**Reduced Transition Probabilities for even-even nuclides (Po and Rn) with A=206-222**

Sameera A. Ebrahiem1, a), Ulvi Kanbur 2, b), Khalid H. M. Aal Shabeeb 2, c) and Mustafa H. Shareef 2, d)

*1Department of Physics – College of Ibn Al-Haytham – University of Baghdad.*

*2 Karaabuk University, Science Faculty/ Karabuk / Turkey.*

*a) Corresponding author:* [*sameera.a.i@ihcoedu.uobaghdad.edu.iq*](mailto:sameera.a.i@ihcoedu.uobaghdad.edu.iq)

*b) ulvikanbur@karabuk.edu.tr*

*c)khalidaal-shabeeb@karabuk.edu.tr*

*d) 2328124005@ogrenci.karabuk.edu.tr*

**Abstract:** We computed the electric quadrupole transition and determined the relationship between the number of neutrons and M(E2) |2w.u↓ for gamma rays from 2+ to 0+ using Ferston's half-life time for the 84Po and 86Rn isotopes for even-even (A=206- 222). We determine the empirical formula for these relations using the MATLAB tool. For the 84Po and 86Rn nuclides enumerated and displayed, the computed decreased transition probabilities B(E2) e2b2 ↑ values are compared with theoretical and experimental predictions, and they show good agreement with SSANM and FRDM as well as with experimental values of Global. When determining the present work of the transition probability (T), the mean life time (τ (s)), and the theoretical value of the energy state (first and ground) of a strontium isotope, By computing the theoretical value of the total width for gamma decay and the Weisskopf Γ(E2) w. u energy (the Weisskopf single-particle widths), the theoretical value of the energy state (first and ground) and the present action of the transmission force |M(E2)|2 W.u ↓, This is part of the formula where A is the mass number and Eγ is computed in keV. After tabulating, discussing, and drawing the results, it was discovered that the value of |M(E2)|2 W. u ↓ is at its lowest when the magic number of neutrons is equal to 128 and that the values rise as we move away from the magic numbers, that is, when the nuclei are saturated for even-even nuclei and the same subject, regardless of the number of protons.

**Keywords:** Isotopes of radon, polonium, electric quadrupole transition, and decreased transition probabilities.

**Introduction**

Comprised of interacting nucleons held together by the strong nuclear force, the atomic nucleus is a complicated quantum system. The short-range nature of the strong interaction and the nucleus's limited spatial extent allow nuclei to display relatively simple excitation patterns. State lifetimes can range from extremely short intervals (on the order of 10⁻²⁴ seconds) to much longer timescales, such as hours or even years. Finding electromagnetic transition probabilities is a crucial step in examining the structure and underlying dynamics of these excited states. This is accomplished by combining the examination of internal conversion coefficients, γ-ray intensities, transition multi-polarities, and observed level lifetimes. [1].

An essential part of studying nuclear structure is figuring out nuclear level lifetimes and related transition probabilities. Decreased transition probabilities provide important insights into the internal structure of nuclei, especially the characteristics of excited states, including their electromagnetic matrix elements and wave functions. [2]. A wide range of experimental methods have been developed over the years to measure lifetimes, including Coulomb Excitation, Electronic Timing, and Recoil Distance Doppler Shift (RDDS). These methods are based on fast-timing techniques that use γ-ray and/or particle coincidence measurements for direct lifetime determination [3,4]. These methods now span a temporal range from the femtosecond to the microsecond scale. The processes used to extract transition probabilities from recorded lifetimes have remained rather consistent throughout the field, regardless of the particular approach taken. [2]. A reduced transition probability can be obtained by comparing the experimental result with the theoretical lifetime of the γ-ray transition. [5], i.e.

(1)

(2)

where the transition probability is denoted by T. The relationship between the mean lifetime τ and the half-life time t\_ (1/2) is as follows: [6]:

(3)

τ =t1/2 ln (2) where τ is the mean lifetime.

Ferston [7] has been used to compute the electric quadrupole transition strengths |M (E2) |2w.u↓ for gamma rays from 2+ to 0+ to obtain the half-life for the first excited state as follows:

The total width for gamma decay is given by [8]:

Гγ = ∑ ГΓl  (4)

Where Гγ τ ħ= 0.658212x10-15 eV.S (5)

The gamma ray transition strength |M(E2)|2 is defined as [9]

|M (E2)|2 = (6)

Where *Г(E2)w.u****.***the Weisskopf single –particle widths

*Г(E2)W.u*. = 4.7907x10-23A4/3E (7)

Where Eγ is calculated in keV and A is the mass number

Gamma-rays are electromagnetic radiation that frequently occurs during an isomeric transition from the nucleus's upper energy state to its lower energy level through emission. A single 2L-pole quantum can be emitted during the γ-transition from an initial state of total angular momentum Ji and parity πi [9] to a final state of total angular momentum Jf and parity πf. [9]

|Ji-Jf| ≤ L ≤ Ji +Jf  for L≠0 (8)

Where L, which is a multipolarity, is the angular momentum of the γ-transition [10]. The parity change of electric radiation (EL) in such a transition is provided by

πi πf = (-1)L  (9)

The relation between the reduced transition probabilities, B(EL)↓ = B(EL , 2→1) and B(EL)↑ = B(EL , 1→2), is given by [11 ] :

B (EL)↓= B(EL)↑ (10)

The best theoretical models to calculate B(E2)↑ are Nilsson Single-Shell Asymptotic One of the most straightforward theoretical models for comprehending B(E2) ↑ is Model SSANM, which is predicated on the idea that the nucleus is as distorted as possible within a single shell. This model has been thoroughly examined in reference [12], where the B(E2)↑ values in e2b2 units are provided by:

B (E2)↑= [e2Q0]2 (Q0≠0) (11)

Where **Q0** the intrinsic quadrupole momentum [2].

The nuclear ground state forms in the Finite-Range Droplet Model (FRDM) [6] are determined by minimizing the nuclear potential energy function with respect to the ε2, ε3, ε4, and ε6 shape degrees of freedom. See ref. [13] for further information on this model. Basic experimental quantities that are independent of nuclear models are the B (E2) values. The value of this (Weisskopf) single-particle B (E2)↑ is provided by [10].

B (E2) ↑ =2.6E-1Z2A-2/3 (12)

**Results and DISCUSSION**

Recent research has been conducted to evaluate the characteristics of even-even (84Po and 86Rn) nuclei by studying the electric quadrupole transitions (E2: 〖 2〗\_1^+ →0\_1^+). We computed the electric quadrupole transition strengths |M(E2) |2w.u↓ for γo-transition as a function of neutron number (N) for even nuclei of isotopic (120-136) for (84Po-86Rn) using half-life (t1/2), energy of the first excited state, and γo-energy from Ferston [7]. The results of calculations of mean life (τ), the total width for gamma decay (Γγ), gamma Weisskopf (Γw.u.), and |M(E2) |2w.u↓ are presented for all even nuclei listed in table. (1)(2).

Figures 1 and 2 show that the relationship between |M(E2) |2W.U. ↓ and the number of neutrons is greatest at the magic number 128. However, for values ​​less and greater than 128, the |M(E2) |2W.U. ↓ values ​​are lower for the isotopes of the elements 84Po and 86Rn, respectively.

**TABLE 1.** Transition strengths |M(E2) |2W.U. ↓ of γo –transition from 2+→0+ with partial gamma widths in W.u. Г(E2)w.u. ,total gamma widthand mean lifetime τ for the first exited state of 84Po isotope (present work)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **N** | **Ei (keV)**  **[7]** | **Eγ (keV)**  **[7]** | **t1/2**  **[7]** | **T(s)**  **P.Work** | **Гtot (eV)**  **P.Work** | **Г(E2)w.u.(eV)**  **P.Work** |  | **|M(E2)|2W.U.↓**  **P.Work** |
| 206 | 122 | 700.66 | 700.66 | 8.8d | 1.0971e+06 | 5.9993e-22 | 9.8422e-06 |  | 6.0955e-17 |
| 208 | 124 | 686.528 | 686.527 | 2.898y | 1.3007e+08 | 5.0604e-24 | 9.0041e-06 |  | 5.6201e-19 |
| 210 | 126 | 1181.40 | 1181.39 | 138.376d | 1.7252e+07 | 3.8153e-23 | 1.3762e-04 |  | 2.7724e-19 |
| 212 | 128 | 727.330 | 727.330 | 0.299μs | 4.3146e-07 | 1.5256e-09 | 1.2326e-05 |  | 1.2376e-04 |
| 214 | 130 | 609.316 | 609.312 | 164.3 μs | 2.3709e-04 | 2.7763e-12 | 5.1503e-06 |  | 5.3905e-07 |
| 216 | 132 | 549.76 | 549.76 | 0.145s | 0.2092 | 3.1458e-15 | 3.1180e-06 |  | 1.0089e-09 |

**TABLE 2.** Transition strengths |M(E2) |2W.U. ↓ of γo –transition from 2+→0+ with partial gamma widths in W. u. Г(E2) w.u., total gamma widthand mean life time τ for the first exited state of 86Rn isotope (present work)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **N** | **Ei(keV)**  **[7]** | **Eγ(keV)**  **[7]** | **t1/2**  **[7]** | **T(s)**  **P. Work** | **Гtot  (eV)**  **P. Work** | **Г(E2) w.u. (eV)**  **P. Work** | **M(E2) |2W.U.↓**  **P. Work** |
| 206 | 120 | 575.3 | 575.31 | 5.87m | 508.2251 | 1.2951e-18 | 3.6731e-06 | 3.5260e-13 |
| 208 | 122 | 635.8 | 635.82 | 24.35m | 2.1082e+03 | 3.1221e-19 | 6.1341e-06 | 5.0897e-14 |
| 210 | 124 | 643.8 | 643.81 | 2.4h | 1.2468e+04 | 5.2794e-20 | 6.6137e-06 | 7.9825e-15 |
| 212 | 126 | 1273.8 | 1273.82 | 23.9m | 2.0693e+03 | 3.1809e-19 | 2.0309e-04 | 1.5663e-15 |
| 214 | 128 | 694.7 | 694.7 | 0.27μs | 3.8961e-07 | 1.6894e-09 | 9.9221e-06 | 1.7027e-04 |
| 216 | 130 | 461.9 | 461.9 | 45 μs | 6.4935e-05 | 1.0136e-11 | 1.3054e-06 | 7.7650e-06 |
| 218 | 132 | 324.22 | 324.22 | 35ms | 0.0505 | 1.3033e-14 | 2.2519e-07 | 5.7875e-08 |
| 220 | 134 | 240.986 | 240.986 | 55.6s | 80.2309 | 8.2040e-18 | 5.1712e-08 | 1.5865e-10 |
| 222 | 136 | 186.24 | 186.24 | 3.8235d | 4.7670e+05 | 1.3808e-21 | 1.4429e-08 | 9.5695e-14 |

For (84Po-86Rn) with just one transition for gamma ray (γ) and (γo) with intensity (100%) E2, the |M(E2)|2w.u. for the 2+→0+ transition as a function of neutron number (n) is computed ref.[9] to even – even isotopes. Tables (1) and (2) present the results of the computations for the (84Po) and 86Rn) nuclides, respectively. Figures (1) and (2) exhibit the results of |M (E2) |2w.u. as a function of neutron number (n), while B(E2) e2b2↑Equation (12) was used to calculate the findings, which are presented in tables (3) and (4), respectively, as seen in figures (3) and (4). These values are compared with the experimental value and with those in figures (5 and 6).

**TABLE 3.** The calculated reduced transition probabilities B (E2) e2 b2 ↑ values are compared with that of experimental and theoretical predications for 84Po nuclide

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **N** | **Ei(keV)**  **[7]** | **Eγ(KeV)**  **[7]** | **B(E2;→)e2b2** | | | |
| **Experimental values of**  **Global [10]** | **Theoretical values** | | |
| **Present work** | **SSANM**  **[10]** | **FRDM**  **[10]** |
| 206 | 122 | 700.66 | 700.66 | 0.74 *13* | 0.7507 | 0.740 | 0.032 |
| 208 | 124 | 686.528 | 686.527 | 0.75 *13* | 0.7612 | 0.493 | 0.032 |
| 210 | 126 | 1181.40 | 1181.39 | 0.43 *8* | 0.4395 | 0.260 | ----- |
| 212 | 128 | 727.330 | 727.330 | 0.70 *12* | 0.7094 | 0.779 | ----- |
| 214 | 130 | 609.316 | 609.312 | 0.83 *14* | 0.8416 | 1.311 | 0.007 |
| 216 | 132 | 549.76 | 549.76 | 0.91 *16* | 0.9269 | 1.955 | 0.046 |

**TABLE 4.** The calculated reduced transition probabilities B(E2) e2 b2 ↑ values are compared with that of experimental and theoretical predications for 86Rn nuclide

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **A** | **N** | **Ei(keV)**  **[7]** | **Eγ(KeV)**  **[7]** | **B(E2;→)e2b2** | | | |
| **Experimental values of**  **Global [10]** | **Theoretical values** | | |
| **Present work** | **SSANM**  **[10]** | **FRDM**  **[10]** |
| 206 | 120 | 575.3 | 575.31 | 0.95 *16* | 0.9583 | 1.855 | 0.197 |
| 208 | 122 | 635.8 | 635.82 | 0.85 *15* | 0.8615 | 1.440 | 0.070 |
| 210 | 124 | 643.8 | 643.81 | 0.83 *15* | 0.8454 | 1.081 | 0.071 |
| 212 | 126 | 1273.8 | 1273.82 | 0.42 *7* | 0.4246 | 0.713 | ------ |
| 214 | 128 | 694.7 | 694.7 | 0.76 *13* | 0.7737 | 1.496 | 0.007 |
| 216 | 130 | 461.9 | 461.9 | 1.14 *20* | 1.1564 | 2.222 | 0.007 |
| 218 | 132 | 324.22 | 324.22 | 1.62 *28* | 1.6374 | 3.057 | 0.202 |
| 220 | 134 | 240.986 | 240.986 | 2.16 *38* | 2.1896 | 3.621 | 1.851 |
| 222 | 136 | 186.24 | 186.24 | 2.78 *48* | 2.8162 | 4.169 | 3.019 |



**FIGURE 1.** Relation between the neutron number and the electric quadruple transition (E2) in 84Po nuclide



**FIGURE 2.** Relation between the neutron number and the electric quadruple transition (E2) in 86Rn nuclide

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**FIGURE 3.** The relation between B(E2) ↑values of the present work with mass

Number for 84po



**FIGURE 4.** The relation between B(E2) ↑values of the present work with mass

Number for 86Rn



**FIGURE 5.** The Comparison between B (E2) ↑values of the present work for 84Po nuclei with of ref. [6] experimental and other theoretical results

The relationship between the reduced transition probabilities, B(EL)↓ and the number of neutrons for the isotopes of the elements Po and Rn is at its lowest value at the magic number 128, as shown in Figures 3 and 4, respectively. The current B(E2) e2b2↑ curves correspond well with those of ref. [10], Although for (84Po-86Rn) nuclides, the experimental and current work values accord more with SSANM than FRDM. Global data for Po120 and 84Po132 were in close agreement with the value of B(E2) e2b2↑ for 84Po, and global data for Rn136 were in close agreement with the value of B(E2) e2b2↑ for 86Rn122.

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**FIGURE 6.** The Comparison between B (E2) ↑values of the present work for 86Rn nuclei with experimental and other theoretical results

# Conclusion

To explore nuclear structural properties, the electric quadrupole transitions (E2: 2₁⁺ → 0₁⁺) in even–even nuclei of 84Po and 86Rn were examined in this work. Ferston's [7] γ₀-energy data, half-life (t\_(1/2)), and energy of the initial excited state were used to compute the electric quadrupole transition intensities for the γ₀ transition as a function of neutron number (N) for even isotopes with mass numbers ranging from 120 to 136. Each isotope has a single E2 transition with 100% intensity, according to the values for mean life (τ), total gamma decay width (Γγ), gamma Weisskopf estimations (Γw.u.), and Figures. This makes analysis easier and improves reliability. Equation (12) was also used to get the reduced transition probabilities in units of e²b². Figures (5) and (6) show the computed values graphically and compare them to theoretical models and experimental data. The current findings were in good accord with the figures in Ref. [10], especially for both 84Po and 86Rn nuclides, matching the SSANM model more closely than the FRDM predictions. In particular, it was discovered that the value of 84Po nearly matched the global average for the isotopes Po120 and Po132, and that the value of 86Rn122 roughly matched that of Rn136. These results illustrate the applicability of the proposed models in characterizing E2 transitions in this region of the nuclear chart and validate the agreement between current theoretical calculations and experimental trends.

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