**Physics-Based Modeling of Cocoon Reeling Dynamics   
for Industrial Silk Production**

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**Abstract.** The article investigates the process of reeling mulberry silkworm cocoons with the aim of improving silk thread production technology and increasing the efficiency of textile manufacturing. The study examines the complex interactions between various physical factors, such as silk thread tension, cocoon gravity, and hydrostatic buoyancy forces. The paper presents mathematical models of static and dynamic conditions that ensure effective cocoon reeling. Special attention is given to the conditions of buoyancy, submersion, and dynamic behavior of cocoons during reeling. The influence of cocoon shape, density, and degree of water saturation on process stability is revealed. An analysis of the conditions necessary for the stable state of cocoons in water is conducted, and methods for optimizing the reeling technology are proposed. The obtained results can be applied to enhance equipment productivity, improve silk thread quality, and reduce manufacturing defects. Directions for further research have been proposed, focusing on the experimental refinement of the developed models and the development of automated reeling technologies.

**Keywords:** silkworm, cocoon, cocoon thread, sericin, pupa, silk reeling technology, raw silk, reeling, cocoon reeling machines, cocoon reeling dynamics.

**INTRODUCTION**

The study of technological processes involved in reeling silkworm cocoons is an important scientific and practical task of significant relevance to the textile industry. Cocoons represent a complex biologically formed system consisting of a silk shell and a silkworm pupa. The process of cocoon reeling encompasses both static and dynamic interactions of various forces acting on the cocoon, such as thread tension, gravity, and buoyancy resulting from the cocoon's immersion in water. Moreover, the characteristics of the cocoon itself, including its shape, density, mass, and degree of water saturation, are of great importance. A proper understanding of the physics behind the reeling process will enable improvements in the quality of the resulting silk thread, increase equipment productivity, and reduce the likelihood of product defects. This work examines the mathematical and physical models for describing the silkworm cocoon, the conditions of its buoyancy and submersion, as well as the static and dynamic aspects of the silk thread reeling process [1-5].

The process of reeling silkworm cocoons is a complex multifactorial task, involving thermo-hydromechanical impact and stochastic cocoon thread defects. Current literature identifies four research clusters structural and technological parameters of reeling and defect formation, energy and resource intensity, modeling of silk structure and strength and cyber-physical modeling of industrial processes as the basis for transferring methods to silk reeling.

Statistical analysis of raw silk defects by meters of controlled length demonstrates that "middle dropped cocoons" and thick-thin heterogeneities are systematically linked to technological deviations. Regression models show a significant influence of reeling temperature and cooking time on the frequency of end breakages. At the micro- and nano-level, silk protein network models reproduce the redistribution of stresses and the effect of preliminary stretching, which increases the strength of raw silk during reeling [6-10].

In related industries - continuous steel casting and rolling, polymer extrusion - physical and CFD models are widely used to predict the distribution of forces and dynamic instabilities. For steel, bench physical simulators validated by heat flows and structure, as well as mathematical models for predicting temperatures and mill vibrations have been developed. In polymer rheology, the mechanisms of instability occurrence at the die exit and methods for suppressing them have been substantiated. This methodology is naturally transferred to silk reeling as a process with controlled tension and local transient modes.

Physical and computational modeling allows for a priori estimation of force distribution, heat transfer, and the occurrence of dynamic instabilities, thereby optimizing machine settings before full-scale tests. In metallurgy, physical simulators and 3D models link thermal maps of the form with the structure and vibrations of the mills. Rheological models explain "sharkskin" and periodicities during slip, while controlled temperature gradients at the die exit stabilize the flow. For silk reeling, a similar approach includes the mechanics of thread tension and its oscillations during passage, the thermohydrodynamics of cocoon steaming accounting for sericin dissolution, as well as a probabilistic model of thread breakage.

**METHODS**

Theoretical and experimental approaches were used in the study. The main methods are mathematical and physical modelling aimed at the analysis of static and dynamic conditions of cocoons reeling. The laws of continuum mechanics, hydrostatics and buoyancy principles, including Archimedes' law, were used. Mathematical models describing the shape of cocoons were considered, including ellipsoid of rotation, spheres and hyperboloids. Conditions of equilibrium of forces acting on the cocoon were determined, including thread tension, gravity and ejective forces. Methods of analytical solution of equations describing immersion and buoyancy conditions were applied. In considering the dynamics, inertia forces, rotational moments, and oscillatory motions of cocoons in multicocoon reeling systems were taken into account. To further refine and verify the theoretical conclusions, experimental studies are planned, including laboratory measurements of process parameters and dynamic testing of models under conditions as close as possible to real life. The obtained theoretical and experimental data will be used to improve cocoon reeling technologies and equipment.

**RESULTS AND DISCUSSION**

The cocoons of the mulberry silkworm are a complex system consisting of a sheath 1 and a silkworm pupa 2 (see Fig. 1). The cocoon shell is a paired silk thread produced by the silkworm caterpillar, up to 1500-1800 metres long, with linear density of 0.18-0.33 tex, glued with sericin. The shape of the cocoon ranges from elipsoid of rotation (a) to complex, with two spherical surfaces connected by a saddle (b).

1

|  |  |
| --- | --- |
|  |  |
| a) | b) |

**FIGURE 1.** Form and components of silkworm cocoons: a) waistless b) deep waist. 1 - cocoon shell, 2 – pupa.

Mathematically, the first type of cocoon can be described using a simple ellipsoid of rotation

 (1)

For the second form it is necessary to describe two spheres (or rather their part) together

 (2)

and a hyperboloid of rotation.

Accordingly, it is possible to determine their volumes

 (3)

and also from the known mass *M* their average density

 (4)

Considering the hygroscopicity of cocoons, the air and water permeability of their shell, and the possible presence of water in the internal volume, cocoons are subdivided into:

(a) Dry or conditionally dry with a density of

 (5)

b) soaked for reeling on the float, at

 (6)

such cocoons are called semi-submerged, as is the method of reeling them accordingly:

c) soaked for reeling in a submerged state, when

 (7)

where

*mc* is the mass of fibre in the cocoon;

*mc* is the mass of sericin;

*mcuk* is the mass of the pupa;

*mc1* - mass of water in the cocoon cavity;

*V* is the volume of the cocoon.

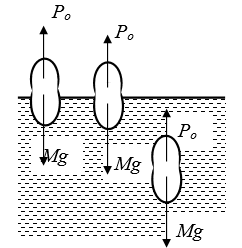
Inequality (7) is possible for one reason - the density of cocoon thread (ρ =1.37) and sericin is always greater than one, and if the cocoon cavity is filled - it sinks

While in a tub of water, the cocoon can:

(a) Stay afloat, when, according to Archimedes' law, the weight of *Mg* and the expulsive force balance each other, and the regulator of this ratio is the depth of immersion of the cocoon in water - the lighter it is, the shallower the depth to which it is immersed (see Fig. 2).

b) sink when the expulsive forces become less than the total weight of the water-filled cocoon, having displaced all the air with water.

These alphabetical concepts will become essential when considering the statics and dynamics of the cocooning process.

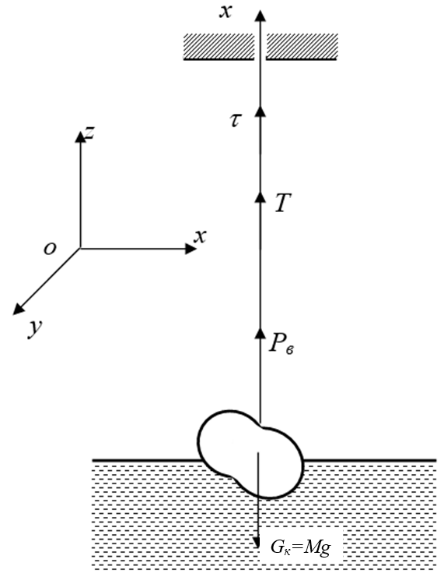


**FIGURE 2.** View of the state in the water during the reeling of cocoons

Let the process of reeling a single cocoon (see Fig. 3) proceed.

In statics (not considering the fluctuations of the thread tension *T*, and the ejection force *Pc*,) the system has one degree of freedom and is described by Eq

 (8)



**FIGURE 3.** The schematic view of the cocoon reeling process

The determinant in the system is the tension value *T*, which ensures the thread detachment from the cocoon shell and depends on the degree of sericin boiling, the location of the point of silk thread descent from the cocoon and, most importantly, does not remain constant in time - *T(t*), which is connected with the dynamics of the process, as discussed below.

Let us now point out that reeling is possible if the following condition is fulfilled

 (9)

where [*Pfrom*] is the force of thread detachment from the shell.

If condition (9) is not fulfilled, then at any *Mg* and *Pc*, the cocoon begins to rise above the water mirror - there is a stretching of *T* from initial to maximum:

 (10)

and if

 (11)

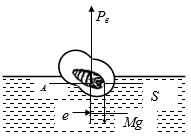
then the cocoon is removed from the water and transported upwards. This case should be mentioned in particular - the cocoon is *not sufficiently* steamed, it is transported to the obstacle and the thread is broken.

Given that in the case of reeling of cocoons submerged in water, the values of *Mg* are much higher than in the case of semi-submerged, and even more so in the case of dry cocoons, the probability of fulfilment of condition (11) becomes lower (apparently by several times), and cocoon pulling out of water does not occur.

This is or approximately this is how the static process of reeling a single cocoon would look like if the cocoon had a ball shape. Its moment of inertia during rotation would be negligibly small, and any position would be stable relative to the water mirror. In practice, these conditions are absent, in a semi-submerged or submerged state due to the displacement of the centre of gravity (pupa) cocoon has one *stable* position on the water mirror, to which it tends under the action of forces *Mg* and *Pc* due to the appearance of the moment with a shoulder *l* in general case

(12)

Of course, when there is no tension on the silk thread *T=0* (see Fig. 4), the value of *l is* always zero if the centre of gravity lies at the point through which the expulsive force *Pc* passes



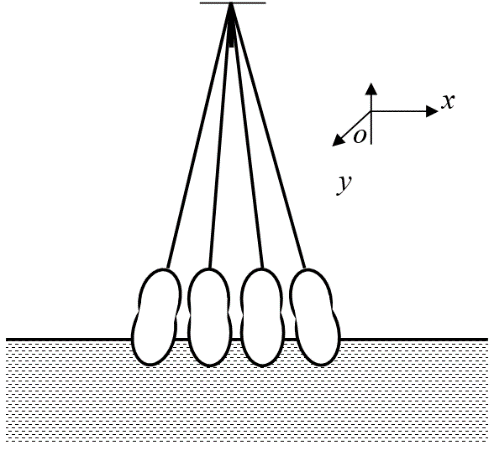
**FIGURE 4.** The forces of acting on the reeling process

When the shape of the floating cocoon is complex and the centre is displaced from the line of action of the ejective force, the moment of forces (12) rotates the cocoon until it appears that *l=0*. There are two such positions for any cocoon shape:

(a) Stable, when the centre of weight application *Mg-* point; *S-* mode below the point; *A-* centre of application of the pushing force;

b) unstable - in the second case, when *A is* below *S.* In the theory of ship design, this case is the most frequent one, due to the consideration of minimal draft of ships. The special shape of the vessel body provides *conditional stability,* when the shape of the side surfaces of the vessel makes this position of the floating vehicle relatively equilibrium.

The figure (see Fig. 5) shows a static picture of reeling of a single cocoon (one can imagine a very slow reeling), which leads to the conclusion (see Fig. 3) - the silk thread must necessarily have a vertical position. So, about the rosette of cocoons without friction of cocoons against each other, pressure of one on another, is out of the question - threads inevitably form a complex multi elementary system (Fig. 5), where each cocoon deviates from the vertical (in *xoz* plane) as a result of contact interaction of cocoons in the rosette, not always desirable because of random violation of purity of topological braid [1], which can be a bundle of fibres from 7-10 and more cocoons being unwound.



**FIGURE 5.** The static scheme of reeling single cocoon

Here the picture is different - spatial in *z,x,y* axes, inevitably movement of cocoons relative to each other, i.e. the process becomes dynamic, consideration of which is possible only according to the Dalembert principle - to all existing forces (influences) it is necessary to add *fictitious forces of inertia* - linear and rotational, dependent on masses, moments of inertia of elements, linear and angular accelerations.

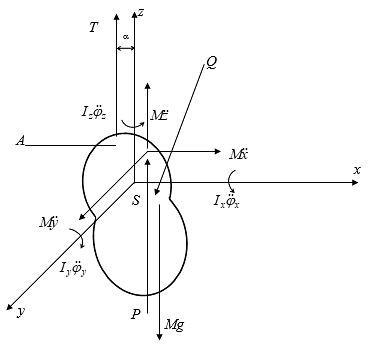
Being consistent, the diagram of Fig. 6 and to the described real forces *T* (tension), *Mg* (weight), *Pc* - (Archimedean ejection force), we add the forces of inertia of the cocoon along the vertical axis , and along the *x* and *y* axes  and  (if they arise - discussed below).

Moments of inertial forces in planes:

**;

**;

*.*



**FIGURE 6.** Directions of inertia moments in the reeling process

In the above notations we have extracted and newly derived:

*constant* - *M* is the mass of the cocoon (the mass changes in the considered time interval and therefore it can be considered constant);

*g* is the acceleration of free fall;

*variables* - *Pc(z)* - Archimedean ejection force, a function of the *z* coordinate (depth of immersion), for small oscillations of the cocoon can be considered linear

 (13)

in the general case it's non-linear

*T(t)* - tension, - in dynamics a function of time, i.e. *T=T(t)*, is caused by the force of thread detachment from the cocoon according to - acceleration of the centre of gravity of mass *M of* the cocoon relative to the axes of the same name;

- moments of inertia of the cocoon at rotation relative to *x, y, z* axes are variable, because the cocoon rotates and its orientation relative to coordinates changes. In mechanics it is proved that anybody has principal moments of inertia - for the cocoon these are axes of symmetry - passing through the centre of gravity;

- angular accelerations of the rotating cocoon relative to the corresponding axes;

*Q* –( (external) (force -) con the part of other elements of the system (other cocoons, jet flow, etc.))

 - (represented) in the figure components (vector) (vector)(s) (tension) (*T)* and forces (along the coordinate axes);

 (14)

 (15)

Let's orient them by angles with corresponding axes

 (16)

 (17)

Now it is possible by Dalembert to make equations of equilibrium in the most general case, covering the whole range of possible cocoon positions (though, except for the special case of cocoon rotation in a circulating water flow, which is discussed separately, although this case is included in the general problem).

Another important circumstance, partly derived from the reeling scheme -

 (18)

Whence it follows

 (19)

and also

 (20)

Or

 (21)

Taking into account the sign of the inertial force (*Fn=-ma*), the system of equations of equilibrium of the cocoon (according to Dalembert) will be written down (in the order of axes designation), taking into account (14):

 (22)

 (23)

 (24)

 (25)

 (26)

 (27)

In the system of equations it is accepted to provide the arms of forces:

(a) *Tz* - *z(zx), zzy* with respect to the axes *oh* and *ou*;

b) *Ty* - z*(yx), z(yz)*with respect to the axes *oh* and *ohz*;

c) *Tx* - *z(xy), zxz* with respect to the *ou* and *oz* axes;

*(Pw - az) - ax, ay* with respect to the axes *ox* and *ou*;

*Qz -ρ xz,ρ yz* with respect to the *och* and *ou* axes;

*Qy -ρ (xy),ρ zy* with respect to the axes *oh* and *ohz*;

*Qx -ρ (yx),ρ zx* with respect to the *ou* and *oz* axes*.*

The signs± at the moments from the force *Q* mean the variation of the direction of rotation of the moments. The signs in the equations of systems (22) - (27) are taken in accordance with the scheme of Fig. 6 - only in three planes at the moments of external forces their variation at change of points of *Q* application to the cocoon is given.

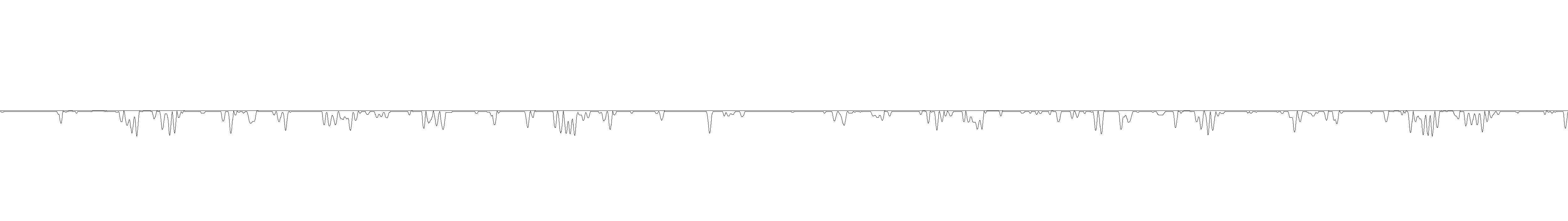
Let us also add that the external force *Q* may be absent, or there may be several of them, which will change only the equations written in the above system. The analysis proposed below is more qualitative than quantitative in its essence - for all applied forces of statistics have their own solution, which, however, is not so important. Each cocoon moves according to its own regularity - our task is to determine the main moments of this movement.

The complex system at time *t* can be dissected into three groups of motions:

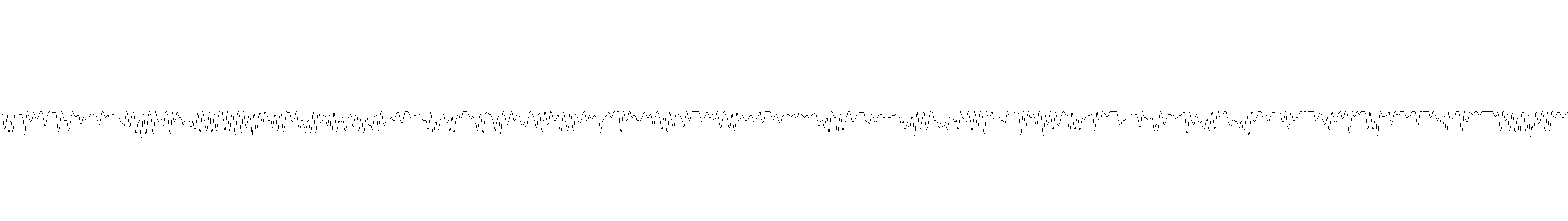
(a) Floating of the cocoon on the water mirror - (22) - (27);

b) rotation of the cocoon on the water surface around some axis (instantly changing directions - (25) - (27));

c) with the previously mentioned probability of cocoons immersion at thread breakage or detachment from the water mirror - equation (24).



a)



b)

**FIGURE 7.** a) Graph of oval-shaped cocoon slippage without interception, b) Graph of oval-shaped cocoon slippage with deep interception.

The conducted research on the static-dynamic conditions of silkworm cocoon unwinding revealed key factors determining the stability and efficiency of the process. Mathematical models based on the laws of continuum mechanics, hydrostatics, and the Dalamber principle showed that the relationship between silk thread tension, cocoon weight, and pushing force plays a decisive role in successful unwinding. The influence of the steaming degree is especially significant, and it has been established that submerged cocoons have more stable characteristics, which reduces the risk of their premature rise above the water surface and thread breakage. Comparison of different cocoon shapes (ellipsoidal and combined) made it possible to establish that the shell geometry is directly related to the moments of inertia and, consequently, to the dynamics of movement during unwinding. The established relationships between thread tension and the Archimedean pushing force are of particular interest. This result opens up new possibilities for process management.

**CONCLUSION**

In conclusion, it should be noted that the conducted research allowed us to identify the main regularities of formation and behavior of silkworm cocoons. The mathematical model successfully describes the distribution of forces affecting the process of cocoon reeling and is confirmed by analytical calculations. The results obtained demonstrate a significant dependence of cocoon stability on the distribution of weight and inertial forces. The analysis showed that changes in the density and humidity of the cocoon significantly affect its buoyancy and dynamics of movement. Application of the Dalamber principle allowed to take into account inertial effects, which significantly improved the description of dynamic processes. The revealed dependence between silk thread tension and Archimedean ejection force opens up new possibilities for regulating the technological process. The developed modelling technique provides a tool for optimization of reeling processes in industrial production. The obtained data allow us to estimate the critical conditions under which the system transitions from a stable to an unstable state. Comparison of theoretical calculations with experimental observations confirmed the correctness of the proposed model.

The study demonstrates the relevance of an integrated approach in the study of biological and mechanical systems.

Further research may expand the scope of the model by considering additional physical and biochemical factors.

Thus, the work makes a significant contribution to the development of theoretical and practical aspects of the study of cocoon dynamics.

Generalization of the obtained results contributes to the improvement of the efficiency of technological processes in silk breeding.

Thus, the analysis shows that the reeling process of mulberry silkworm cocoons depends on a whole complex of factors, among which the most significant are silk thread tension forces, cocoon weight and hydrostatic ejection forces. The static stability and dynamic behavior of the cocoon depend on its shape, weight distribution and degree of soaking. Submerged cocoons were found to have more stable reeling conditions compared to dry or semi-submerged cocoons, due to the increased weight and reduced risk of unwanted lifting from the water. The dynamic interaction of cocoons in a multi-conductor reeling system significantly complicates the process, requiring the inertia and rotational forces to be taken into account. The obtained mathematical dependencies can be applied to optimize cocoon reeling technology, improve product quality and reduce production costs. Promising directions of further research are experimental specification of model parameters and development of software tools for process automation.

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