Mathematical Model of Dynamic Problems of a Viscoelastic Shell Considering the Interrelation of Mechanical and Thermal Fields

Ummatali Akbarov1, Jalolxon Nuritdinov1, a), Abdugaffar Tashxodjayev2, Maftuna Yakubjanova1

1Kokand State University, Kokand 100700, Uzbekistan

2Kokand University, Kokand 100700, Uzbekistan

*a)Corresponding author:* [*nuritdinovjt@gmail.com*](mailto:nuritdinovjt@gmail.com)

**Abstract.** In this paper, the derivation of the equation of nonlinear vibrations and dynamic stability of a viscoelastic orthotropic shell is presented, based on the Kirchhoff–Love model, taking into account the interrelation between temperature fields and deformation fields. In addition, nonlinear integro-differential equations were derived based on the conditions of thermoviscoelasticity and the theory of thin shells, and their complete solution was presented. The effects of thermal and mechanical loads, as well as the curvature of the shell’s middle surface, were analyzed for various boundary and geometric conditions. These results can be applied in engineering, particularly in controlling and predicting the dynamic stability and thermo-mechanical behavior of advanced composite and anisotropic shell structures.

# Keywords: Viscoelastic shell, thermo-mechanical coupling, nonlinear vibrations, dynamic stability, Kirchhoff–love model, orthotropic materials, integro-differential equations, heat conduction

# Introduction

The advanced composite materials (ferroconcrete, glass-fiber-reinforced plastic (GFRP), carbon-fiber-reinforced polymer (CFRP)) have found wide application in various sectors of engineering. For this reason, the mechanics of composite materials has developed intensively – a branch of mechanics that emerged in response to the demand for materials possessing a predetermined combination of characteristics that are optimal for specific extreme operating conditions. Within the framework of contemporary and complex economic conditions, one of the principal vectors for the accelerated development of the national economy consists of the extensive implementation of polymer materials, various composites, resource-efficient technologies, and design solutions in modern technical apparatus. This is directly related to the reduction of material consumption in building structures. Solving this problem is closely linked to improving structural analysis methods through a more comprehensive consideration of material properties – that is, by making the mathematical model of a solid body closer to reality. One such property is viscoelasticity of the structural material – the relevance of the stress and strain condition of the configuration on time under the influence of weight.

By viscoelastic materials, we mainly understand polymers (e.g., rubber, plastics), biopolymers (e.g., human tissue, cartilage), wood, and even some metals at high temperatures. It should be noted that structural metals at normal temperatures behave like elastic bodies, while at higher (above 200°C) temperatures they exhibit viscoelastic characteristics. At 0 degrees Celsius, plastics have viscoelastic properties, but they are very weak, meaning they are close to elastic bodies. However, at 50 degrees Celsius, their true viscoelastic properties can be observed [1, 2].

Hence, the analysis of deformation and strength characteristics of structural components functioning under severe dynamic conditions, while considering temperature and various influencing factors, is of significant importance. Specifically, the issue of field interaction holds major theoretical and practical value in continuum mechanics, as it focuses on exploring the relationships among mechanical, thermal, electromagnetic, and other types of effects. This is motivated by both applied demands and the inherent principles underlying the advancement of continuum mechanics.

The consideration of the mutual influence of the mentioned sectors is of profound theoretical significance, as it allows for a penetrating, wider in scope, and an accurate quantitative representation of viscoelastic medium dynamics, reveals a number of qualitatively new effects, and makes it possible to assess the limits of applicability of theories that neglect coupling effects [3-7].

All the aforementioned factors highlight the significance of the present study, which focuses on nonlinear vibration and dynamic stability issues of viscoelastic thin-shell structures, formulated within the framework of the Kirchhoff–Love theory, both considering and neglecting temperature effects, as well as analyzing interrelated and independent mechanical and thermal fields [8-11].

# MethodS

Let the shell be non-uniformly heated through its thickness and along its central surface to a temperature , which varies with time. We align the - axis perpendicular to the central surface toward the curvature center and set the coordinate origin at a point on the median surface. Let the axes  and  coincide with the principal curvature directions of the shell. Denote the shell’s thickness by , and its dimensions along the  and axes by  and , respectively.

The correlation between temperature , the strain components , , and the stress components  in the case of a plane is expressed as follows







where  - the elastic moduli of the material in the directions of the axes  and , correspondingly;  - the Poisson’s coefficient in the  direction under tension along the  direction; ,  - the coefficients of thermal expansion in the directions of the axes and, respectively.

The following relationship holds between the material characteristics:

;

are integral operators with relaxation kernels :



The strain components, the displacements of the middle layer, and the changes in curvature of the middle surface are interrelated through the following mathematical relationships [11–15]









in here  is the initial deflection.

The strains  for a layer situatedunits away from the central surface, based on hypothesis of straight normals, may be expressed in the form of

.

# RESULTS AND DISCUSSION

Let us calculate  - the normal forces,  - the shear force,  and  are moments of bending and  - the moment of torsion

























replacing these expressions in the motion equations [13–15]

; (1)

; (2)



; (3)

As a result, the following system of nonlinear integro-differential equations is derived.



































 (4)

where





.

Proceeding in the same manner as in works [3,5,15–17], we derive the governing heat conduction equation for orthotropic materials in the following form



where  are the parameters of heat conductivity in three perpendicular orientations, and is the specific thermal capacity.

When the right-hand side of this equation is expressed in terms of displacement, it can be written as follows











. (5)

# Conclusion

The obtained system is quite general.

Let us consider some particular cases.

1. In the absence of inertial effects, the dynamic process can be simplified. Under this assumption, equations (1)–(3) take a reduced form. The justification for neglecting inertial loads is supported by the findings reported in [5]. The corresponding simplified system is therefore omitted here.

2. For the case where  and , with  denoting the radius of curvature of the middle surface, the equations correspond to those describing a circular cylindrical shell.

3. If , the obtained relations describe a spherical shell.

The systems of equations presented in (4) and (5) are mutually connected. Hence, this mathematical model represents the deformation behavior of a viscoelastic orthotropic shell under non-stationary mechanical and thermal loads, as well as the reciprocal phenomenon – the modification of its temperature field caused by deformation. This type of formulation is referred to as a coupled dynamic thermo-viscoelasticity problem.

# References

1. Ilyin, V. P., Maltsev, L. E., & Sokolov, V. G. (1991). *Calculation of building structures made of viscoelastic materials.* Leningrad: Stroyizdat.
2. Bratukhin, A. G., Sirotkin, P. F., Sabodash, P. F., & Egorov, V. N. (1995). *Materials of the future and their remarkable properties.* Moscow: Mashinostroenie.
3. Karnaukhov, V. G., & Kirichok, I. F. (1986). *Coupled problems in the theory of viscoelastic plates and shells.* Kiev: Naukova Dumka.
4. Bolotin, V. V. (1961). Dynamic problems of thermoelasticity for plates and shells in the presence of radiation. In *Proceedings of the 3rd Conference on the Theory of Plates and Shells*. Kazan.
5. Ilyushin, O., & Pobedrya, B. E. (1970). *Fundamentals of the mathematical theory of thermo-viscoelasticity.* Moscow: Nauka.
6. Nuritdinov, J., Khaydarov, I., Turdaliyev, S., & Djuraev, I. (2025, October). Application of Minkowski difference to optimal control tasks. In *AIP Conference Proceedings,* 3377(1), 030001. AIP Publishing LLC. <https://doi.org/10.1063/5.0299492>
7. Akbarov, U. I. (1997). Vibrations of a viscoelastic rod considering coupling of deformation and temperature fields. *Uzbek Journal “Problems of Mechanics,”* (1), 10–17.
8. Badalov, F. B., Eshmatov, K., & Akbarov, U. I. (1991). Stability of a viscoelastic plate under dynamic loading. *Soviet Applied Mechanics,* 27(9), 892–898. <https://doi.org/10.1007/BF00887982>
9. Mamatov, M. S., Nuritdinov, J. T., Turakulov, K. S., & Mamazhonov, S. M. (2024). Geometric properties of the Minkowski operator. *Bulletin of the Karaganda University. Mathematics Series,* 116(4), 127–137. <https://doi.org/10.31489/2024M4/127-137>
10. Akbarov, D., Akbarov, U., & Nuriddinov, J. (2025). On the development of an algorithm for symmetric block encryption over the Feistel network. *Journal of Applied Mathematics & Informatics,* 43(4), 1171–1179. <https://doi.org/10.14317/jami.2025.1171>
11. Ambartsumyan, S. A. (1961). *Theory of anisotropic shells.* Moscow: Fizmatgiz.
12. Ogibalov, P. M., & Suvorova, Yu. V. (1965). *Mechanics of reinforced plastics.* Moscow State University.
13. Volmir, A. S. (1972). *Nonlinear dynamics of plates and shells.* Moscow: Nauka.
14. Ogibalov, P. M., & Gribanov, V. F. (1968). *Thermal stability of plates and shells.* Moscow State University.
15. Kovalenko, A. D. (1975). *Thermoelasticity.* Minsk: Vysshaya Shkola.
16. Lykov, A. V. (1967). *Theory of heat conduction.* Minsk: Vysshaya Shkola.
17. Carslaw, H. S., & Jaeger, J. C. (1964). *Conduction of heat in solids.* Moscow: Nauka.