**Study The Effect of Neutron Numbers on The Reduced Transition Probability and Deformation Parameter for Even-Even 56Ba Isotopes**

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**Abstract.** Neutron numbers play a crucial role in determining both the reduced electric quadrupole transition probability, B(E2), and the overall geometric shape of the nucleus. In the present study, the evolution of nuclear shape with varying neutron numbers has been investigated for the even–even ⁵⁶Ba isotopes, spanning the neutron range N = 62–92. To examine this dependence, the calculated values of B(E2) and the corresponding quadrupole deformation parameter, , have been systematically analyzed. Both quantities—B(E2) and —were plotted as functions of the neutron number in order to visualize and assess how increasing or decreasing neutron content influences the degree of collectivity and deformation. The observed trends reveal that the nucleus tends to become more spherical as the neutron number approaches the magic neutron number, reflecting the stabilizing effect of shell closures and the corresponding reduction in deformation and collective motion.

**Keywords:** Magic number, Reduced transition probability, Nuclear deformation.

INTRODUCTION

At first, it was supposed that the shapes of nuclei were spherical. Later, Wolfgang Pauli proposed that there were other shapes for nuclei when the nucleus was excited. Then, Bohr and Fritz Kalkar found that the shapes of nuclei could be experimentally probed through the measurement of gamma-ray photons emitted during nuclear de-excitation [1]. Nuclei deviate from the spherical shape, undergoing deformation, when the number of constituent nucleons (protons Z and neutrons N) not equal to any of the magic numbers (2, 8, 20, 28, 50, 82, and 126). That is, the nuclei will be more abundant and stable if the number of nucleons (neutrons N and protons Z) in them equals one of the magic numbers [2]. The underlying cause of nuclear deformation stems from the collective ordering of valence nucleons in incompletely filled major shells. Consequently, deformation is primarily observed in nuclei where both the neutron and proton levels are partially filled [3]. The easiest and most common type of non-spherical distortion is the quadrupole deformation; in this case of deformation, the shape of the nuclei is usually either elongation or flattening [4]. The quadrupole deformation parameter is a fundamental quantity for characterizing and quantifying these nuclear shape transitions [5]. For even-even nuclei, moving from the first excited level, typically the 2+ state, to the 0+ ground state is of particular importance [6]. Electromagnetic transitions, specifically the electric quadrupole (E2) transitions, serve as a vital source of information for studying nuclear structure [7]. (B(E2; 0+→2+)) incorporates crucial nuclear structure information, including the energy of low-lying collective levels. The values of B(E2; 0+→2+) are a direct indicator of quadruple distortion in nuclides [8]. Many studies have focused on the form of the nuclide & examined the degree of nuclear distortion [9-13]. In this work, the effect of the number of neutrons on the shape of the nucleus and the degree of distortion in its shape was studied. The deformation parameter (β2) for the 56Ba isotopes, which have neutron numbers (62-92), was calculated. To obtain this coefficient, we need to calculate the probability of an electrical transition B (E2; 0+→2+) from the Global Best fit equation. Finally, our results are compared with theoretical data.

**MATERIALS AND METHODS**

**Reduced Transition Probability B(E2)↑**

B(E2) is a fundamental quantity that can be calculated theoretically or derived from experimental data, serving as a basis for comparison with experiment [14]. The nuclear model does not affect on these fundamental experimental quantities. To calculate the B(E2) value, based on Global Best Fit (GLOBAL) we need only to know the energy of the gamma rays for the 2+ level [15].

(1)

Where: :is the ɣ energy of transition in (keV) units.

z: proton numbers

A: proton and neutron numbers of a nucleus

**Quadruple Deformation Parameter (𝛽2)**

The (β2) quantifies the extent of the deflection of the nuclear shape from circular symmetry, representing the degree of nuclear quadrupole deformation (Elongation or flattening), which is calculated from the equation below [16,17].

𝛽2 = (2)

R0 is the average nuclear radius calculated by using the following equation

*=* 0.0144 A2/3 barn . (3)

**METHODLOGY**

In the present study, both the transition probability B(E2)↑ for the (0+→2+) transition and the deformation coefficient (β2) were calculated. To apply equation 2, the average radius of the nucleus calculate from equation 3. These parameters were calculated for even-even 56Ba isotopes which have neutron numbers (62-92). The study also addressed the effect of the number of neutrons on both B(E2)↑ and (β2) through a graph that illustrates the effect of this relationship between them.

Finally, the relationship between B(E2)↑ and (β2) of the nucleus was clarified by drawing a graph of (β2) as a function of B(E2)↑.

**RESULTS AND DISCUSSION**

The calculated B(E2)↑ transition probabilities for the even–even ⁵⁶Ba isotopes are presented in Table 1. These calculated values are compared with the corresponding theoretical predictions, which are listed side-by-side in the same table to facilitate direct evaluation of the agreement between experiment and model.

Similarly, the calculated values of the mean-square nuclear radius and the quadrupole deformation parameter for the even–even ⁵⁶Ba isotopic chain are presented in Table 2. The computed values are compared with the available theoretical estimates, which are provided in column 3 of the table, allowing for a clear assessment of deviations and trends across the isotopes.

To better illustrate the nuclear-structure evolution along the isotopic chain, the values of B(E2)↑ and are plotted as a function of the neutron number (N) in Figures 1 and 2, respectively. These graphical representations highlight the dependence of collective behavior on neutron content.

In Figure 3, the deformation parameter is plotted as a function of B(E2)↑, providing a direct visualization of the correlation between electric quadrupole transition strength and nuclear deformation.

**TABLE 1.** Atomic Number A, Neutron Numbers N, Gamma Energy (), Transition Probabilities B(E2) e²b²↑ for ₅₆Ba Isotopes.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **N** | **Theoretical Values** | | **Present Work** |
| **(KeV)**  **[ 18]** | **B(E2) ↑e²b²**  **Global Best Fit [15]** | **B(E2) ↑e²b²** |
| 118 | 62 | 194 | 1.72 | 1.7470 |
| 120 | 64 | 183 | 1.81 | 1.8314 |
| 122 | 66 | 196 | 1.67 | 1.6912 |
| 124 | 68 | 229 | 1.41 | 1.4319 |
| 126 | 70 | 256 | 1.25 | 1.2673 |
| 128 | 72 | 284 | 1.11 | 1.1304 |
| 130 | 74 | 357 | 0.88 | 0.8900 |
| 132 | 76 | 464 | 0.67 | 0.6778 |
| 134 | 78 | 604 | 0.51 | 0.5155 |
| 136 | 80 | 818 | 0.37 | 0.3769 |
| 138 | 82 | 1435 | 0.210 | 0.2128 |
| 140 | 84 | 602 | 0.50 | 0.5023 |
| 142 | 86 | 359 | 0.82 | 0.8344 |
| 144 | 88 | 199 | 1.47 | 1.4914 |
| 146 | 90 | 181 | 1.60 | 1.6247 |
| 148 | 92 | 141 | 2.03 | 2.0668 |

**FIGURE 1.** Reduced transition probabilities B(E2) for even-even 56Ba isotopes

**TABLE 2.** Atomic Number A, Neutron Numbers N, Average Radius Nuclear () & Deformation Parameter (β₂) for 56 Ba Isotopes.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **A** | **N** | **Theoretical Values β₂ [15]** | **Present Work** | |
|  | **β₂** |
| 118 | 62 | - | 0.3464 | 0.2854 |
| 120 | 64 | - | 0.3503 | 0.2889 |
| 122 | 66 | 0.35418 | 0.3542 | 0.2746 |
| 124 | 68 | 0.3027 | 0.3581 | 0.2500 |
| 126 | 70 | 0.2737 | 0.3619 | 0.2327 |
| 128 | 72 | 0.2496 | 0.3657 | 0.2174 |
| 130 | 74 | 0.218315 | 0.3695 | 0.1910 |
| 132 | 76 | 0.1866 | 0.3733 | 0.1650 |
| 134 | 78 | 0.16099 | 0.3771 | 0.1424 |
| 136 | 80 | 0.125812 | 0.3808 | 0.1206 |
| 138 | 82 | 0.093318 | 0.3845 | 0.0897 |
| 140 | 84 | 0.12628 | 0.3883 | 0.1365 |
| 142 | 86 | 0.159542 | 0.3919 | 0.1743 |
| 144 | 88 | 0.1946 | 0.3956 | 0.2309 |
| 146 | 90 | 0.218039 | 0.3993 | 0.2388 |
| 148 | 92 | - | 0.4029 | 0.2669 |

**FIGURE 2.** Deformation parameter (β2) for even-even 56Ba isotopes.

This study focused on studying the effect of neutron numbers on the B(E2)↑ as well as the shape of the nuclide. To determine how the shape of the nuclide is affected by the changing neutron numbers, B(E2)↑ and (β2) which studied for 56Ba isotopes, that have neutron numbers (62-92).

From Figure 1, notice that as a result of increasing the number of neutrons, it well decreased in the B(E2)↑ values until it reaches the lowest value when the number of neutrons reaches the magic number (82) (i.e. the relationship is inverse), but after increasing the number of neutrons far from neutron magic number, the relationship becomes directly proportional between the number of neutrons and the B(E2)↑, so we notice a clear increase in the probability of transition with the increase in the neutron numbers.

In Figure 2, the relationship between the calculated values of (β₂) and the number of neutrons was studied by plotting this parameter as a function of the neutron numbers. Here too, there was an inverse relationship between the (β₂) and the number of neutrons until reaching the magic number of neutrons, where the isotope (¹³⁸Ba) has the least distortion, which means obtaining a semi-spherical shape.

For isotopes with N > 82 (¹⁴⁰Ba to ¹⁴⁸Ba), the B(E2)↑ and (β₂) values begin to increase again. This indicates that as neutrons are added beyond the closed shell, they begin to occupy a new, higher-energy shell. These valence neutrons exert a polarizing effect on the spherical core, inducing quadrupole deformation and re-establishing collective rotational behavior. The nucleus moves away from sphericity and becomes progressively more deformed.

On the other hand, nuclei having a deformed charge distribution (deformed shape) when numbers of neutrons move away from a magic number. Nucleus with a magic number of neutrons (full levels) result in a circular symmetry of their charge distribution and circular form of nucleus.

The very low values of B(E2)↑ and (β₂) for ¹³⁸Ba are a clear and direct signature of this transition to a near-spherical shape. Also, the gamma-ray energy transition for the nucleus () is very high (1435 keV), and the collective motion is minimized, which is characteristic of magic nuclei.

Table-2 explain computed (present work) and theoretical values (earlier work) of (β2) [15], these values show a clear difference between them, the reason of this variation resulting from adoption Global Best Fit equation for our compute of the B(E2)↑ which is used in computing (β2), while the reference data marks approved values [15] of B(E2)↑.

**FIGURE 3.** Bar chart of the deformation parameter (β2) with reduced transition probabilities B(E2)↑ for even-even 56Ba isotopes.

Figure 3 shows that the lower value of (β₂) (0.0897) is obtained at the minimum value of B(E2)↑(0.2128) for the (138Ba) nucleus, that has a magic neutron number (82). As well as, we can observe the effect of (β2) by B(E2).

**CONCLUSION**

This study successfully demonstrated the profound influence of neutron numbers on the collective properties of even-even 56Ba isotopes. By applying the Global Best Fit method, we calculated B(E2)↑ and (β₂) for 56Ba isotopes from which have neutron numbers (N=62-92).

The primary conclusion is that the nuclear structure undergoes a significant transformation at the neutron magic number N=82. The minima in both B(E2)↑ and (β₂) at ¹³⁸Ba provide strong evidence for a transition from deformed, collective nuclei to a more spherical, non-collective configuration at the closed shell. The subsequent increase in these values for N > 82 confirms the re-emergence of deformation as neutrons are added to the next shell. This systematic analysis reinforces the fundamental concepts of nuclear shell theory and collective motion, highlighting the delicate balance of forces that dictate the shape of the atomic nucleus.

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