A Fuzzy TOPSIS Framework With Chebyshev Distance and Entropy Weights for Multi-Criteria Decision-Making

P. N. Kanimalar1,a), R. Balakumar1,b)

1Department of Mathematics, Periyar Maniammai Institute of Science & Technology (Deemed to be University),   
Thanjavur, Tamil Nadu, India -613 403

Corresponding author: a)[balakumar@pmu.edu](mailto:balakumar@pmu.edu), b)[kanimalar3011@gmail.com](mailto:kanimalar3011@gmail.com)

Abstract: Evaluating transportation policies often encounters challenges due to uncertain and imprecise data, which can limit the effectiveness of traditional multi-criteria decision-making (MCDM) methods. To tackle this issue, this research presents a fuzzy MCDM framework that utilizes triangular fuzzy numbers (TFNs) to capture ambiguous or unclear expert opinions. The model applies entropy-based weighting to objectively assess the significance of each criterion, and it utilizes the Chebyshev distance measure to enhance the reliability of alternative comparisons. The proposed fuzzy TOPSIS method is tested in a transportation policy evaluation scenario, showcasing its strength in ranking options amid uncertainty. The findings suggest that the combination of entropy weights with the Chebyshev distance offers consistent and dependable decision support, positioning this framework as an effective tool for intricate transportation planning challenges.

Keywords: Triangular fuzzy numbers, Fuzzy TOPSIS, Fuzzy multi-criteria decision making, Closeness coefficient.

# Introduction

Decision-making in transportation systems often involves multiple conflicting criteria under uncertainty. Classical Multi-Criteria Decision-Making (MCDM) techniques, while powerful, may not adequately handle the vagueness and imprecision inherent in real-world problems. Fuzzy set theory, first introduced by Zadeh in 1965, provides a systematic approach to integrating uncertainty into decision-making processes. This makes fuzzy extensions of MCDM particularly relevant for applications such as transport policy evaluation, airport planning, and sustainable logistics.

Over the years, researchers have developed numerous fuzzy MCDM approaches to address decision-making challenges under uncertainty. Among the methods, the technique for order preference by Similarity to the Ideal Solution (TOPSIS) has been extensively adapted for use in fuzzy contexts (Chen, 2000; Jahanshahloo, Lotfi, & Izadikhah, 2006). Fuzzy TOPSIS allows decision-makers to assess options by considering how closely they align with the ideal solution, as well as their capability to handle uncertain assessments.

Recent studies have emphasized the importance of integrating fuzzy MCDM into transportation planning. For instance, Ertuğrul and Karakaşoğlu (2009) used fuzzy TOPSIS for facility location decisions, while Saleh, Jaber, and Alsyouf (2023) applied fuzzy techniques for sustainable airport site selection. These applications highlight how fuzzy decision models can capture both technical and environmental aspects in complex transport systems. More broadly, fuzzy MCDM has been recognized as an effective decision-support tool in energy management, supply chain optimization, and sustainable development (Mardani et al., 2015; Aasa, Phoya, Monko, & Musonda, 2025).

The foundation of fuzzy decision-making was laid by Bellman and Zadeh (1970), who first formulated a decision framework under fuzzy environments. Their work inspired subsequent researchers to adapt classical MCDM methods into fuzzy versions. Chen (2000) introduced the fuzzy TOPSIS method, which has subsequently emerged as one of the most commonly utilized fuzzy decision-making frameworks. Chu and Lin (2003) extended fuzzy TOPSIS to supply chain contexts, while Jahanshahloo et al. (2006) formalized algorithms for handling interval-valued fuzzy data. These contributions illustrate the method’s flexibility in capturing uncertainty across different application domains.

Entropy weighting has often been integrated into fuzzy MCDM to derive objective criterion weights. This approach minimizes subjectivity by measuring the amount of information embedded in decision data (Parkash, Biswas, & Mahapatra, 2008). More recently, Chen (2021) extended entropy measures to intuitionistic fuzzy sets, further enhancing decision-making robustness.

Another stream of research has focused on distance measures used in fuzzy TOPSIS. While Euclidean and Hamming distances are common, Chebyshev distance has been increasingly adopted due to its ability to emphasize maximum deviations (Dubois & Prade, 1980; Chen, 2020). This property makes it particularly suitable for risk-sensitive decision contexts such as air traffic management.

In transportation, fuzzy MCDM methods have been applied to airport location selection (Ertuğrul & Karakaşoğlu, 2009; Saleh et al., 2023), airline policy evaluation (Mufazzal & Muzakkir, 2021), and sustainable logistics planning (Dong, Xu, & Li, 2021). These studies consistently demonstrate the value of fuzzy models in addressing uncertainty and complexity, strengthening the case for their application in modern transportation systems.

The paper is structured to provide a comprehensive understanding of the proposed fuzzy TOPSIS model with the Chebyshev distance metric for air traffic management. Section 1 is the introduction followed by preliminaries in section 2. Section 3 is about the mathematical formulation. Section 4 is about the theoretical analysis of the formulas given the previous section followed by numerical example in section 5 which provides solution for the synthetic data created. Section 6 (Uniqueness of the Proposed Method) highlights the methodological advancements over traditional approaches, and Section 7 (Conclusion and Future Work) summarizes the findings while suggesting future research directions, including hybrid fuzzy optimization models.

# Preliminaries

This section introduces the fundamental mathematical concepts required for the proposed fuzzy TOPSIS model with the Chebyshev distance metric. The foundation of the proposed fuzzy TOPSIS model rests on concepts from fuzzy set theory, triangular fuzzy numbers, entropy-based weighting, and distance measures. As originally proposed by Zadeh (1965), fuzzy sets allow elements to have degrees of membership between 0 and 1, rather than the binary values of classical sets. According to Zimmermann (2010), “unlike classical sets, where membership is binary (0 or 1), fuzzy sets allow partial membership values between 0 and 1,” which makes them suitable for representing uncertainty in real-world decision problems.

To represent uncertain numerical values, triangular fuzzy numbers (TFNs) are widely used. A TFN is expressed as a triplet , where denotes the lower bound, is the most likely value, and is the upper bound. Tayal et al. (2025) describe the TFN membership function as:

This representation is simple yet effective in capturing imprecision in expert judgments (Dong et al., 2021) .

## Entropy-Based Weight Determination

To assign objective importance to each criterion, we use the fuzzy entropy method, which measures the uncertainty in decision data. The entropy of the criterion is:

where ​ represents the normalized membership value of the alternative under criterion , and number of alternatives. The final weight of each criterion is:

A lower entropy value indicates that a criterion carries more useful information, leading to a higher assigned weight (Chen, 2021) .

## Chebyshev Distance Metric

o capture the greatest deviation across parameters of fuzzy numbers, the Chebyshev metric is applied as the distance measure. For two TFNs and , the Chebyshev distance is given by:

Unlike Euclidean distance, which averages deviations across dimensions, the Chebyshev metric considers the largest deviation, making it more robust for decision-making. This ensures that alternatives with significant differences in any single criterion are properly distinguished (Chen, 2020).

## Fuzzy TOPSIS Method

The TOPSIS method is extended into the fuzzy domain to account for uncertainty in decision-making. For each alternative ​, the Chebyshev distances to FPIS and FNIS are denoted by ​ and ​, respectively. The closeness coefficient is calculated as (Chu & Lin, 2003):

A larger ​ reflects a better alternative. This adaptation integrates entropy-based weights and a robust distance metric to ensure that ranking results are both mathematically rigorous and interpretable.

# Mathematical Formulation

This section presents the mathematical formulation for the Fuzzy TOPSIS-based transportation decision-making model, designed to handle multiple conflicting criteria under uncertainty using fuzzy numbers, entropy-based weighting, and closeness coefficient calculations.

## Problem definition

Consider a set of transportation alternatives that need to be evaluated based on a set of decision criteria (Zadeh, 1965). Each alternative represents a possible transportation solution, such as choosing between different public transport routes, selecting an optimal freight logistics plan, and evaluating airport congestion reduction strategies. Each criterion can either be beneficial (e.g., reliability, safety) or non-beneficial (e.g., cost, congestion). Due to the uncertainty in transportation planning, the performance of each alternative under each criterion is modelled as a fuzzy number rather than a crisp value (Chen & Hwang, 1992).

Let and . These indices will be used throughout the following formulations. Let be the -th transportation alternative and be the -th evaluation criterion. Let ​ be the fuzzy performance rating of the alternative under criterion ​, represented as a Triangular Fuzzy Number (TFN):

where:

* ​ = Lower bound (pessimistic estimate)
* ​ = Most likely value (expected performance)
* ​ = Upper bound (optimistic estimate)

The fuzzy decision matrix is then given by:

The decision matrix is structured as:

where each entry ​ is a triangular fuzzy number representing the performance of alternative under criterion ​.

## Normalization of the Decision Matrix

Because evaluation criteria may be measured in varying units, normalization is required.

For beneficial criteria (where higher values are preferred):

For non-beneficial criteria (e.g., cost, congestion):

Where, ​ (for beneficial criteria) and ​ (for non-beneficial criteria) (Mardani et al., 2015, Sałabun, 2020).

## Fuzzy Entropy Weighting

The fuzzy entropy of each criterion measures the degree of fuzziness or uncertainty in its normalized decision values. A higher entropy means the criterion is less informative (more uncertain), and a lower entropy means the criterion is more important (Parkash et al., 2008).

The fuzzy entropy for the criterion ​ is defined as:

Where, is the entropy for the criterion ​, represents the normalized fuzzy performance score of the alternative under the criterion ​ and is the total number of alternatives.

This entropy calculation is done for all criteria.

Once the entropy values ​ are computed, The ambiguous importance of each criterion is established as:

​​

Where, is the weight of the criterion ​. ensures that all weights sum to 1.

This weight allocation ensures that criteria with lower entropy receive higher weights, while more uncertain criteria receive lower weights.

3.4. Fuzzy TOPSIS Model

Fuzzy TOPSIS extends the classical TOPSIS method into handling uncertainty in decision-making. It is useful in air traffic management, where multiple criteria need to be optimized under uncertain conditions (Chen, 2000, Kim, S. Y., & Thuc, L. D. 2020, Saleh et al., 2024, Shannon, 1948).

The first step is to determination of the FPIS and FNIS

The FPIS and FNIS are calculated as:

where:

* represents the best performance.
* represents the worst performance.

The next step is to compute the distance using Chebyshev distance metric and the relative closeness coefficient.

The Chebyshev distance measures the worst-case deviation and is defined as:

Where, is the distance between the alternatives and the FPIS and is the distance between the alternatives and the FNIS (Kim, S. Y., & Thuc, L. D., 2020).

The relative closeness coefficient is given by:

Where, is the final score for ranking alternatives. Higher values indicate better options (Chen, 2020, Dubois & Prade, 1980, Mahalanobis, 1936).

The last step is to rank the alternatives

Sort all alternatives ​ in descending order of and the alternative with the highest value is the best choice (Aasa, O. et al., 2025).

# Theoretical Analysis

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# Numerical example

Table 1, given below, presents the fuzzy decision matrix for five air traffic alternatives, each evaluated based on Passenger Traffic, Average Delay, Fuel Consumption, Air Traffic Density, and Operational Cost. This matrix acts as the basis for utilizing fuzzy TOPSIS with the Chebyshev distance metric to identify the most effective alternative.(All values are in respective units: millions for passenger traffic, minutes for delay, litters per flight for fuel, flights per hour for traffic density, and million USD for cost.)

The five alternatives (A1 to A5) represent distinct strategic policy scenarios for air traffic management under uncertainty:

* **A1**: *Hub-Focused Policy* — Prioritizes increasing passenger traffic through major airport hubs by optimizing flight schedules and gate utilization, with moderate attention to delay and fuel usage.
* **A2**: *Delay-Minimization Strategy* — Focuses on reducing average flight delays through airspace re-routing, priority sequencing, and enhanced air traffic control coordination.
* **A3**: *Fuel Efficiency-Oriented Plan* — Emphasizes minimizing fuel consumption per flight through aircraft load optimization, route smoothing, and descent profile adjustments.
* **A4**: *Congestion-Responsive Framework* — Targets reduction of air traffic density in critical sectors using flow management tools and sector capacity balancing.
* **A5**: *Cost-Conscious Operation Model* — Designed to lower overall operational expenditures by optimizing maintenance schedules, resource allocation, and low-cost carrier incentives.

Table 1: Fuzzy Decision Matrix for Air Traffic Alternatives

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Alternative | Passenger Traffic (millions) | Average Delay (minutes) | Fuel Consumption (litters/flight) | Air Traffic Density (flights/hour) | Operational Cost (million USD) |
| Alternative 1 | (68.73, 195.07, 273.19) | (79.93, 115.60, 215.59) | (52.90, 186.61, 260.11) | (85.40, 102.05, 296.99) | (91.62, 121.23, 218.18) |
| Alternative 2 | (59.17, 130.42, 252.47) | (71.60, 129.12, 261.18) | (56.97, 129.21, 236.63) | (72.80, 178.51, 219.96) | (75.71, 159.24, 204.64) |
| Alternative 3 | (80.38, 117.05, 206.50) | (97.44, 196.56, 280.83) | (65.23, 109.76, 268.42) | (72.00, 112.20, 249.51) | (51.71, 190.93, 225.88) |
| Alternative 4 | (83.13, 131.17, 252.01) | (77.34, 118.49, 296.96) | (88.76, 193.95, 289.48) | (79.89, 192.19, 208.84) | (59.80, 104.52, 232.53) |
| Alternative 5 | (69.43, 127.13, 282.87) | (67.84, 128.09, 254.26) | (57.05, 180.22, 207.45) | (99.34, 177.22, 219.87) | (50.28, 181.55, 270.68) |

The normalization process depends on the type of criterion:

* Benefit Criteria: Passenger Traffic and Air Traffic Density
* Cost Criteria: Average Delay, Fuel Consumption, Operational Cost

Table 2, below, shows the normalized fuzzy decision matrix after calculation using the equation explained in the previous section

Table 2. Normalized fuzzy decision matrix

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Alternative | Passenger Traffic  (L, M, U) | Average Delay  (L, M, U) | Fuel Consumption  (L, M, U) | Air Traffic Density (L, M, U) | Operational Cost  (L, M, U) |
| A1 | (0.2429, 0.6896, 0.9658) | (0.3147, 0.5869, 0.8487) | (0.2034, 0.2835, 1.0000) | (0.2876, 0.3436, 1.0000) | (0.2305, 0.4147, 0.5488) |
| A2 | (0.2092, 0.4611, 0.8925) | (0.2597, 0.5254, 0.9475) | (0.2236, 0.4094, 0.9286) | (0.2451, 0.6011, 0.7406) | (0.2457, 0.3158, 0.6641) |
| A3 | (0.2842, 0.4138, 0.7300) | (0.2416, 0.3451, 0.6962) | (0.1971, 0.4820, 0.8110) | (0.2424, 0.3778, 0.8401) | (0.2226, 0.2633, 0.9723) |
| A4 | (0.2939, 0.4637, 0.8909) | (0.2284, 0.5725, 0.8772) | (0.1827, 0.2728, 0.5960) | (0.2690, 0.6471, 0.7032) | (0.2162, 0.4811, 0.8408) |
| A5 | (0.2454, 0.4494, 1.0000) | (0.2668, 0.5296, 1.0000) | (0.2550, 0.2935, 0.9273) | (0.3345, 0.5967, 0.7403) | (0.1858, 0.2769, 1.0000) |

Table 3. Final entropy-based weights for each criterion

|  |  |
| --- | --- |
| Criterion | Weight |
| Passenger Traffic | 0.1523 |
| Average Delay | 0.1809 |
| Fuel Consumption | 0.1945 |
| Air Traffic Density | 0.2624 |
| Operational Cost | 0.2099 |

Table 4. FPIS and FNIS

|  |  |  |
| --- | --- | --- |
| Criterion | Fuzzy Positive Ideal Solution (FPIS) | Fuzzy Negative Ideal Solution (FNIS) |
| Passenger Traffic | (0.2939,0.6896,1.0000)  (0.2939, 0.6896, 1.0000)  (0.2939,0.6896,1.0000) | (0.2092,0.4138,0.7300)  (0.2092, 0.4138, 0.7300)  (0.2092,0.4138,0.7300) |
| Average Delay | (0.2284,0.3451,0.6962)  (0.2284, 0.3451, 0.6962)  (0.2284,0.3451,0.6962) | (0.3147,0.5869,0.9475)  (0.3147, 0.5869, 0.9475)  (0.3147,0.5869,0.9475) |
| Fuel Consumption | (0.1827,0.2728,0.5960)  (0.1827, 0.2728, 0.5960)  (0.1827,0.2728,0.5960) | (0.2550,0.4820,1.0000)  (0.2550, 0.4820, 1.0000)  (0.2550,0.4820,1.0000) |
| Air Traffic Density | (0.3345,0.6471,1.0000)  (0.3345, 0.6471, 1.0000)  (0.3345,0.6471,1.0000) | (0.2424,0.3436,0.7032)  (0.2424, 0.3436, 0.7032)  (0.2424,0.3436,0.7032) |
| Operational Cost | (0.1858,0.2769,0.9723)  (0.1858, 0.2769, 0.9723)  (0.1858,0.2769,0.9723) | (0.2457,0.4811,1.0000)  (0.2457, 0.4811, 1.0000)  (0.2457,0.4811,1.0000) |

To determine the significance of each criterion, we calculate entropy-based weights as given in the table below (Table 3). Using the normalized fuzzy decision matrix, we determine the FPIS and FNIS for our air traffic dataset which is displayed in the table below (Table 4).

Table 5. Chebyshev distances and ranking

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Alternative |  |  |  | Rank |
| A1 | 0.5123 | 0.6721 | 0.5678 | 2 |
| A2 | 0.4834 | 0.721 | 0.5983 | 1 |
| A3 | 0.5297 | 0.6438 | 0.5482 | 3 |
| A4 | 0.5589 | 0.6123 | 0.5235 | 4 |
| A5 | 0.581 | 0.5892 | 0.5032 | 5 |

The computed distances are provided in the above table (Table 5), and it indicate that Alternative A2 has the highest relative closeness coefficient (), making it the optimal choice among the given alternatives, which emphasizes delay minimization, demonstrates a well-balanced performance across all considered criteria, particularly excelling in delay reduction and maintaining moderate operational costs. Alternative A1, focusing on hub-based passenger throughput, ranks second, suggesting strong performance in traffic volume but relatively less efficiency in fuel or cost metrics. Alternatives A3 to A5, while effective in specific dimensions such as fuel optimization (A3) and cost control (A5), fail to achieve performance. This suggests that A2 achieves the best balance across key air traffic criteria, such as passenger traffic, operational costs, and air traffic density. The results demonstrate the robustness of the proposed fuzzy MCDM approach in handling uncertainty and complex decision-making scenarios in air traffic management. This ranking enables policymakers to make data-driven choices for optimizing air transportation efficiency while considering multiple conflicting factors.

# Highlight

The proposed method combines the fuzzy TOPSIS with the Chebyshev distance metric, creating a more robust and stable decision-making framework. Unlike traditional Euclidean-based fuzzy TOPSIS methods, the Chebyshev metric accounts for the maximum deviation across all criteria, making it more sensitive to the worst-performing criterion. This prevents any single criterion from disproportionately influencing the ranking, leading to a more balanced and consistent result. Additionally, the use of fuzzy entropy-based weighting enhances objectivity by deriving weights directly from data rather than relying solely on expert judgment. The proposed approach ensures a unique ranking order through the strict monotonicity and boundedness of the Chebyshev distance, ensuring each alternative is distinctly ranked. This distinctiveness makes the method stable and computationally efficient, making it well-suited for handling uncertain and multi-dimensional air traffic data in real-world scenarios.

# Conclusion

This work presented a fuzzy TOPSIS model enhanced with entropy-based weights and the Chebyshev distance measure to improve stability in decision-making for air traffic management. The approach demonstrated consistency in ranking policies and offered a robust alternative to classical fuzzy MCDM methods. By applying fuzzy entropy-based weight calculation, the model objectively assigns weights to different criteria without solely depending on subjective expert opinions. The incorporation of the Chebyshev distance helps ensure stability by considering the largest variation among all criteria, preventing any single factor from having an outsized influence on the ranking. Through a detailed numerical example using simulated air traffic data, the proposed approach accurately identifies the best alternative while remaining efficient and valid for decision-making. The theoretical analysis confirms the model by proving the existence, boundedness, and uniqueness of the ranking order. This work advances mathematical optimization in transportation systems and provides policymakers with a structured, data-driven method for making decisions that account for uncertainty. Future research can explore fuzzy optimization and hybrid models based on deep learning to achieve better predictability.

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