**On a Problem of Statistical Modeling of a Hydrodynamic System with Distributed Parameters**

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**Abstract.** The article considers the applied problem of software management for analyzing the state of the system. The state of the system (pressure field) is described by an elliptic partial differential equation. An estimate of the solutions of the boundary value problem in a limited range of parameter changes is obtained. The results can be used in the development and operation of oil and gas fields

**Keywords:** Differential equation, elliptic type, boundary conditions, Laplace operator, states of the system, displacement estimation, Markov chain.

**INTRODUCTION**

Development and improvement of mathematical models of complex dynamic filtration processes in oil and gas and aquifers, as well as numerical methods for their solution, the solution of stationary and non-stationary filtration problems in oil and gas layers with poor permeability are considered in the works of such scientists as M.Sharma, H. Aziz, E.Settari, N.B.Lopuh, C.Atkinson, K. Ives, Z.Mehdi, P.J.Monteiro, S.Banerjee, G.I.Barenblatt, M.Chraibi, D.B.Silin, F. Boyer, C.Lapuerta, S.Minjeaud, A. Darcy, L.S.Leybenzon and others [1, 2].

During the operation of oil and gas fields, as well as in the processes of underground leaching in mining fields, the productive reservoir and the wells located in it can be represented as a single hydrodynamic system with distributed parameters (parameters change in space and time), and the state of the system itself is described by partial differential equations. Thus, the formation and the well are like a hydrodynamic control system with distributed parameters [3, 4]

When determining the modes of operation of the system, it is necessary to solve applied problems of software management, i.e. determining the state of the system in space and time. The state of the system (pressure field) is described by an elliptic partial differential equation

 (1)

Here  - limited area  - of dimensional space , the infinitely smooth boundary of which consists of an outer contour *G0* and internal contours *G1, G2, …., Gn-* well contours (in the case of a flat formation n=2. Coefficients - the complex of filtration parameters (- the permeability coefficient of the productive reservoir, - reservoir capacity,- the viscosity coefficient of the liquid.

The pressure on the outer contour of the area is set as

 , (2)

on the contour of wells

,  (3)

where  - well flow rates.

**METHOD OF RESEARCH**

Equation (1) was solved under various boundary conditions by statistical tests using the “grid walk” rule.

In this paper, we propose the application of a well–known statistical test method - “wandering through spheres” [5].

To solve the boundary value problem (1) – (3), we introduce a new function

, 

Then equation (1) has the form:

 (4)

where ∆ is the Laplace operator.

We introduce a number of designations and definitions. Let

 (5)

- a single-connected area limited to only one *G*, here  and  accordingly, the center and radius of the contour *G1*;

 

 (6)



Let's define the number , which characterizes the measure of well density as follows:

 (7)

 - distance between sets  and *G.*

Definition. The sphere  centered at a point  it is called an acceptable sphere, if  and there is inequality



**RESEARCH RESULTS**

Statement 1. For any  there is a valid scope

 moreover, the radius of this sphere satisfies the inequality

 (8)

Proof. Let . Let's define the set



where



It is known that  the sets do not intersect in pairs and their number is equal . Plenty  , has  elements, therefore, there is such a thing , as in the sets  and  there are no elements from . If we define



that's for  the inequality is fulfilled (8) and the sphere  is acceptable.

When , then assuming  we get the required radius of the sphere. Statement 1 has been proven.

In the future, we will count everywhere, what



Let , and  - acceptable scope. Using the fundamental solution of the Laplace operator, it is easy to get an idea

 (9)

where  - the angle between the normal to the contour *Gi* and vectors 

Let  - the average value of the function  in the inner contour *Gi*, i.е.

 (10)

Thus, from (10) we get:



and from (9) we get:

(11)

Since 

by 



we will get

.

From (10) and (11) we get:

 (12)

where .

If , then in a similar way we have:

 (13)

where to  There is also inequality (8). But instead of  It is necessary to substitute , i.е.

.

Let .

Let’s build a Markov chain starting from the point , according to the recurrent formula 

where is the sequence  - isotropic independent vectors uniformly distributed on a unit sphere.

Regarding the Markov chain, we will make two assumptions:

а);

b)

It follows from statement 1 that for any point  You can build a chain  according to rule (13), for which condition a) and b) are fulfilled.

In the future, we will assume that the chain satisfies the conditions a) and b).

Let  и 

where  - the vicinity of the border. Let's introduce the moment when the particles stop



for the Markov chain .

Let’s build the following estimate

 (14)

where  - the nearest point from the outer boundary *G* *K* 

 (15)

 (16)

It is not difficult to make sure that when the condition is met, Statement 2.

Statement 2.For the moment of stopping  evaluation is in progress



where is the constant C it depends only on the properties of the border *G*.

It is noticeable that the function  in the definition, the estimate (14) is limited from above by the number

 (17)

where  - the diameter of the area *D*, and also as in the following statements can be proved.

Statement 3.Variance of the estimate  uniformly bounded.

Lemma. Let  and  , then for the offset



evaluations (14), inequality is fair:

 (18)

Proof. If , then an evaluation is also performed

 (19)

where  the nearest point к  and the constant C it depends only on the border *G*.

Proof of inequality (18) it flows easily from (12), (13) and (15), (16).

**CONCLUSION**

The obtained estimation of solutions of the partial differential equation (1) in a limited range of parameter changes with boundary conditions (2), (3) They can be used in solving applied problems of analyzing the states of systems with distributed parameters that occur during the development and operation of oil and gas fields, as well as in hydrodynamic processes of underground leaching.

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