**The Application of Physical Models in Economic Dynamics: from Statics to Econophysics**

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**Abstract.** This article investigates the potential and limitations of applying models and methods from physics to the analysis of economic dynamics. It examines how fundamental concepts such as statics (system equilibrium) and dynamics (the evolution of systems over time), originally developed in mechanics and thermodynamics, are reflected in economic theory. Particular attention is paid to the shift from traditional economic equilibrium models to dynamic analysis of economic processes using the mathematical apparatus of differential and difference equations, analogous to those used in physics. The paper analyzes examples of transferring physical principles and mathematical models into the economic domain, forming the basis of econophysics—an interdisciplinary field. It discusses how these approaches provide deeper insights into complex, nonlinear, and non-equilibrium phenomena, such as financial markets, wealth distribution, and macroeconomic fluctuations. The objective of this work is to systematize existing approaches, demonstrate their practical relevance, and outline the future prospects for physically informed modeling for forecasting and management within the contemporary economy.

**Keywords:** Law of Motion, Capital accumulation, Stocks and Flows, Economic Dynamics, Comparative Statics, Dynamics, Economic Equilibrium, Non-Equilibrium Processes, Physical Models, Interdisciplinary Analysis.

**INTRODUCTION**

During its development as a scientific discipline, economic theory incorporated a wide range of approaches. Each approach emerged within a particular academic field, and its methodology was subsequently applied to economics. As a result of influences from the social sciences—such as sociology, political science, history, and geography—and from the natural sciences, including biology, physics, and mathematics, as well as interdisciplinary fields like statistics, various branches of economic thought were formed, including [1-5]:

* Normative economics;
* Positive economics;
* Neoclassical and Keynesian schools;
* Marginalism;
* Behavioral economics, among others.

Physics, as a systematic discipline focused on studying material phenomena, has influenced not only natural sciences such as chemistry and biology but also economic theory [6-8]. This influence is visible in two main ways:

1. Many early contributors to economic theory were scholars with backgrounds in physics and mathematics.
2. Because economics studies the production, exchange, distribution, and consumption of both tangible and intangible assets, it naturally required models grounded in physical and mathematical principles.

The field that examines the direct application of physical methods to economics is known as *econophysics*. The term was first introduced by H. Eugene Stanley. In 1998, the first conference on econophysics was held in Budapest, and in 2000, R. N. Mantegna and H. E. Stanley published the first book dedicated to this discipline [9-11].

Econophysics developed primarily through the use of probabilistic and statistical methods. Its main directions include quantum mechanics–inspired models, quantum finance, and quantum economics. Even prior to the formal emergence of econophysics, physical modeling played a significant role in marginalism, economic dynamics, financial engineering.

Economic dynamics concerns changes in the economy and its indicators over time. Economic growth and growth rates, technological change and wealth accumlation are central topics of economic dynamics. While economic growth refers to the transformation of the economy and its structure from one state to another, the process of transitioning between these two static states is inherently dynamic. Indicators that do not account for this transition are referred to as comparative statics.

In this work, we explore the correspondence between physical and economic concepts.

**METHODS**

Since the methods shared by economic and physical concepts are fundamentally mathematical, we employ tools such as difference and differential equations, real analysis, measure theory, probability theory, and mathematical statistics.

The main challenge lies in defining, formalizing, and appropriately transforming variables. The similarity between economic variables and physical units becomes evident when they are classified in the following way:

Stock variables are used in comparative statics and represent quantities measured at a specific point in time—essentially accumulated values. An example is the stock of physical capital.

Flow variables depend on a unit of time and correspond to physical processes such as velocity or acceleration. In economics, examples include investment (the rate of capital addition) and capital depreciation (the rate at which capital wears out).

Particularly, as in Figure 1 the inflow of capital as the investment and outflow of capital as the depreciation result the accumulation of capital in a stock. This is similar to the concept of fluid flow through the pipe and its stock in a tank.



**FIGURE 1.** The process of capital accumulation through stock and flows

To express displacement, distance, velocity, speed, and acceleration, we use scalar and vector variables, as well as real- or vector-valued functions. Hence, we write functions such as r(t) and f(x) for a vector r and real-valued function respectively. To minimize error, we move from discrete to continuous time intervals. For concave (or quasiconcave) paths, the displacement may approach the distance in the limit. In particular,

(1)

Here,

r – a vector-valued function, that represent the displacement;

s – the parameter that describe the distance.

Displacement may be defined in Euclidean, metric, vector, or topological spaces, or in any normed space. In ℝn, a general norm can be represented by the Lp norm

(2)

Distance along a curve, s(t), at time t is then generalized through the arc length or the integral of the velocity norm,

(3)

Here,

γ – parameter function;

τ – parameter variable.

The equivalence between physical and economic models can be demonstrated only when the underlying quantities are well-defined and measurable. Therefore, we require that the spaces of both physical and economic variables admit meaningful measures. Particularly, in the equations (12) and (13) the probability space and random variables, through which we express the Brownian motion and the random walk are considered to be measurable. This is because ℱ is a σ-algebra.

Thus, we generally assume that all functions under consideration are continuous, piecewise smooth, sufficiently differentiable, and measurable on their domains.

We can now express these variables in terms of their corresponding units.

In discrete time, both physical and economic laws are typically expressed using difference equations. In continuous time, the corresponding laws are formulated using differential equations, where changes occur with respect to real time. In this context, physical quantities are defined as follows:

(4)

Here,

k is an arbitrary variable in ℝn.

Displacement, velocity, and acceleration are related through successive differentiation:

(5)

The work done by a force is given by the following physical law:

(6)

Here, *W* – the work done by applying a force or energy to an object;

*F****(x)*** – the force applied as a function of displacement (x);

***x***– the bundle of displacement variables;

*a, b* – the initial and final points;

*∆K* – the change in kinetic energy.

In ℝ2 the equation (6) can be illustrated as in Figure 2. In ℝn we can generalize (6) using identities (1) thorough (3).

Growth and (radioactive) decay processes, in which current quantity, N, is proportional to the initial quantity, N0, through the rate of change, k, are described by the following equations:

(7)

These differential equations model a wide range of physical and natural phenomena, including radioactivity, population growth, and other processes that exhibit a constant proportional rate of decay or growth.

A more refined representation of growth dynamics is given by the logistic growth model, where the growth rate initially increases, reaches an inflection point, and then decreases until the system approaches an asymptotically stable level:

(8)

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**FIGURE 2.** Work done by force which is function of displacement

This model applies, for example, to the spread of viruses, Newton’s Law of Cooling under certain constraints, and various biological and ecological processes.

A classical example of a pressure–volume (P–V) relationship in physics is the ideal gas law. Under isothermal conditions, this static relationship is expressed as:

(9)

The work done by an ideal gas during an isothermal expansion from volume (V1) to (V2) is given by:

(10)

where

N – the number of gas molecules,

k – the Boltzmann constant,

T – the absolute temperature.

In a scalar field, such relationships are represented by isotherms (see Figure 2).



**FIGURE 2.** The isotherms as constant in the P-V scalar field

According to Isaac Newton’s Law of Universal Gravitation, the gravitational force of attraction between two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between them:

(11)

Here,

F – the gravitational force of attraction;

m1 and m2 – masses of the two bodies;

r – the distance between their centers of mass;

G – the gravitational constant.



**FIGURE 3.** Gravitation between two objects, where m is mass of an object, r – distance between two objects

Stochastic changes in the motion of a particle, where the number of collisions becomes infinitely large, are modeled using Wiener processes. A discrete approximation can be written as:

(12)

Here,

ξ – independent and identically distributed random variables with zero mean and unit variance;

xt(N) – represents the displacement of the particle at time (t) after (N) collisions.

Depending on the context, the random variables may follow Normal, Lognormal, Binomial, Exponential, or other probability distributions. Regardless of the specific distribution used, we define a random variable rigorously as follows.

Let (Ω, ℱ, ℙ) be a probability space. A measurable function X: Ω →ℝ is called a random variable. If *X(Ω) = {xi: i ϵ ℕ}* is a countable set of values, then its expectation is

and in the continuous case,

(13)

Harmonic motion, which describes oscillatory phenomena such as pendulum swings and wave behavior, can be represented as a periodic function determined by amplitude, angular frequency, and phase. The displacement is given by:

(14)

Here,

*xt* – the displacement at time t;

*A* – the amplitude of motion;

ꞷ - the angular frequency;

φ – the phase shift.

Such equations describe economic dynamics involving periodic fluctuations. For example, in business cycle theory, phases such as boom, stagnation, and recession exhibit cyclic patterns. The amplitude, frequency, and phase parameters determine the intensity of economic expansions, the depth and duration of crises, and the overall performance of the economy.

In physics, weight is defined as mass multiplied by gravitational acceleration and is measured in Newtons (N). For instance, if g = 9,81 m/s2, then a mass of 1 kilogram corresponds to a weight of approximately 1 N. Analogous principles can be applied in economics—for example, adjusting nominal values using social discounting or deflators to measure real values—where a “force-like” adjustment transforms one scale of measurement into another.

**RESULTS AND DISCUSSION**

The work done in economics is modeled as an output of an economy, that is typically represented by a production function. The factors of production that contribute to output—such as physical capital, labor, and human capital—are treated as variables. Many forms of production functions exist, but a common representation through constant elasticity substitution (CES) is as follows:

(15)

Here,

*0<a<1, 0<b<b, ψ<1* are parameters;

*F* – the production function;

*K* – the capital stock;

*L* – the labor unit;

*A* – the elasticity of capital

In comparative statics the Utility function, U, dynamically described in the equation (25) is also a CES function. The level curves of this function in the static and scalar field is known as the Indifference curves.

A dynamic version of the empirically supported Cobb–Douglas production function, incorporating technology and two factors (capital and labor), can be written as:

(16)

Here,

*F* – the production function;

*K* – the capital stock;

*L* – the labor unit;

*A –* technology or total factor productivity

*a* – the elasticity of capital,

*et* – the exponential time-dependent factor of technology.

The physical characteristics of production factors imply diminishing marginal returns relative to one another. For instance, if physical capital is held constant, an increase in labor reduces capital per worker and therefore lowers labor productivity. Similarly, if labor is constant, the additional output gained from increases in physical capital diminishes over time. This principle, known as diminishing marginal returns, can be expressed mathematically as:

(17)

The static relationship between inputs can be illustrated in a scalar field using isoquants, which represent combinations of labor and capital that yield the same level of output.

Since physical capital is considered the foundation of economic growth theories, the conditions for its per-capita accumulation play a central role. The dynamics of capital accumulation—expressed in both discrete and continuous time—is known as the law of motion for capital. In the methodology of economic dynamics and growth theory, this law is represented by the following differential equation:

(18)

Here,

– the time derivative of per capita capital;

s – saving rate;

f(k) – per capita production;

k – per capita capital stock;

n – population growth rate;

– depreciation rate.

According to this law of motion, the capital stock per capita increases through investment, i.e. the term s·f(k), but decreases due to the combined effects of population growth and depreciation, i.e. the term . This interaction explains why an economy’s growth rate eventually declines and why countries with lower initial capital stocks tend to converge toward more developed economies.

The speed of convergence*, β*, of these countries is computed as follows:

(19)



**FIGURE 3.** Isoquants: K- capital, L - labor

The identity (19) is similar to the physic concept, in which two objects move along the curve as the projectile motion, not necessarily at the same speed. The later one tends to catch up the former because the acceleration of both objects is negative as shown in the equation (17).

The level of capital at which accumulation ceases—that is, where 0, — is called the “Steady state”. In Figure 4, this steady-state capital stock is shown as k\*. From both sides, starting from initial values, k(0) and k'(0), the trajectories converge asymptotically toward k\*. For this reason, the steady-state equilibrium k\* is globally asymptotically stable.

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**FIGURE 4.** The law of motion k(t)-k(t+1) field, where k\* is steady state level [1]

The depreciation of physical capital (including reductions in environmental pollution) can, in some cases, be modeled analogously to the decay of radioactive materials. Depending on the decay rate of existing material capital, its half-life or terminal value can be calculated using the formulation presented in equation (2).

Next, consider equations (9) and (10). One of the central issues in macroeconomics is maintaining the balance between the money supply and real output. Disruptions to this balance can lead to phenomena such as inflation. In static form, general equilibrium is expressed as:

(20)

Here,

M – money supply;

V – velocity of money circulation velocity

Y – real output, (GDP);

P – price level.

In dynamic settings, this equilibrium may temporarily shift before a new equilibrium is established. Taking the total differential of the logarithmic form gives:

Since the velocity of money is treated as constant, i.e. V = const., its percentage change is zero. As a result, we obtain:

(21)

here,

m – percentage change in the money supply;

g – economic growth rate;

p – inflation rate.

Using the analogy with the gravitational force equation (11), we can apply a similar logic to international trade. Trade flows between two countries depend positively on the size of their economies and negatively on the distance between them. This relationship is expressed through the gravity model:

(22)

Here,

*Tij* – trade between countries *i* and *j*;

*A* – constant;

*Y* – *i* and *j* countries GDP

*Dij* – distance between countries *i* and *j.*

*a,b* and *c* – adjusting parametres.

Asset prices in financial markets often follow stochastic processes. Depending on economic conditions—boom, stagnation, or recession—prices may rise, remain constant, or fall. These changes occur according to a probability distribution. Recalling the Wiener process used to describe the displacement of a particle in equations (12) and (13), the evolution of a stock price can be written as:

(23)

here,

μ – mean rate of return, that is a random variable;

σ – standard deviation (volatility).

More generally, a random walk describes the cumulative movement of stock prices as the sum of independent and identically distributed (i.i.d.) random variables. This is also equivalent to the Weiner process of a moving particle considered in the equations (12) and (13).

(24)

where,

w(t) – value of the random walk at step (n);

ξ(n) – sequence of n independent random variables.

In many practical applications, it is necessary to compute an interest rate weighted by risk-neutral probabilities. A two-scenario version of such a rate is illustrated in Figure 5, where the risk-neutral probability of the upper interest rate, u, is p\* and the risk-neutral probability of the lower interest rate, d, is 1-p\*.

In economics, as in physics, many modern theories are formulated not in absolute terms but in relative terms. This approach became widespread after David Ricardo’s theory of comparative advantage. One of the most general frameworks based on relative relationships is Walras’s General Equilibrium Theory. Also known as Walras’s Law, it states that aggregate demand and supply balance in the economy as a whole, even if equilibrium has not been reached in every individual market.

In general, the production function introduced in equations (15) through (17) represents the supply side of the economy, while the utility function represents the preferences and willingness of the demand side. Compared with the production function, the utility function is much more abstract. In comparative statics, however, its general shape is assumed to be similar to that of the production function and, by analogy to physics, comparable to pressure–volume (P–V) relationships. Instead of isotherms, preferences and trade-offs are represented by level sets of utility—known as indifference curves—in a scalar field.

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**FIGURE 5.** Barycentric interpretation of risk-neutral probability [6]

In dynamic settings, utility is adjusted by a discount factor known as the social discount rate. Determining the appropriate value of this rate remains a central problem in welfare economics. The continuous-time utility maximization problem can be formulated as:

Subject to

(25)

Here,

*U* – intertemporal (lifetime) utility;

*u* – individual utility function at time t;

*c(t)* – individual consumption at time t;

– parameter of time preference;

– asset growth per time;

w – wage;

*at* – individual asset holdings at time t.

To solve this system of differential equations, the Hamiltonian (optimal control) method is typically employed. Even if temporary disequilibria or surpluses arise, variables such as interest rates and prices create incentives—analogous to pressure gradients in physics—that push the system back toward equilibrium.

In the field of economic dynamics, many economic problems are analyzed using physics- and mathematics-based modeling techniques. These include discrete and continuous dynamical systems, difference and differential equations, optimal control theory, and chaos theory. Such methods are applied in areas including:

* Demand and supply models;
* Dynamic theory of oligopoly;
* Closed economy dynamics;
* The dynamics of inflation and unemployment;
* Open economy dynamics: sticky price models;
* Flexible price models;
* Population models;
* Harvesting and dynamics of fisheries and others [11].

To advance the field of econo-physics, it is necessary to consider more sophisticated principles from physics and use richer mathematical tools. While many applied economic problems have traditionally relied on exponential and real-valued functions, a broader range of results can be achieved by incorporating trigonometric and vector-valued functions, linear and nonlinear transformations, Fourier and complex analysis, topology, and measure theory—methods widely used in physics and its subfields.

**CONCLUSION**

In this article, we examined several examples of how economic laws can be formulated and interpreted through the lens of physical laws and mathematical models. In particular, we highlighted how physical concepts such as work and energy, ideal gas relations, the law of gravitation, and Wiener processes have meaningful economic analogues. These physical ideas map respectively onto theories of production, capital accumulation, decision-making under constraints, financial market behavior, and comparative advantage. Such analogies not only clarify the structure of economic relationships but also provide powerful tools for modeling dynamic and uncertain environments.

Looking forward, the continued development of econophysics as an interdisciplinary field presents both challenges and opportunities. Advancing this discipline requires the deeper incorporation of modern mathematical frameworks—such as measure theory, probability theory, stochastic calculus, quantification techniques, and nonlinear dynamical models—into economic analysis. Likewise, drawing on advanced physical principles, including thermodynamic optimization, network theory, and statistical mechanics, can lead to richer insights into market interactions, distributional patterns, and large-scale economic fluctuations.

Ultimately, the systematic integration of physical reasoning and mathematical rigor into economic theory offers a promising direction for future research. By expanding the analytical toolbox and strengthening interdisciplinary connections, econophysics may contribute to more realistic, predictive, and comprehensive models of economic systems.

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