNU-University of Science, 334 Nguyen Trai, Thanh Xuan, Ha Noi 100000, Viet Nam*

a), b) Corresponding author: [buidinhtu@gmail.com,](mailto:buidinhtu@gmail.com) [namnhat@gmail.com](mailto:namnhat@gmail.com)

**Abstract.** We present in this article the description of the magnetic responses of induction coils immersed in aqueous solutions of ions of different types. This study aims to interpret the results obtained from recent measurements of coils of different geometries and sizes immersed in ion solutions, in order to detect water contamination by specific ions by scanning the inductive responses of the coil when changing the frequency of the applied current.

# INTRODUCTION

Polluted water often contains many types of dissolved organic compounds and metallo-organic ionic complexes. Because metal ions generally produce large electromagnetic responses, detecting the presence of metal ions in an aqueous environment is not difficult, but ion identification, that is, selective detection of ionic compounds or complexes, is often difficult due to large interference with salts and soils presented.

Recently, techniques used for water pollution detection involve various spectral methods such as UV-Vis, infrared absorption, Raman scattering, gas chromatography and/or wet chemical analytical methods using different chemical indicators. The variation of the inductive responses of coils immersed in an aqueous solution of ions has been known for a long time, but this method has not been developed for the detection of a particular type of ion due to the low selectivity caused by the averaging effect of the relaxation times of many ionic complexes. Therefore, we present in this article the analysis of the inductive responses of coils immersed in an aqueous solution of chlorids to provide a solid basis for the interpretation of ionic solutions. The theoretical model is applied to aqueous solutions of NaCl, CuCl2 and FeCl3 to show good agreements in all cases.

As far as is known for elementary magnetism, the self-inductance of a coil is defined as:

*n* *n i* *r* 2 







*L*  *n* 

*l*

*i*

 *nBA* 

*i*

*n**n i**r* 2    

*i i*

0

 *n*2 *S*

*l*

, (1)

where ** is the magnetic permeability, ** ** , **  4**107 is the magnetic permeability of vacuum (*Tm*/*A*),

*r* 0 0

*r* the relative permeability, *r*  1  **, **is the magnetic susceptibility (no unit), *n* a number of loops of wire, *n0*

the length density of loops, *r* the coil diameter, *S* the intersection of the coil, and *l* the coil length.

Some calculated values for the inductance of the coil in air or pure water are given in TABLE 1.

**TABLE 1.** Self inductance *L* (*H*) of a coil of different size, ** = 1 (air, water)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *n* | *r = 0.3 cm l = 5 cm* | *r = 0.5 cm l = 5 cm* | *r = 0.3 cm l = 10 cm* | *r = 0.5 cm l = 10 cm* |
| 100 | 7 | 20 | 4 | 10 |
| 200 | 28 | 79 | 14 | 39 |
| 500 | 178 | 493 | 89 | 247 |
| 800 | 455 | 1263 | 227 | 632 |

Naturally, the self-inductance of coil depends linearly on the magnetic flux and inversely on the self-inductive

current

*L*  *n* / *i*

, giving

1*Henry*  1*Wb* /1*A*  1*T* 1*m*2 /1*A*

. Another useful equation is

*Ldi* / *dt*  *nd* / *dt*  ** *Ldi* / *dt*

1 *Henry*  1*V* 1*s* /1*A* .

, because the self-inductive voltage is defined as

# THEORY AND ANALYSIS

** *nd**dt* ,

#### Magnetic responses of ions in water solution

By immersing the coil in a solution of ions of type *k* (as in a typical NaCl solution), the Brown motion of ions of



charge *q* and velocity *v* in a magnetic field *B R*,*t*  of the coil induces the magnetic moment *q*,*v* of the ions. Under

 

*k*



applied magnetic field *B R*,*t* , orbits tend to form closed curves with their sections oriented against the direction of

##  

    

the field *B**R*,*t* . At any given point *R* , a direction of *B**R*,*t*  forces a given ion to move in an orbit, which is equivalent to a current generated by a loop of wire of radius *r* under applied current *i*, *qv = (it)v = i(vt) = ix*, with *x* as

## 

a distance traveled in a unit of time, therefore, *i = qv/x.* If the speed of ion and distance travelled at a position *R* are

known than the value of *q*,*v* can be estimated for each ion of type *k* (note that *v*  *r* ):

*k*

*q*,*v* 

 **0*ik*

 **0 *q*

*vk*  **0*qkk*

*k k k*

(2)

*k R*,*t*

2*rk*

2*rk k*

2*xk*

*x*

*k*

If *nk* is the total number of *k*-type ions contributing to the inductive magnetic moment *q*,*v* then the total

*k*

contribution of *k-*type ions to the final magnetic moment is:

*M q*,*v*  *n q*,*v*  *n*

**0*qkk*

(3)

*k k k*

*k* 2*x*

Naturally, the total magnetic moments generated by all types of ions in a solution is:

*k*

*k*

*M q*,*v*   *M q*,*v*  ** 

*k*

*niqii*

(4)

*i*

*i*1

0

*i*1

2*xi*

If the *k-*type ions (such as *Fe3+*) possess the intrinsic electronic magnetic moment *e* , then these electronic

*k*

magnetic moments also contribute to the final magnetic moment of the solution:

*e*

*k*



*Q*

*M Q*  *M*

*q*,*v*

*k*

 **0  *i i*

*n *





*e*

 *qii*

0 *i*  2*x*

*k*

*n* 

 *i*    *k*

(5)

*i*1

*M*

*i*1  *i*

 *i*1

Here, *Q*

*q*,*v*



*k*

*e*  *qkk*

*e*  is the magnetic moment of all *k-*type ions*.* For the ions with

*Mk*  *Mk*

 **0*nkk*  **0*nk*  2*x*

 *k* 



no electronic magnetic moments (such as Na+, Cl) only the orbital magnetic moments contribute to the final magnetization of the solution. Naturally, the magnetic responses from these two contributions, orbital and electronic, are different in the frequency domain because they possess different recovery times *q*,*v* and *e* . From (5), the magnetic susceptibility of ion solution is a sum from all ion types:

*Q k Q k M Q*

**  *i* 

*i*1

*i*1 *B R*,*t*

*i* 

(6)

This contributes to the final inductance of immersed coil:

*k*

*LQ*   *LQ*  

*k*

*n*2 ** 1 *Q*  *S l*

(7)

*i*

*i*1

*i*1

0 *i* 0 *i i*

Here,

*Si* , *n*0*i*

and *l* are the average intersection of ion orbits, length density of *i-*type ions, and length of the

coil. From this, the final susceptibility of immersed solenoid is:

*L*  *LN*  *LQ*

(8)

where *LN* is the contribution when measured in air. Because *LN* varies only a little in the frequency domain, while *LQ* depends strongly on frequency, there will be two limit cases. The first case is that the final inductance *L* differs from *LN* only by a background caused by constant *LQ* , or it varies slowly in the given frequency domain. The second case appears when the values of *LN* are much smaller than that of *LQ,* and contribute as a background.

#### Enhancement effect at the coil surface





*B t*

For all solenoids,

 

*B R*,*t*

 is usually constant inside the coil

 

### *B R*,*t*

  , and depends only on time and applied

current. But when immersed in salt water, a small magnetic field will appear right on the surface of the coils. For examples, for the listed thickness of insulation coatings in Vietnamese Standard # IEC 60317/ TCVN 7695 (https://cadivi-vn.com/vn/day-trang-men.html) of several tens m, ~5% of the wire diameter, the appeared magnetic

field may be large enough to cause ions to move in spirals along the wires, therefore the dipped solenoids will become



toroidal tubes with *B R*,*t*  increased as the number of ions moving in the toroidal tubes increases.

 

In the particular case of 0.2 mm diameter wires with an applied current of 0.1A, this surface effect causes the

magnetic field *B*  **0*i* 

0 2*R*

4**107  0.1

## 2** 0.00011

 1.8*G* to appear immediately on the surface of the coil (within a distance

of 0.1mm from the surface), while the magnetic field at the center of the solenoid is

*B*  *n i*  4**107  10

 0.1  1.26*G* (assuming 10 loops / 1 cm).

0 0 0

## 0.01

Evidently, this effect depends on the number of ions **at the surface, which is equal to the outer area of coil (*i.e.*

2*Rl* , *R* is a radius, *l* a coil length) multiplied by the ion density **0 at the surface: ****0 2*Rl* . If each ion causes

an induced current *i* then the magnetic contribution of this ion is *b*

 **0*i* , therefore the total contribution of *k-*

**

type ions to the increase of



**

 inside the solenoid is

2*R*

*B* *b*

** **0*i*

. It follows that

### *B R*,*t*

*k k k*

*k*

*k* 2*R*

*B* ** 2*Rl *0*i*

*k*

*li* . Now, the contribution from all types of ions will be:

*k* 0 2*R*

0 0 *k*

*k k*

**0*i* *k*

*B*   *B*  ** 2*Rl i* *l**i* *lI*

(9)

*i*

*i*1

*k*

0

*i*1

2*R* 0

0 *i* 0 0

*i*1

where

*I*  *ii* is the total current by all types of ions. Naturally, this final contribution depends on the

*i*1

frequency of applied current. Because the contribution of each types of ions depends physically on their motions, with different recovery times **, the final magnetic field inside the coil will vary according to the frequency:

*B* **  *n i*  *B* **  *n i* *lI*

**

(10)

 0 **

*l* 0 0 **

#### Response of orbital magnetic moments of ions

Under applied current (at the given frequency) the induced orbital magnetic moments act against the change of the magnetic field. The process of reversing the direction of orbital magnetic moments depends directly on direction of movement of ions. Therefore, it depends on the interplay between the energy of magnetic field and the kinetic energy

of movement of the ions. Now we assume that the distribution of kinetic energy *p()* of ions follows the Maxwell- Boltzmann distribution of gas:

*p***  *e*

****



*kBT*

(11)

For the *k-*type ions under applied magnetic field we have:

1 *m v*2 *B*

*p* **  *e*

*Wi* *B*

*kBT*

 2

 *e*

*i i*

*kBT*

(12)

*i i i*

Adopting the notation of moment of inertia *I*  *mr* 2 and angular velocity *v*  *r* :

1 *m* *r* 2 *B*

*i*

 2 

* *i i*  1 *Iii* *B* 

2

*p* **  *e*

2





*kBT*  *e*

 *kBT* 

(13)

Now, put *xi* *i*

*i i*

*Ii* , ** , **

*B*

*i*

, we have:

*kBT*

1  *x*2 **2 **2 **2 2 *x*2 *x*

2

1 *x* ** 2 2

1 *x* ** 2

*  *i*

2



**

**2

*i i* 

*  *i*

 **  *xi*

  *i* 

with

**

2

*pi* *xi*   *ie*





**2  *xi*



  *ie*

*B* *i Ii B*



 *e *2

 

 *Aie* 

(14)

*A*  *e *2  *e*

*k BT*

(15)

*i i i*

60 2



p(x)

dp(x)

40

20

**pi(x i)**

0

-20

**FIGURE 1.** *Distribution function pi(xi) and its derivation for given parameters*

It can be seen from in Eq. (11)-(15) that while the density function of *i-*type ions depends only on energy difference between kinetic energy of ions and energy of magnetic field, its magnitude *Ai* depends also on the interplay between the ions and the applied field. We can now derive *pi()* according to *xi* , that is, according to the frequency *i*. The final formula is:

2

1  *x* **2

* *i*

*dp*

  *d*

**2  *x*

*i*

**2  *x d*

*i*

1  *x* **2

 

* *i*

**

1  *xi* **2 

*i*



**2  *xi* 

**2  *xi*  1  *xi* **

*i* 2 ** **2

2

**

**2

2 ** 

2 **  ** 2

2 2 **  

1   *xi* ** 1

 *e*

*dxi*

*dx ie*

*ie*

**2  *x*

*i*

*e*

*dxi*

1  *x* **2

 *e* 

 **

 *i*



**2 *e*

 *e  e* 

 *i*



 



2  

  ** 

*dp* 2

  *i*   **

*x* **  2** *x* 

*i*  *e *

*i*

*e* 2 **  

 *i*   *p* *i* 

(16)

*dxi*

**2

**2  *i*  **2 

Another useful formula is:

1

*pi*

*dpi* 

*dxi*

*d dxi*

ln *pi*

 2** *xi*

**2

(17)

**Fig. 1** illustrates *p* *x*  and *dpi* *xi*  with the following parameters ** 0.2, ** 0.161, **  1 (equivalent to

*i i*

setting *T = 300 K*).

*i*

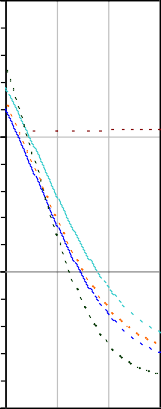
*dxi*

# RESULTS AND DISCUSSION

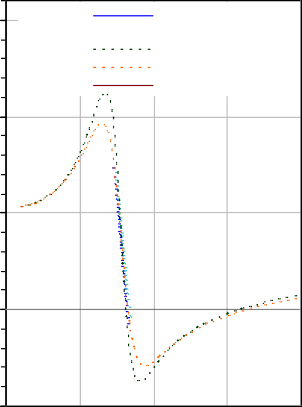
To show the difference of coil self-inductances experimentally measured and the modeled values derived from the distribution function, **Fig. 2** shows the data obtained for the mixture of NaCl and FeCl3 at various concentrations. What is interesting from this figure is that all measured values of *L* reduce to 0 at certain frequency around 825 kHz, with ~20 kHz difference for NaCl and FeCl3, the differences are also seen between the mixtures of both salts and individual cases. The physical origin of this behavior lies in the cancelation of induced magnetizations of positive and negative ions at given frequencies. This cancelation should occur for various water solutions of salts at different frequencies. For the data given in **Fig. 2** the cancelation frequency for pure NaCl occurs at 820 kHz, pure FeCl3 at

* 1. Hz. It is also evidently from this figure that the contribution of the positive ions prevails below 820 kHz and that of the negative ones above 850 kHz. Above 1500 kHz the contribution of ionic solution to *L* differs from that of the pure water only by a constant background of 50 H. Also, it is not difficult to understand why at the low frequency (~200kHz) the measured values of *L* for both ionic solution and water are almost equal 50 H; this 50 H may be considered as the background for all water solutions.

100



(b)



(a)

Fe Cl 3 (2)

(1) 3g+ (2) 3g

(1) 0.5g + (2) 3g H2O

Na Cl (1)

150

100

50

50

**L (mH)**

**L (mH)**

0

0

-50

0 500 1000 1500 2000

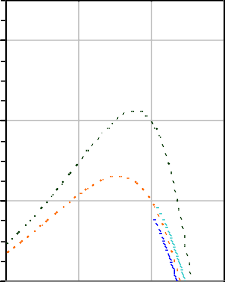
**Frequency f (kHz)**

-50

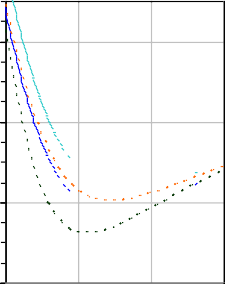
750 800 850 900

**Frequency f (kHz)**

130 10



(c)



(d)

110 -10

**L (mH)**

**L (mH)**

90 -30

70

500 600 700 800

**Frequency f (kHz)**

-50

800 900 1000 1100

**Frequency f (kHz)**

* + 1. (b)

**FIGURE 2.** *Data for the mixture of NaCl / FeCl3. (a) full range of frequency; (b) middle segment of frequency; (c), (d) peak positions at the lower and the higher frequency ranges*

**Fig. 3** shows the decomposition of contributions of positive and negative ions to derivation of distribution functions (part (a)) and the concentration dependence of the modeled *L* (part (b)). In this figure we center the theoretical signals at 0 kHz for clarity, but it can be seen that if the graphs in Fig. 3(b) are shifted to 825 H then the background will be moved to 50 H. The fitted *L* agree excellently with the measured ones and as showed in the part (b) all zero-signals are located around 825 kHz. The question is why there is a remain gap of 50 H after magnetization when frequency increased from 0 to 2000 kHz. The answer may lie in the magnetization effect itself, that at higher frequency above 1500 kHz the lighter ions Cl- react faster to the magnetization of the solenoids than the heavier positive ions Na+.

120

100

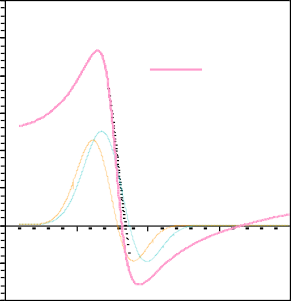
80

60

** p(f)/df**

200

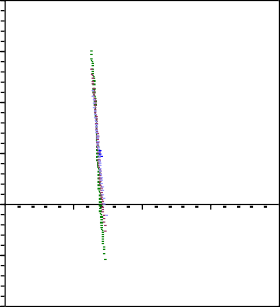
150



dp(f)/df

dp(f)/df Cl

dp(f)/df Na NaCl



3g NaCl

2.5g NaCl

2g NaCl

1.5 NaCl

1g NaCl

100

40 50

** p(f)/df**

20

0

-20

-40

0 500

1000 1500 2000

***f (kHz)***

(a)

0

-50

-100

0 500 1000 1500 2000

***f (kHz)***

**FIGURE 3.** *NaCl solution at varying concentration*. (a) *decomposition for cation and anion; (b) concentration dependence*

**TABLE 2** shows the parameters used in fitting the measured data of NaCl, CuCl2 and FeCl3, and as well as of their mixtures, to the derivation of density function of Eq. 16-17. We only need the atomic mass of ions as the inputs, together with their concentration. The obtained fitted parameters include (showed in **TABLE 3**): (*i*) the average diameter of orbit of motion *ri* (*m*) of each type of ions (for the first attempt this was set at 420000 *m*), from which the values of the moment of inertia of ions were deduced; (*ii*) it is assumed that  itself depends on frequency by two parameters *a1* and *a2* (see Eq. 16-17), these two are set fixed at 0.25 (*a1* cations), 0.245 (*a1* anions) and 30000 (*a2*);

1. the half-width parameter  was set at 0.161; and (*iv*) the amplitude parameter *i* at 1.010-1; (*v*) finally the temperature was 300 *K*. It is naturally that these data were obtained for chlorides and may vary for the other ionic solutions such as nitrides, or sulfides, NO3-, SO 2- ... etc.

4

**TABLE 2** Sample input data for various chlorides

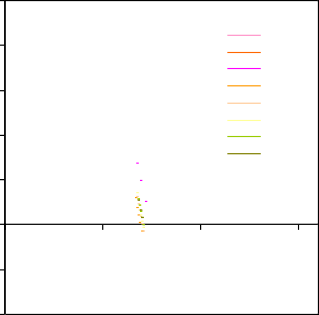
|  |  |  |  |
| --- | --- | --- | --- |
| **Type of Ion** | **FeCl3** | **CuCl2** | **NaCl** |
| *m*(metal) (*kg*)\*) | 9.27210-26 | 1.05510-25 | 3.81710-26 |
| *m*(anion) (*kg*) | 5.88610-26 | 5.88610-26 | 5.88610-26 |
| *c* (*mol/l*)\*\*) | 0.10-0.9 | 0.10-0.9 | 0.10-0.9 |

\*) *m* is an atomic mass of cation and anion in kg respectively

\*\*) *c* is concentration of ion in *mol/liter*

**Note**: 1*J =* 6.241511018 *eV*, *kB=*1.3806510-23 *J/K*, the background offset is 0

The theoretical curves derived from these fitting parameters are showed in **Fig. 4**. By comparison of Fig.4(a) with Fig. 3(a) one can observe that the change of ion concentration is clearly resolved by values of *L*. The change occurs for the maxima, the minima of *L*, as well as for the zero signals of *L*, and is ~25 kHz per 0.1 *mol/l* by average. It is interesting for the cations Cu2+ and Fe3+ that while the fitting curves show only small changes of maxima, the large changes are seen for the minima as frequency increases, resulting at about 20-50 kHz shifts of the minima. This agrees well with the previous observation (**Fig. 3**) that the chloride anion Cl- reacts faster to the frequency than the heavier cations (Na+, Cu2+, Fe3+). The different recovery times of chloride anion and the cations also result well in the larger splits of the signal zero (*L = 0*). The recovery time does not explicitly occur in the formula for the density distribution but is inversely proportional to the mass of each ionic type (see the values of moments of inertia listed in **TABLE 3**).



**NaCl**

0.1

0.3

0.4

0.2

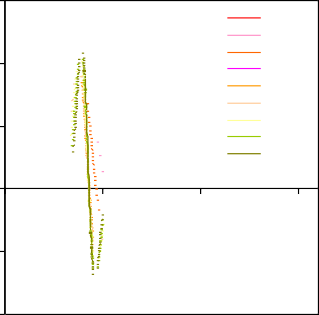
0.5

0.6

0.7

0.8

0.9



**CuCl2**

0.1

0.3

0.4

0.2

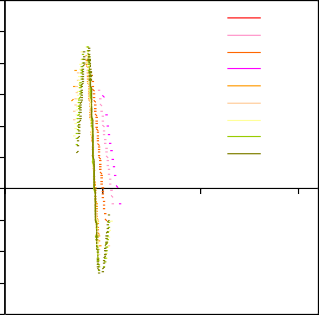
0.5

0.6

0.7

0.8

0.9



**FeCl3**

0.1

0.3

0.4

0.2

0.5

0.6

0.7

0.8

0.9

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 500 | 1000 | 1500 | 0 | 500 | 1000 | 1500 | 0 | 500 | 1000 | 1500 |
|  |  | ***f (kHz)*** |  |  |  | ***f (kHz)*** |  |  |  | ***f (kHz)*** |  |

* 1. NaCl (b) CuCl2 (c) FeCl3

** dp/dx) (H)**

** dp/dx) (H)**

** dp/dx) (H)**

**FIGURE 4.** *Fitting curves for chlorides at various concentrations*

**TABLE 3** Fitting constants for chlorides

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Fitting** |  |  | **Metal cation** |  |  | **Anion** | **Note** |
| **constant** |  | NaCl | CuCl2 | FeCl3 | NaCl | CuCl2 | FeCl3 |
|  | *ri* (*m*) |  | 420000 | 420000 | 420000 | 420000 | 420000 | 420000 *Diameter of orbit of motion* |
|  | *Ii*  *mi ri* |  | 8.20610-8 | 1.36410-7 | 1.27910-7 | 1.01910-7 | 1.01910-7 | 1.01910-7 *Moment of inertia of ion* |
|  | *a1* (**) |  | 0.25 | 0.25 | 0.25 | 0.245 | 0.245 | 0.245 *Parameter of self-*  *induction L* |
|  | *a2* (**) |  | 30000 | 30000 | 30000 | 30000 | 30000 | 30000 *Decrease of L according*  *to frequency* |
|  | ** |  | 0.161 | 0.161 | 0.161 | 0.161 | 0.161 | 0.161 *Half-width correction*  *constant* |
|  | *T*(*K*) |  | 300 | 300 | 300 | 300 | 300 | 300 *Temperature* |
|  | *i* |  | 1.010-1 | 1.010-1 | 1.010-1 | 1.010-1 | 1.010-1 | 1.010-1 *Scale constant (no unit)* |

# CONCLUSION

In this paper, with fitting to the measured data of coil self-inductance *L* obtained for the chloride solutions NaCl, CuCl2 and FeCl3 we show that the proposed theoretical model of self-inductance fits well to describe the measured data of *L*. It was demonstrated that the density distribution function explained all changes of locations of the maxima, minima as well as of the occurrences of zero signals. Therefore, both positive and negative ion and its concentration can be detected by measuring the self-conductance *L* of the solenoid immersed in the water solution containing those ions.

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