Estimating Lifetime of Pressure Vessels Subjected to Mechanochemical Corrosion: Effect of Hydrostatic Pressure

Yulia Pronina [1, 2]

[1]*St. Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034 Russia.*
[2]*Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences, 61A, Bolshoi pr-t V.O., St. Petersburg, 199178 Russia.*

Corresponding author: y.pronina@spbu.ru

Abstract. Simple unified analytical solution is presented for the lifetime of cylindrical, spherical, and toroidal vessels subjected to one- or double-sided mechanochemical corrosion under internal and external pressure. This solution provides sufficiently accurate results for perfect cylindrical and spherical vessels, while for toroidal vessels it gives approximate estimates based on the weakest cross-section principle. The obtained solution reflects the difference in the hoop stresses at the inner and outer surfaces of the vessels and the effect of hydrostatic pressure (i.e. minimum of the internal and external pressures) on the vessel lifetime. It is shown that high hydrostatic pressure may noticeably reduce the durability of the vessels or, on the contrary, slightly increase it. The need for an integrated approach – using different methods – to solving such problems is emphasized.

INTRODUCTION

Most structures are routinely exposed to both chemically active environment and mechanical loads. This often causes general or localized corrosion of materials, facilitated by mechanical stresses. Some other stress-assisted processes of material degradation were discussed, e.g., in [1–6]. The present paper concerns mechanochemical corrosion that is general anodic dissolution of metals, accelerated by elastic stresses [7], assuming electrochemical homogeneity of the entire surface.

Corrosion Kinetics Models

One of the first models of mechanochemical corrosion kinetics is the phenomenological model of Dolinskii [8]:

$$v(s) = a + m\sigma(s) \qquad (1)$$

where v is the corrosion rate at point s of the corroding surface, a (the corrosion rate of unstressed metal) and m are empirical constants, σ is an equivalent stress which is usually accepted as either the first principal stress (or that of the maximum absolute value) or the von Mises stress.

Well-known Gutman's model [7]

$$v = v_0 \exp\left(\frac{V\sigma}{R_g T}\right) \qquad (2)$$

was obtained from theoretical considerations and confirmed by direct experimental measurements; here, v_0 is the corrosion rate of unstressed metal, V is the molar volume of the material, R_g is the universal gas constant, and T is the absolute temperature. Later, this model was combined with the Arrhenius type law (referring to [9]) to more accurately reflect the temperature effect [10]:

$$v = v_0 \exp\left(\tilde{E}_{c0} \left(1 - \frac{\tilde{E}_c}{\tilde{T}} \right) + \frac{V\sigma}{R_g T} \right) \tag{3}$$

where $\tilde{E}_{c0} = E_{c0} / R_g T_0$, $\tilde{E}_c = E_c / E_{c0}$, $\tilde{T} = T / T_0$; E_c and E_{c0} are the effective activation energy of the corrosion process and its reference value, respectively, T_c and T_0 are the absolute temperature and its reference value; v_0 is the corrosion rate of non-stressed material at $E_c = E_{c0}$ and $T = T_0$.

The Dolinskii model was also extended to reflect the temperature effect when temperature T exceeds a certain, experimentally determined, threshold T^{th} [11]:

$$v = \begin{cases} a + m\sigma, & T \le T^{th} \\ (a + m\sigma)\exp\left(\beta\left[T - T^{th}\right]\right), & T > T^{th} \end{cases} \tag{4}$$

Here, β is an empirical constant.

To reflect the effect of the corrosion inhibition with time t, an exponential factor (e.g., with an empirical coefficient b) is often introduced [12]; in combination with model (1) this yields:

$$v(s) = \left(a + m\sigma(s)\right)\exp(-bt) \tag{5}$$

Pavlov [12] also established the existence of the threshold stress σ^{th} such that $v(s) = v_0$ at $|\sigma| \le |\sigma^{th}|$ and (1) holds at $|\sigma| > |\sigma^{th}|$, where $a = v_0 - m\sigma^{th}$, v_0 is the corrosion rate of unstressed metal; he emphasized that, in general, the constants m and σ^{th} are different for tensile and compressive stresses and $\operatorname{sign} m = \operatorname{sign} \sigma$ [12]. However, according to Gutman, the concept of "a threshold stress" for the general corrosion may be raised not for scientific reasons but because of the limited sensitivity of measuring equipment. Most authors accept it being equal to zero. This does not contradict with the existing solutions involving arbitrary σ^{th} (e.g., [13]): this constant can be set equal to zero or treated as an adjustable parameter in approximations of non-linear dependences of corrosion rate on the stress (e.g., introduced in [7,14], or other ones describing experimental data) by a pies-wise function.

Several models were developed incorporating the effect of anti-corrosion protective coatings. Among them is the model [15]

$$v = \begin{cases} 0, & t \le \tau_c \\ d\exp\left(-[t - \tau_c]/\Theta\right)/\Theta, & t > \tau_c \end{cases} \tag{6}$$

where d, τ_c, and Θ are the long-term thickness of the corrosion wastage, coating life, and the transition time, correspondingly. Comparing (6) with Pavlov's factor $\exp(-bt)$ yields $b = 1/\Theta$. In this and other models (e.g. [16]), it is assumed that corrosion does not occur for a certain period τ_c, whereas fuzzy model [17] of corrosive wear, accounting for the gradual decrease in coating protective properties, assumes that the wear occurs even under coatings. Some issues related to the stability of coatings are considered in [18,19]. Authors of [16] generalized model (6) :

$$v = \begin{cases} 0, & t \le \tau_c \\ d\dfrac{\lambda}{\Theta}\left(\dfrac{t - \tau_c}{\Theta}\right)^{\lambda - 1} \exp\left[-\left(\dfrac{t - \tau_c}{\Theta}\right)^{\lambda}\right], & t > \tau_c \end{cases} \tag{7}$$

which may reflect non-monotonic behavior of corrosion rate with time (although it was originally intended to describe somewhat different effects).

All the considered factors may be combined in the following generalized model:

$$v = \begin{cases} 0, & t \le \tau_c \\ f(\sigma, T)g(t), & t > \tau_c \end{cases} \tag{8}$$

where $g(t)$ is defined by the second line of the right-hand side of eq. (7) if $\Theta < \infty$, or $g(t) = \text{const}$ (a or v_0) if corrosion does not decrease with time ($\Theta = \infty$); in a particular case of monotonous decay of corrosion rate ($\lambda = 1$), one can use Dolinskii and Pavlov's designation: $g(t) = a\exp\left(-b[t - \tau_c]\right)$. Function $f(\sigma, T)$ is defined by the right-hand side of one of eqs. (1), (4) normalized to a or eqs. (2), (3) normalized to v_0 [20]. The constant β entering (4) may also be a function of the temperature (since formation of the protective layer of corrosion products or precipitates may be accelerated by higher temperature).

Note that the linear dependence (1) of corrosion rate on stress may be considered as a linear approximation of dependence (2), where $a = v_0$ and m is determined from the expansion of the right-hand side of (2) in a Taylor series [21].

There also exist several models taking into account plastic deformations, but since we focus on the elastic behavior of pressure vessels, we do not list them here.

SOLUTIONS FOR THE LIFETIME OF PRESSURE VESSELS SUBJECTED TO MECHANOCHEMICAL CORROSION

Usually, mechanochemical corrosion of structures is simulated using various numerical procedures. However, for some special shapes, closed-form solutions may be found. Due to structural instability of the problems of general stress-assisted corrosion, they are often solved assuming a specific shape of a corroding surface or under the assumption of the constancy of the mid-surface of elements [10,11,22–28]. For local corrosion, the shape of pittings is also often predefined [29–32] (note that the models discussed in the present paper are only applicable to general corrosion). Modeling corrosion without such assumptions significantly complicates the problem [33,34].

Analytical Solutions

First closed-form solutions for thin-walled cylindrical and spherical vessels under pressure, utilizing models (1) or (2) were reported (in Russian) by Gutman, Karpunin and Kornishin, Ovchinnikov and Petrov with co-authors [22,23,35]. Elegant solutions for thick-walled cylindrical and spherical vessels using model (2) were obtained in [35], with σ being the mean hydrostatic stress. Paper [10] presents the solution for a thin-walled sphere utilizing model (3), thermoelastic stresses not being involved in the analysis. The above-mentioned solutions do not reflect the change in stresses across the shell thickness and therefore its effect – albeit weak – on the corrosion rates on the internal and external surfaces. The approach to accounting for these changes, as well as some other factors, in analytical solutions was proposed by Pronina [36,37]; we mention some of these solutions and their features:

- Accurate solutions [36,38] for thick-walled cylindrical and spherical vessels with additional account for the elasto-plastic transition and corrosion inhibition, utilizing model (5) with the von Mises stress for the equivalent one;
- Unified accurate solutions [39] for thick-walled cylindrical and spherical vessels with additional account for various temperature effects (including thermoelastic stresses), presence of anti-corrosion coatings, and possible corrosion inhibition, utilizing combination of models (4) and (6) with the maximum principal stress for the equivalent one;
- Unified simplified solutions [20] for thin-walled cylindrical and spherical vessels with additional account for various temperature effects (including thermoelastic stresses and thermal softening of the material), anti-corrosion coatings, non-monotonic corrosion inhibition, and effect of hydrostatic component of the internal and external pressures, utilizing general model (8) (various combinations involved) with the maximum principal stress for the equivalent one.

These results include previously obtained solutions as simple special cases.

There also exist a couple of approximate closed-form estimates for more complex shapes of pressure vessels. Closed-form solution for a short thin-walled cylindrical vessel with elliptical cross-section and variable wall thickness was obtained by Gutman with co-authors [24], utilizing model (2). Ilyin and Pronina [34] found approximate analytical solutions for toroidal shells under pressure, using model (1).

Since the present article focuses on the effect of hydrostatic pressure (minimum of the internal and external pressures) on the lifetime of pressure vessel, the equivalent stress entering into eq. (1) should be chosen as the maximum, in absolute value, principal stress (using the von Mises stress in eq. (1) does not allow one to account for the effect of hydrostatic pressure [40]). For comparative analysis we consider simplest solutions that do not take into account effects of inhibition, protective coatings and various temperature effects.

Formulation of the problem

We consider a thin-walled pressure vessel of cylindrical, spherical or toroidal shape under internal p_i and external p_o pressure. Let the instantaneous thickness of the vessel be denoted by $h = h(t)$ and the mean radius of the cross-section by r_c, which is assumed to be constant (the effects of this assumption was studied in [34,39]). Mean toroidal radius (radius of curvature of the axis of the torus pipe) is denoted by R. Edge effects are not taken into account. Thickness h decreases with time t due to double-sided mechanochemical corrosion with the rates described by eq. (1), where a_i, a_o m_i, and m_o are the corrosion kinetics constants for the inner (marked with subscript *i*) and outer (marked with subscript *o*) surfaces of the vessels, correspondingly. Important that for cylindrical and spherical vessels, the thickness decreases uniformly over the entire mid-surface of the shell, whereas the decrease in the thickness of the toroidal shell depends on the circumferential coordinate of its cross-section. To obtain closed-form estimates of its lifetime (approximate solution), the principle of "the weakest cross-section" is used, considering cases when the maximum, in absolute value, principal stress is at the torus intrados (points closest to the axis of the torus revolution); the range of physical parameters satisfying this condition is determined in [34]. The maximum, in absolute value, principal stress is used as the equivalent stress in eq. (1), considering only the cases when it is the hoop stress.

Unified solution of the problem

Detailed solutions of these problems and discussion of the range of their applicability are presented in [20] for cylindrical and spherical vessels and in [34] for toroidal ones. Here we present the combination of these solutions for vessels of the three types in the unified form:

$$t = t_0 + \frac{h_0 - h}{A} + \frac{M}{A^2} \ln \frac{Ah + M}{Ah_0 + M} \tag{9}$$

where

$$A = a_i + a_o + (m_i + m_o)\Delta\sigma^i - m_o \Delta p, \quad M = Y\left(m_i + m_o\right)\Delta p r_c, \quad \Delta p = p_i - p_o; \tag{10}$$

For a tube

$$\Delta\sigma^i = -p_o, \quad Y = 1; \tag{11}$$

For a sphere

$$\Delta\sigma^i = -\frac{p_i + 3p_o}{4}, \quad Y = \frac{1}{2}; \tag{12}$$

For a torus ("thin-shell" solution)

$$\Delta\sigma^i = -p_o, \quad Y = \frac{2R - r_c}{2(R - r_c)}; \tag{13}$$

For a torus ("thick-shell" solution)

$$\Delta\sigma^i = -p_o - \frac{\Delta p\, r_c\, \nu}{4R(\nu - 1)} - \frac{\Delta p (r_c)^2 (\nu^2 + \nu - 2)}{8R^2(\nu^2 - 1)}, \quad Y = 1 + \frac{r_c}{2R} + \frac{(r_c)^2 (7\nu - 10)}{8R^2(\nu - 1)}; \tag{14}$$

ν is the Poisson ratio.

In cases of one-sided corrosion, the corresponding corrosion kinetics constants should be equated to zero (a_i and m_i for external corrosion; a_o and m_o for internal one). For the stress-independent corrosion ($m_i = m_o = 0$), the last term in the right-hand-side of eq. (9) disappears and the solutions are the same for all the vessels with the same cross-section geometry.

Equation (9) provides one-to-one correspondence between the thickness h and time t. Stresses at any time t can then be calculated by the refined formulas of the shell theory presented in [20,34]; for example, maximum stresses at the internal surface can be written in the unified form as follows

$$\sigma = \frac{\Delta p\, r_c}{h} Y + \Delta\sigma^i \tag{15}$$

where Y and $\Delta\sigma^i$ are defined by the above equations for cylinder (11), sphere (12), and torus (13) or (14).

The lifetime of the vessel may be estimated by substituting for h in eq. (9) – the critical value of the thickness equal to $\max\{h_{\min}, h^*\}$, where $h_{\min}$ is the minimum allowable thickness and h^* is the thickness corresponding to the maximum allowable stress σ^* in the vessel. The latter is given by the equation

$$h^* = Y \frac{\Delta p r_c}{\sigma^* - \Delta\sigma^i} \tag{16}$$

which is obtained by equating (15) to σ^*. Note that there is a misprint in similar eq. (67) in [20]: the factor Y is lost.

Important that these solutions reflect the effects of the hydrostatic pressure $p = \min\{p_i, p_o\}$ and the difference in stresses on the inner and outer surfaces.

Results based on these solutions for cylindrical and spherical vessels are practically coincide with the accurate solutions [39], while solutions for toroidal vessels are approximate, reflecting its behavior only at the intrados and neglecting gradual increase in stress concentration and possible local bending (relative errors for both solutions are discussed in [34]; note, however, that computational results for mechanochemical wear in [34] provide noticeably underestimated lifetime, and the difference between the numerical and analytical predictions is mostly caused by inaccuracy of the numerical solution; our refined computational scheme gives the results more close to the analytical one).

Notes on Numerical Solutions

Due to structural instability of the problems of mechanochemical corrosion, computational procedures often do not converge, unless additional simplifying hypotheses (for example, the ones of thin shell theory) are accepted. Modeling the behavior of compound pressure vessels including toroidal part, based on the numerical solution of the equations of thin-shell theory, is presented, e.g., in [28]. In general cases, since corrosion kinetics models are very sensitive to stress fluctuations caused by both physical reasons and inaccuracies of numerical methods (e.g., FEM), the challenging problem is separating the “true” (physically justified) mechanochemical effect from the “false” one (caused by inherent errors of a numerical method). Accumulation of errors for thousands of time steps may result in the false loss of stability of the shape of the corroding surface (local intensification of corrosion – see Fig. 1, or even formation of loops on the boundary). Some problems of FEM-based corrosion modeling using as an example toroidal vessels under pressure are discussed in [34]. Even relatively minor changes in computational procedures incorporating FEM may result in different predictions for the same problem (even when reproducibility of results for each scheme was observed on several meshs). Another major factor adding to unreliability of results is human factor (which manifested itself in the numerical results in [34]). This emphasizes the role of analytical solutions which provide quantitative estimates of parameters of interest, and highlights the need for an integrated approach – using different methods – to solving such problems. Some approaches to modeling stress-dependent dissolution without accepting simplifying assumptions were proposed in [33,41]. Note, however, that models of general corrosion are not applicable to simulation of microscale defects or roughness in metals. For this reason, the models considered in the present paper may be used until the noticeable localization of corrosion develops.

We add that in scientific literature, the term localization may be understood in different meanings:

- Chemical reaction occurs only on part of the surface (e.g. pitting corrosion [42]);
- Reaction covers the entire surface, but is more intense on some parts of it [43,44].
- Volumetric chemical reactions are localized at the front of the phase transformation zone [5].

FIGURE 1. Example of false loss of stability of the shape of an initially circular corroding surface.

RESULTS OF COMPUTATIONS

Figure 2 shows the dependences of the vessels thickness and the maximum, in absolute values, stresses on time for spherical, cylindrical, and toroidal vessels at constant $|\Delta p|$ and various $p = \min\{p_i, p_o\}$. The following parameters were used for calculations: $a_i = a_o = 0.1[l_c / t_c]$; $m_i = m_o = a / 200[l_c / (t_c p_c)]$; $r_c = 80[l_c]$, $h_0 = r_c / 20$, and $\sigma^* = 300[p_c]$, where l_c, t_c, p_c are conventional units of length, time, and stress.

In contrast to solutions based on conventional formulas of thin shell theory, the unified analytical solution presented in the previous section reflects the effect of hydrostatic component of internal and external pressures $p = \min\{p_i, p_o\}$. Pressure $p = \min\{p_i, p_o\}$ shortens the lifetime of the vessels at the negative Δp and prolongates at the positive one, however, at small p this effect is weak. As one can see from the comparison of dependences $h(t)$ and $\sigma(t)$ in Fig. 2, under the considered conditions, the ultimate stress σ^* is reached in the vessels when their thickness is relatively small, therefore, their lifetime should be defined by the criterion of minimal allowable thickness, i.e. by points of intersection of the dependences $h(t)$ with the line $h = h_{\min}$ (in Fig. 2 $h_{\min} = h_0 / 2$).

Another important circumstance is that at $\Delta p < 0$, buckling of shells may occur. According to the classical theory of shell stability, under the considered conditions, spherical and toroidal vessels are stable at $h > h_0 / 2$, but at smaller $h \in [h^*, h_0 / 2]$, the toroidal shells will lose their stability. Straight cylindrical vessel is unstable even at $h > h_0 / 2$. Note that the critical minimum shell thickness corresponding to the loss of stability of shells of non-uniform thickness may be larger than that calculated for shells with uniform wall thickness. Thus, the lifetime of a shell should be determined as the minimum

$$\min\{t^*, t^h, t^{st}\}, \tag{17}$$

where t^*, t^h, and t^{st} are times corresponding to the maximum allowable stress $|\sigma^*|$, the minimum allowable thickness $h_{\min}$, and buckling of the shell, correspondingly. Estimates of the lifetime in accordance with any other criteria may be added into (17), including manufacturer's warranty period.

The presented solutions are sufficiently accurate for perfectly circular cylindrical and spherical vessels, but in case of even slight imperfections, lifetime of the vessels may be essentially shorter [45]. For toroidal vessels, lifetime determined by this solution is also overestimated, because of the non-uniformity of the corrosive wear and increasing stress concentration [34]. Despite the fact that the estimated lifetime of toroidal vessels of relatively large mean toroidal radii R are close to the lifetime of pipes of the same cross-section (which may be considered as tori of an infinite radius R), it is reasonable to evaluate it also using numerical methods (e.g. by FEM-based procedures) to assess the possibility of corrosion localization and bending of the vessel wall.

CONCLUSION

Simple unified analytical solution is presented for the lifetime of cylindrical, spherical, and toroidal vessels subjected to one- or double-sided mechanochemical corrosion under internal and external pressure. Since the stress state of a torus depends on the circumferential coordinate, this solution gives an approximate estimate of its lifetime based on the principle of the weakest cross-section, considering its behavior in the intrados and neglecting gradual stress concentration increase and possible local bending. For perfectly circular cylindrical and spherical vessels, this solution provides sufficiently accurate results. The obtained solution reflects the difference in the hoop stresses at the inner and outer surfaces of the vessels and the effect of hydrostatic pressure (i.e. minimum of the internal and external pressures) on the vessel lifetime. It is shown that high hydrostatic pressure may noticeably reduce the durability of the vessels, or, on the contrary, slightly increase it. The need for an integrated approach – using different methods – to solving such problems is emphasized.

ACKNOWLEDGMENTS

This work was supported by the Russian Science Foundation, grant No 25-11-00274, https://rscf.ru/project/25-11-00274/.

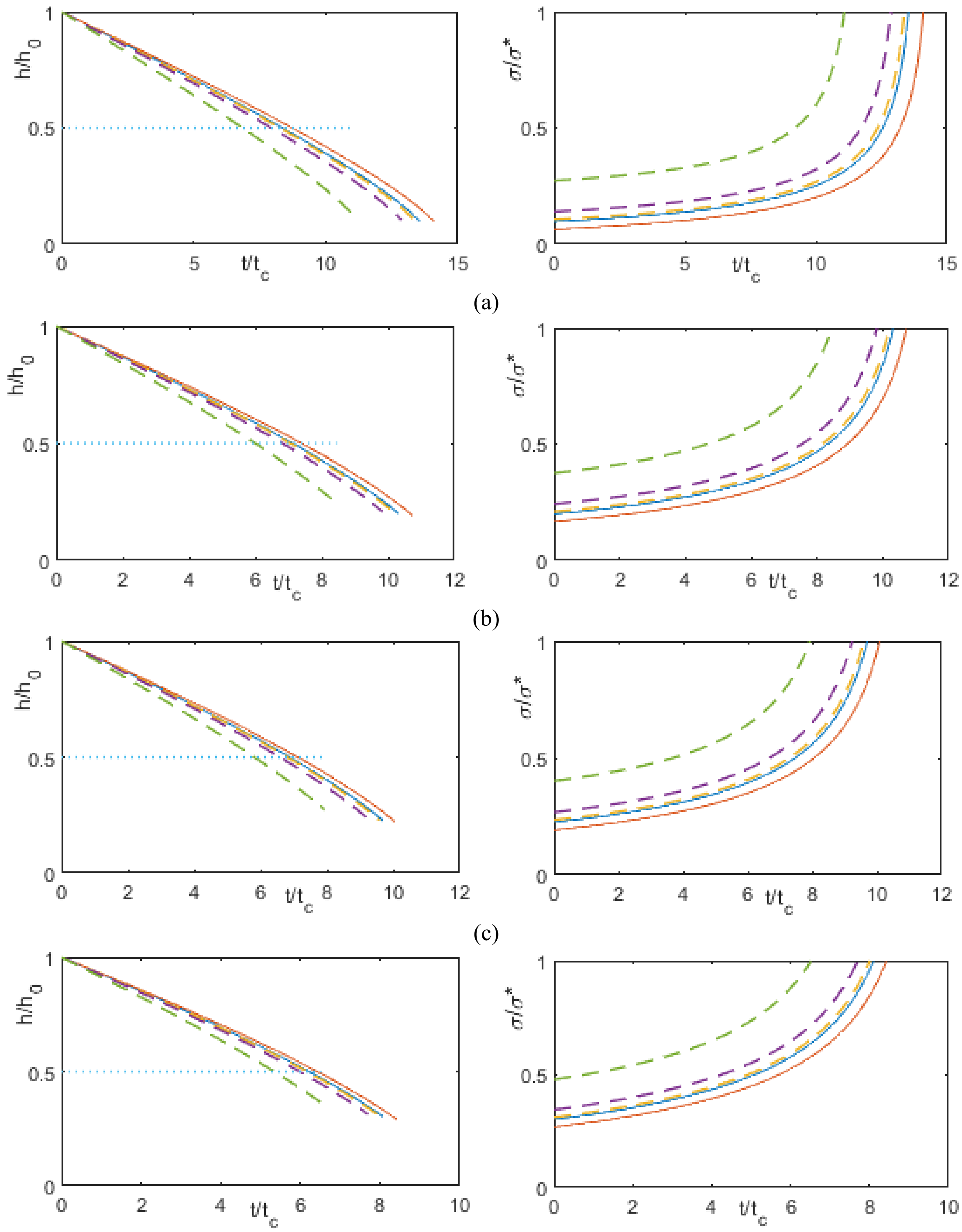

FIGURE 2. Dependences of the vessels thickness h normalized to h_0 and maximum, in absolute values, hoop stresses σ normalized to σ^* on time t for spherical (a), cylindrical (b) and toroidal vessels of $R = 5r_c$ (c) and $R = 2r_c$ (d).

Dashed lines: $\Delta p = -3[p_c]$; $p = p_i = 50,\ 10,\ 0[p_c]$ (for curves from left to right).

Solid lines: $\Delta p = 3[p_c]$; $p = p_o = 0,\ 10[p_c]$ (for curves from left to right).

REFERENCES

1. M. Poluektov, A. B. Freidin, and Ł. Figiel, Modelling stress-affected chemical reactions in non-linear viscoelastic solids with application to lithiation reaction in spherical Si particles, *Int. J. Eng. Sci.* **128**, 44–62 (2018). https://doi:10.1016/j.ijengsci.2018.03.007
2. G. Shuvalov and S. Kostyrko, On the role of interfacial elasticity in morphological instability of a hetero-epitaxial interface, *Contin. Mech. Thermodyn.* **33(**5), 2095–2107 (2021). doi:10.1007/s00161-021-01010-6.
3. I. Argatov, A fractional time-derivative model for severe wear: Hypothesis and implications, *Front. Mech. Eng.* **8**, 905026 (2022). https://doi.org/10.3389/fmech.2022.905026.
4. I. I. Argatov and Y. S. Chai, A theoretical justification of the slip index concept in fretting analysis, *Friction* **11(**7), 1265–1275 (2023). https://doi.org/10.1007/s40544-022-0662-1.
5. M. Poluektov and A. Freidin, Localisation of stress-affected chemical reactions in solids described by coupled mechanics-diffusion-reaction models, *Int. J. Eng. Sci.* **196**, 104006 (2024). https://doi.org/10.1016/j.ijengsci.2023.104006.
6. G. Shuvalov, S. Kostyrko, and H. Altenbach, Interplay of surface and bulk elasticity in morphological stability of ultra-thin film coatings, *Contin. Mech. Thermodyn.* **36(**3), 503–523 (2024). https://doi.org/10.1007/s00161-024-01279-3.
7. E. M. Gutman, *Mechanochemistry of Solid Surfaces* (World Scientific, Singapore, 1994).
8. V. M. Dolinskii, Calculations on loaded tubes exposed to corrosion, *Chem. Pet. Eng.* **3(**2), 96–97 (1967) http://dx.doi.org/10.1007/BF01150056.
9. A. Popova, E. Sokolova, S. Raicheva, and M. Christov, AC and DC study of the temperature effect on mild steel corrosion in acid media in the presence of benzimidazole derivatives, *Corros. Sci.* **45(**1), 33–58 (2003). https://doi.org/10.1016/S0010-938X(02)00072-0.
10. E. M. Gutman, R. M. Bergman, and S. P. Levitsky, Influence of internal uniform corrosion on stability loss of a thin-walled spherical shell subjected to external pressure, *Corros. Sci.* **111**, 212–215 (2016). https://doi.org/10.1016/j.corsci.2016.04.018.
11. G. A. Naumova and I. G. Ovchinnikov, *Strength Calculations of Compound Frameworks and Pipeline Structures with Taking into Account Corrosion Damages* (Saratov: Saratov State University, 2000).
12. P. A. Pavlov, B. A. Kadyrbekov, and V. A. Kolesnikov, *Strength of Steels in Corrosive Environments* (Nauka, Alma-Ata, 1987).
13. Y. Pronina and O. Sedova, Analytical solution for the lifetime of a spherical shell of arbitrary thickness under the pressure of corrosive environments: The effect of thermal and elastic stresses, *J. Appl. Mech.*. **88(**6), 061004 (2021). https://doi.org/10.1115/1.4050280.
14. A. I. Rusanov, Mechanochemistry of dissolution: Kinetic aspect, *Russ. J. Gen. Chem*. **77(**4), 491–502 (2007).
15. C. G. Soares, Y. Garbatov, A. Zayed, and G. Wang, Influence of environmental factors on corrosion of ship structures in marine atmosphere, *Corros. Sci.* **51(**9), 2014–2026 (2009). https://doi.org/10.1016/j.corsci.2009.05.028.
16. S. Qin and W. Cui, Effect of corrosion models on the time-dependent reliability of steel plated elements, *Mar. Struct.* **16(**1), 15–34 (2003). https://doi.org/10.1016/S0951-8339(02)00028-X.
17. M. M. Fridman, An integrated approach to the optimization of plates in plane stress state operated at high temperatures, *J. Mech. Eng.* **24**(3), 52–60 (2021). https://doi.org/10.15407/pmach2021.03.052.
18. M. A. Grekov and S. A. Kostyrko, A film coating on a rough surface of an elastic body, *J. Appl. Math. Mech.* **77(**1), 79–90 (2013). https://doi.org/10.1016/j.jappmathmech.2013.04.010.
19. M. A. Grekov and S. A. Kostyrko, A multilayer film coating with slightly curved boundary, *Int. J. Eng. Sci.* **89**, 61–74 (2015). https://doi.org/10.1016/j.ijengsci.2014.12.001.
20. I. Evstafeva and Y. Pronina, On the mechanochemical dissolution of shells and its temperature dependence: Discussion of different models, *Int. J. Eng. Sci*. **190**, 103889 (2023). https://doi.org/10.1016/j.ijengsci.2023.103889.
21. I. Elishakoff, G. Ghyselinck, and Y. Miglis, Durability of an elastic bar under tension with linear or nonlinear relationship between corrosion rate and stress, *J. Appl. Mech.* **79(**2), 021013 (2012). https://doi.org/10.1115/1.4005564.
22. I. G. Ovchinnikov and Yu. M. Pochtman, Calculation and rational design of structures subjected to corrosive wear (review), *Sov. Mater. Sci.* **27**(2), 105–116 (1992). https://doi.org/10.1007/BF00722977.
23. E. Gutman, J. Haddad, and R. Bergman, Stability of thin-walled high-pressure vessels subjected to uniform corrosion, *Thin-Walled Struct.* **38**(1), 43–52 (2000). https://doi.org/10.1016/S0263-8231(00)00024-0.

24. E. M. Gutman, J. Haddad, and R. Bergman, Stability of thin-walled high-pressure cylindrical pipes with non-circular cross-section and variable wall thickness subjected to non-homogeneous corrosion, *Thin-Walled Struct.* **43**(1), 23–32 (2005). https://doi.org/10.1016/j.tws.2004.08.002
25. M. M. Fridman and I. Elishakoff, Design of bars in tension or compression exposed to a corrosive environment, *Ocean Syst. Eng.* **5(**1), 21–30 (2015). https://doi.org/10.12989/OSE.2015.5.1.021
26. Y. Pronina and S. Khryashchev, Mechanochemical Growth of an Elliptical Hole under Normal Pressure, *Mater. Phys. Mech*. **31**(1–2), 52–55 (2017)
27. D. G. Zelentsov, O. A. Liashenko, and O. R. Denysiuk, Problem of parametric optimization of cross-sections in bent rod corroding elements: Method of solution, in *Application of Mathematics in Technical and Natural Sciences*: 12th International On-line Conference for Promoting the Application of Mathematics in Technical and Natural Sciences - AMiTaNS'20, (Albena, Bulgaria, 2020), p. 120009. (2020). https://doi.org/10.1063/5.0034857
28. S. Kabrits and E. Kolpak, Nonlinear axisymmetric deformation of compound shells of revolution under conditions of mechanochemical corrosion, *Procedia Struct. Integr.* **47**, 513–520, (2023) https://doi.org/10.1016/j.prostr.2023.07.073
29. Yu. M. Pochtman and M. M. Fridman, Effect of the stressed state on the kinetics of corrosion of cylindrical vessels, *Strength Mater.* **30(**1), 59–65 (1998). https://doi.org/10.1007/BF02764421
30. Y. Miglis, I. Elishakoff, and F. Presuel-Moreno, Analysis of a cracked bar under a tensile load in a corrosive environment, *Ocean Syst. Eng.* **3**(1), 1–8 (2013). https://doi.org/10.12989/OSE.2013.3.1.001
31. D. Okulova, L. Almazova, O. Sedova, and Y. Pronina, On local strength of a spherical vessel with pits distributed along the equator, *Frat. Ed Integrità Strutt.* **17(**63), 70–80 (2022). doi.org/10.3221/IGF-ESIS.63.08
32. O. S. Sedova, L. A. Khaknazarova, and Y. G. Pronina, Stress concentration near the corrosion pit on the outer surface of a thick spherical member, in *2014 Tenth International Vacuum Electron Sources Conference (IVESC)* (IEEE, Saint-Petersburg, Russia, 2014), pp. 1–2 (2014). https://doi.org/10.1109/IVESC.2014.6892074
33. W. Mai and S. Soghrati, A phase field model for simulating the stress corrosion cracking initiated from pits, *Corros. Sci.* **125**, 87–98 (2017).
34. A. Ilyin and Y. Pronina, Curved pipes subjected to mechanochemical corrosion under pressure: Analytical and numerical estimates of the lifetime, *Int. J. Eng. Sci.* **215**, 104319 (2025). https://doi.org/10.1016/j.ijengsci.2025.104319
35. G. Gutman E. M., R. S. Zainullin, A. T. Shatalov, and R. A. Zaripov, *Strength of Gas Industry Pipes under Corrosive Wear Conditions* (Nedra, Moscow, 1984).
36. Y. G. Pronina, Analytical solution for the general mechanochemical corrosion of an ideal elastic-plastic thick-walled tube under pressure, *Int. J. Solids Struct.* **50(**22), 3626–3633 (2013). https://doi.org/10.1016/j.ijsolstr.2013.07.006
37. Y. Pronina, Lifetime assessment for an ideal elastoplastic thick-walled spherical member under general mechanochemical corrosion conditions, in *Computational Plasticity XII: Fundamentals and Applications - Proceedings of the 12th International Conference on Computational Plasticity - Fundamentals and Applications*, (CIMNE, Barcelona, Spain, 2013), pp. 729–738 (2013).
38. Y. G. Pronina, Analytical solution for decelerated mechanochemical corrosion of pressurized elastic–perfectly plastic thick-walled spheres, *Corros. Sci.* **90**, 161–167 (2015). https://doi.org/10.1016/j.corsci.2014.10.007.
39. O. Sedova and Y. Pronina, The thermoelasticity problem for pressure vessels with protective coatings, operating under conditions of mechanochemical corrosion, *Int. J. Eng. Sci*. **170**, 103589 (2022). https://doi.org/10.1016/j.ijengsci.2021.103589
40. O. S. Sedova and Y. G. Pronina, On the choice of equivalent stress for the problem of mechanochemical corrosion of spherical members, *Vestn. St. Petersburg Univ. Appl. Math. Comput. Sci. Control Process.*, no. 2, 33–44 (2016). https://doi.org/10.21638/11701/spbu10.2016.204
41. Z. Dong, W. Zhang, X. Li, M. Han, B. Long, and P. Jiang, Corrosion induced morphology evolution in stressed solids, *Metals* **13(**1), 108 (2023). https://doi.org/10.3390/met13010108
42. M. Ghahari *et al.*, Synchrotron X-ray radiography studies of pitting corrosion of stainless steel: Extraction of pit propagation parameters, *Corros. Sci.* **100**, 23–35 (2015). https://doi.org/10.1016/j.corsci.2015.06.023.
43. D. J. Srolovitz, On the stability of surfaces of stressed solids, *Acta Met.* **37**(2), 621–625 (1989).
44. A. Morozov, A. B. Freidin, and W. H. Müller, On stress-affected propagation and stability of chemical reaction fronts in solids, *Int. J. Eng. Sci.* **189**, 103876 (2023). https://doi.org/10.1016/j.ijengsci.2023.103876
45. S. Zhao and Y. Pronina, On the stress state of a pressurised pipe with an initial thickness variation, subjected to non-homogeneous internal corrosion, *E3S Web Conf.* **121**, p. 01013 (2019). https://doi.org/10.1051/e3sconf/201912101013