Vertex Odd Power Mean Labeling of Disconnected Graphs

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**Abstract:** Graph labeling plays a vital role in graph theory. It assigns labels to the vertices, edges or both with certain conditions. If the graph G (V, E) with p vertices and q edges is feasible to label the vertices x ε V with different labelings f(x) from f :V(G) →{1, 3, …, 2q + 1}in such a way that when each edge e = uv is labeled with f\*(e = uv) = or f\*(e = uv) = and the edge labelings are distinct. This labeling is called vertex odd power mean labeling*.*In this paper it is proved that the disconnected graphs such as cycle and path Cr Ս Pr ; comb and path (Pr ʘ K1) Ս Pr ;comb and star (Pr ʘ K1)Ս K1,r; comb and cycle (Pr ʘ K1) Ս Cr satisfy the vertex odd power mean labeling.

# Introduction

In the mid-’60s, the concept of graph labeling was first introduced. In the last 60 years, over 350 types of graph labeling have been studied and almost 3600 articles have been published. Graph labeling is an assignment of integers to vertices or edges, or both, under certain conditions motivated by practical problems [1].

A *graceful labeling* of a finite undirected graph G = (V, E), is a one – to – one mapping f from the set V of vertices of G to the set {0, 1, 2, ..., m}, where m = |E|, such that the induced edge labels are all distinct. Given f, an “induced edge labeling” is one-to-one mapping f\* from the set of edges of G to the set {1, 2, ..., m} given by f\*(uv) = |f(u) − f(v)| [2].

A bijective function f: V (G) ∪ E(G) → {1, 2, ..., p + q} such that f(uv) = |f(u) − f(v)| for every edge uv ∊ E(G) is said to be a *super graceful labeling* [1]*..*

A *harmonious labeling* on a graph G is an injection from the vertices of G to the group of integers modulo k, where k. is the number of edges of G, that induces a bijection between the edges of G and the numbers modulo k by taking the edge [3].

A simple graph G is called *felicitous graph* if there exists a one – to – one function f: V(G) →{0, 1, 2, ..,, q} such that the set of induced edge labels f\*(uv) = (f(u) + f(v)) (mod q) are all distinct [4].

A graph G with p vertices and q edges is called a *mean graph* if there is an injective function f from the vertices of G to {0,1,2,...,q} such that when each edge uv is labeled with if f(u) + f(v) is even, and if f(u) + f(v) is odd, then the resulting edge labels are distinct [5].

If the graph G (V, E) with p vertices and q edges is feasible to label the vertices x ε V with different labelings f(x) from f :V(G) →{1, 2, …, q + 1}in such a way that when each edge e = uv is labeled with

f\*(e = uv) = or f\*(e = uv) = 

and the edge labelings are distinct. This labeling is called *power mean labeling* [6].

A function is said to be *Power-3 Heronian odd mean labeling* of a graph G with q edges, if f is a bijective function from the vertices of G to the set {1, 3, 5, …, 2p - 1}such that when each edges uv is assigned the label. The resulting edge labels are distinct numbers [8].



A function f is called a *F-root square mean labeling* of a graph G(V, E) with p vertices and q edges if f : V(G) → {1,2,3,...,q +1} is injective and the induced function f\* defined as

f\*(uv) = 

for all uv ∈ E(G), is bijective [9].

A (p, q) graph G is said to be a power exponential mean graph if there exist a one to one correspondence f : V → {1, 2, 3, . . . , p} such that induced function f∗ : E(G) → N given by

f\*(uv) = or f\*(uv) = 

for every uv ∈ E(G) are all distinct [10].

# Results

## Vertex Odd Power Mean Labeling

The graph G(V, E) with p vertices and q edges, if it is sufficient to label the vertices x ε V with different labelings f(x) from odd numbers of 1, 3,…., 2q + 1 in such a way that when each edge e = uv is labeled with

f\*(e = uv) = 

or

f\*(e = uv) = 

and the edge labelings are distinct. This labeling is called *vertex odd power mean labeling [7]*. This concept of vertex odd power mean labeling is defined by C. Vimala and B. Kavitha in 2020 [7]. They proved that cycle, star, connected graph Cn + 2P2 are vertex odd power mean graphs. In this paper it is proved for the disconnected graphs such as such as cycle and path Cr Ս Pr ; comb and path (Pr ʘ K1) Ս Pr ;comb and star (Pr ʘ K1)Ս K1,r; comb and cycle (Pr ʘ K1) Ս Cr.

## Disconnected Graphs

A graph is said to be *disconnected* if it is not connected, i.e., if there exist atleast one pair of vertices such that these is no path connecting them [12].

**Theorem 1:** *The disconnected graph of cycle and path admits vertex odd power mean labeling*.

**Proof:** The disconnected graph of cycle and path has 2r vertices and 2r edges.

Define the vertex labeling f: V(Cr Ս Pr) →{1, 3, …, 2q +1} as

f(vi) = 2i – 1 for 1 ≤ i ≤ r

f(uj) = f(vr) + 2j for 1 ≤ j ≤ r

**v1**

**v2 e1 er vr**

**e2 er-1  u1  E1 u2 ur-1  Er-1 ur**

**v3  vr-1**

Fig. 1: Disconnected graph of cycle and path

and the induced edge labeling as

f\*(ei) = 2i – 1 for 1 ≤ i ≤ r - 1

f\*(er) = 2

f\*(Ej) = f(vr) + 2(j + 1) for 1 ≤ j ≤ r.

Thus the vertex and edge labels are distinct and the disconnected graph of cycle and path fulfills the vertex odd power mean labeling conditions.

Hence, the graph is a vertex odd power mean graph.

**Theorem 2:** *The disconnected graph of comb and path admits vertex odd power mean labeling*.

**Proof:** The disconnected graph of comb and path has 3r vertices and 3r edges.

Define the vertex labeling f: V(Pr ʘ K1 Ս Pr) →{1, 3, …, 2q +1 } as

**v1ʹ e1ʹ v2ʹ vr-1ʹ er-1ʹ vrʹ**

**e1  e2  er-1  er u1 E1 u2 ur-1 Er-1 ur**

**v1 v2 vr-1  vr**

Fig. 2: Disconnected graph of comb and path

f(vi) = 4i – 3 for 1 ≤ i ≤ r

f(viʹ) = 4i – 1 for 1 ≤ i ≤ r

f(uj) = f(vrʹ) + 2j for 1 ≤ j ≤ r

and the induced edge labeling as

f\*(ei) = 4i – 3 for 1 ≤ i ≤ r

f\*(eiʹ) = 4i for 1 ≤ i ≤ r

f\*(E1) = f\*(er) + 4

f\*(Ejʹ) = f\*(E1) + 2(j – 1) for 2 ≤ j ≤ r -1.

Thus the vertex and edge labels are distinct and the disconnected graph of comb and path satisfies the vertex odd power mean labeling conditions.

Thus, the graph is a vertex odd power mean graph.

**Theorem 3:** *The disconnected graph of comb and star admits vertex odd power mean labeling*.

**Proof:** The disconnected graph of comb and star has 3r + 1 vertices and 3r- 1 edges.

Define the vertex labeling f: V(Pr ʘ K1 Ս K1,r) →{1, 3, …, 2q +1} as

f(vi) = 4i – 3 for 1 ≤ i ≤ r

f(viʹ) = 4i – 1 for 1 ≤ i ≤ r

f(uj) = f(vrʹ) + 2j for 1 ≤ j ≤ r

f(u1ʹ) = f(ur) + 2

**v1ʹ e1ʹ v2ʹ vr-1ʹ er-1ʹ vrʹ u1ʹ**

**e1  e2  er-1  er E1 Er**

**v1 v2 vr-1  vr u1 u2 ur-1 ur**

Fig. 3: Disconnected graph of comb and star

and the induced edge labeling as

f\*(ei) = 4i – 3 for 1 ≤ i ≤ r

f\*(eiʹ) = 4i for 1 ≤ i ≤ r

f\*(Ej) = 5r + (j – 1) for 1 ≤ i ≤ r.

Thus the vertex and edge labels are distinct and the disconnected graph of comb and star fulfills the vertex odd power mean labeling conditions.

Hence, the graph is a vertex odd power mean graph.

**Theorem 4:** *The disconnected graph of comb and cycle admits vertex odd power mean labeling*.

**Proof:** The disconnected graph of comb and cycle has 3r vertices and 3r – 1 edges.

Define the vertex labeling f: V(Pr ʘ K1 Ս Cr) →{1, 3, …, 2q +1} as

f(vi) = 4i – 3 for 1 ≤ i ≤ r

f(viʹ) = 4i – 1 for 1 ≤ i ≤ r

f(uj) = f(vrʹ) + 2j for 1 ≤ j ≤ r.

**v1ʹ v2ʹ vn-1ʹ vrʹ u1**

**u2 ur**

**v1 v2 vr-1  vr u3 ur-1**

Fig. 4: Disconnected graph of comb and cycle

Thus the vertex and edge labels are distinct and the disconnected graph of comb and cycle satisfies the vertex odd power mean labeling conditions.

So, the graph is a vertex odd power mean graph.

**Theorem 5:** *The disconnected path graph admits vertex odd power mean labeling*.

**Proof:** The disconnected path graph has *sr + r* vertices and *sr* edges.

Define the vertex labeling f: V(rPs) →{1, 3, …, 2q +(2r - 1)} as

f(vi) = 2i – 1 for 1 ≤ i ≤ *sr + r*

**v1 v2 vr-1 vr vsr+r-1 vsr+r**

Fig. 5: Disconnected path graph

Thus the vertex and edge labels are distinct and the disconnected path graph fulfills the vertex odd power mean labeling conditions.

Hence, the disconnected path graph is a vertex odd power mean graph.

**Theorem 6:** *The disconnected paths and cycle* rP1 Ս Cs *admits vertex odd power mean labeling*.

**Proof:** The disconnected paths and cycle has *s + 2r* vertices and *s + r* edges.

Define the vertex labeling f: V(rP1 Ս Cs) →{1, 3, …, 2q +(2r - 1)} as

f(vi) = 2i – 1 for 1 ≤ i ≤ *s + 2r*

**vs+2r-1**

**v1 v2 v3 v4 vs+2r**

Fig. 6: Disconnected paths and cycle

Thus the vertex and edge labels are distinct and the disconnected paths and cycle satisfies the vertex odd power mean labeling conditions.

Then, the graph is a vertex odd power mean graph.

# Conclusion

In this paper it is verified that the disconnected graphs such as path, cycle and star satisfied vertex odd power mean labeling.

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