Estimation of Internal Forces of Three-Layer Plates Under Generally Symmetrical Loading

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Abstract. This article examines estimates of internal force factors for the bending of three-layer plates freely lying on an elastic half-space. An elastic filler layer is installed between the plates, which transmits reactive pressures to the plates of the constituent layer. The top plate is loaded with generally symmetrical external loadings related to the middle of the plate. To study the stress-strain state of three-layer plates interacting with elastic half-spaces, a mathematical model and an analytical method for solving the problem based on orthogonal polynomials have been developed. An analytical solution to the problem has been obtained, which has a refined calculation algorithm for calculation and project work. Based on the results of theoretical studies and numerical calculations of the test example, conclusions were drawn about the influence of the filler on the internal forces of the plates.

Keywords. Three-layer plate, elastic filler, half-space, integra-differential equation, orthogonal polynomials, closed system of equations, internal forces.

INTRODUCTION

Research into the stress-strain state of mutually contacting bodies, which depends on many structural elements, is one of the pressing issues of mechanics. A structure interacting with a deformable base belongs to such contacting bodies.

Researchers develop a variety of models and solution methods to assess various factors affecting critical parts of engineering structures. The performance of engineering structures is directly related to the need to improve the level and quality of their design. When constructing any structures, it is necessary to take into account the multifactorial interactions of structural elements given in the relevant design and calculation works. The foundations of industrial and civil buildings, as well as the coverings of airfields, railways, roads, pedestrian roads and many others belong to such structures. The noted shows the need to develop multifactor effective mathematical models and calculation methods for the implementation of design and calculation work, as well as during the construction of objects. The proposed models and calculation methods should lead to more economical solutions for the construction of structures.

In the work [1], the study of the time response of an elastic thin plate interacting with multilayer transversally isotropic soils was carried out. This proposes an effective theoretical method for solving the issue.

In the work [2], a study of nonlinear vibrations of pure polymer plates of three types of polymer composite plates was carried out; in [3, 4], both experimental and numerical analyzes were carried out to study the ultimate strength to model stiffeners and a support plate.

In [5, 6], the issues of bending of multilayer strip-plates lying on an elastic foundation were studied. Estimates of internal forces in plates are given based on the approximation of orthogonal polynomials.

In [7], an interaction function was constructed based on axial compression (tension) and shear loads of orthotropic plates, and in [8] an approach to dynamic modeling of a multi-plate structure connected by nonlinear hinges was presented. Linear modes have been proven to play an important role in dynamic analysis.

In [9], an algorithm and program for the numerical solution of wave issues using the method of characteristics and the finite difference method were developed. In works [10,25], the vibrations of a rod protected from vibrations under the influence of kinematic excitations were studied, and in work [11] the stress-strain state of asymmetrically layered plates with controlled forces interacting with a sandy base was considered.

In [12], the process of free and forced vibrations of two beam systems with intermediate supports was studied. Analytical solutions were obtained to ensure effective results in design work.

[13] investigated the dynamic strain localization of plastic polymer bars under large tensile strains up to failure, and the study [14] proposed a unified procedure to analyze the free and transient vibration behavior of a composite laminated beam subjected to general boundary conditions in a thermal scenario.

In [15], the propagation of oscillatory waves and assessment of the level of their impact with various objects were studied. Mathematical models and methods for assessing the levels of vibration waves at various distances from the soil base have been developed.

In [16,17], a mathematical model was obtained and an analytical method was used to solve the problem. Orthogonal polynomials were used to estimate the internal forces of the plates. Corresponding conclusions are presented on the influence of base and filler pressure on internal forces in plates.

In [18], a mathematical model was developed for assessing the stress-strain state of ground dams using a spatial model based on the variational Lagrange equation, taking into account the real geometry, material properties and heterogeneous design features of structures.

In [19,20], modifications of the structure made from composite laminate plates were studied. The results of experimental and numerical analysis affecting the structural connection relationships are presented.

Article [21] presents a model analysis of a honeycomb structure, structure for various parametric conditions using the finite element method.

The work [22] presents a numerical algorithm for solving odd differential equations using the Runge-Kutte method, which is suitable for dynamic systems.

In [23, 28], based on the bending of the slab, the internal force factors of layer plates in contact with elastic foundations were studied. Using an analytical method, the influence of the filler on the stress-strain states of the plate was determined under various external loads.

It is known that numerous researchers have developed many different calculation methods, which are described by various models. Despite the progress achieved in this area, there is a need to develop analytical calculation methods based on modeling the operation of structures taking into account their interaction with the soil foundation.

METHODS
MATHEMATICAL MODEL

Let us consider two rectangular plates lying on an elastic half-space, located symmetrically, one above the other, in the form of a layered system. We assume that an elastic filler layer is installed between the plates, which transmit reactive pressures to the plates of the constituent layer. Such structures can be called three-layer plates that have contact relationships with elastic half-spaces. For the geometric and mechanical parameters of the plates, we introduce the following notations:

h is height; $2l$ is width; b is length; E is modulus of elasticity; ν is Poisson's ratio.

On the upper (second) plate, from the top there is an external load, a generally symmetrical effect relative to the middle of the plate at a certain distance (Fig. 1, a), and from the bottom - the normal reactive pressure of the filler layer p_z. The lower (first) plate is affected from above by the filler pressure p_z, and from below by the normal reactive pressures of the base p. We assume that the reactive pressure of the filler is proportionally equal to the deflection differences of the plates, i.e.:

$$p_z = k(y_2 - y_1).$$

where y_1, y_2 are accordingly, the deflections of the first and second plates; k is the coefficient of proportionality, which in the future we will call the coefficient of stiffness of the filler.

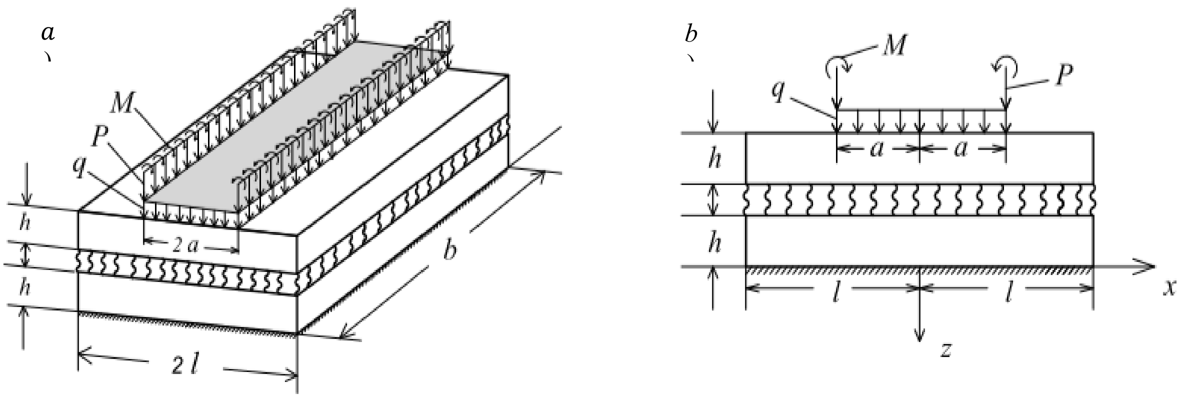

FIGURE 1. Design diagram of three-layer slabs (a) and three-layer beam slabs loaded (b) with symmetrical external loads.

For mathematical modeling of the issue of bending a three-layer plate having continuous, two-sided, contact relationships with an elastic half-space, a fragment of a three-layer plate with a width of one is considered and operating according to the scheme of three-layer beam plates. Then, the task is reduced to the study of the stress-strain state of three-layer beam plates lying on an elastic base when loaded with generally symmetrical external loads (Fig. 1, b).

For convenience, we set the origin of the Cartesian coordinates in the center of the symmetry of the beam plate (Fig. 1, b) and the problem is considered on the segment $[-l, l]$ along the abscissa axis Ox (i.e. on $-l \leq x \leq l$). The deflections of the beam platey_1, y_2, the external load q, as well as the base pressure p, are functions of the variable x.

For deflection of beam plates, the following systems of fourth-order differential equations can be written, under the above assumptions, conditions and notations:

$$Dy_2^{IV}=q-k(y_2-y_1), \quad Dy_1^{IV}=k(y_2-y_1)-p. \tag{1}$$

Where

$$D=\frac{Eh^3}{12(1-\nu^2)}.$$

To determine the precipitation of a homogeneous base V, according to the Gorbunov-Posadov hypothesis, we use the following formula:

$$V=D_0\int_{-l}^{l} ln\frac{1}{|x-s|}\cdot p(s)\,ds. \tag{2}$$

Here, $D_0=\frac{2(1-\nu_0^2)}{\pi E_0}$, E_0 and ν_0 are respectively, the modulus of elasticity and the Poisson's ratio of the base material.

Let's assume that there is a two-way connection between the surface of the first plate and the base. In this case, the relationship between the structure and the base, as contact conditions, is written as:

$$y_1(x)=V(x), \ -l \leq x \leq l\,. \tag{3}$$

Thus, the study of the stress-strain state of a three-layer beam plate, according to the problem statement, leads to the solution of a system of integra-differential equations (1), (2) and (3). Equation (1) and relation (2), (3) constitute a closed system of equations with respect to the unknowns of the issue under consideration.

In the future, when solving the issue, we will use the dimensionless coordinate *x*, which is equal to the ratio of the absolute coordinate to the half-length of the beam plate, i.e. $|x|/l$.

SOLUTION METHOD

The analytical solution of the system of differential equations (1) in a generalized form is presented in the following form

$$y_1=\frac{l^4}{2D}\left\{\sum_{i=1}^{4} C_i\, x^{4-i}+f_q(x)-f_p(x)-\frac{D}{l^4}\left[\sum_{i=1}^{4} B_i\, u_i(\alpha x)+\varphi_q(x)+\varphi_p(x)\right]\right\}; \tag{4}$$

$$y_2=\frac{l^4}{2D}\left\{\sum_{i=1}^{4} C_i\, x^{4\text{-}i}+f_q(x)\text{-}f_p(x)+\frac{D}{l^4}\left[\sum_{i=1}^{4} B_i\, u_i(\alpha x)+\varphi_q(x)+\varphi_p(x)\right]\right\}. \quad (5)$$

Where C_i, B_i are constant integrations determined from the boundary conditions of the issue under consideration;

$$u_1(x)=cshxcosx;\ u_2(x)=snhxcosx+cshxsinx;$$
$$u_3(x)=sinxsnhx;\ u_2(x)=snhxcosx\text{-}cshxsinx;$$
$$f_q^{IV}(x)=q(x);\ \ f_p^{IV}(x)=p(x);\ \ \alpha^4=\frac{kl^4}{2D}\ ; \quad (6)$$
$$\varphi_q(x)=\frac{1}{4\alpha^3}\int_0^x u_4[\alpha(x\text{-}s)]q(s)ds; \quad (7)$$
$$\varphi_p(x)=\frac{1}{4\alpha^3}\int_0^x u_4[\alpha(x\text{-}s)]p(s)ds; \quad (8)$$

Due to the symmetry of the external loads, the reactive base pressure is sought in the form of a series of even terms of Chebyshev polynomials of the first kind [23]:

$$p(x)=(1\text{-}x^2)^{\text{-}\frac{1}{2}}\sum_{n=0}^{\infty} A_{2n}\, T_{2n}(x) \quad (9)$$

Here, A_{2n} - are the unknown coefficients to be determined; $T_{2n}(x)$ – an orthogonal Chebyshev polynomial of the first kind.

The equilibrium equations of the beam plate can be written in the following form:

$$l\int_{\text{-}1}^{1} p(x)\, dx=P_s;\quad l^2\int_{\text{-}1}^{1} xp(x)dx=M_s. \quad (10)$$

Here through P_s and M_s – indicated accordingly, the sum of all vertical forces and the sum of their moments relative to the middle of the beam plate.

Substituting (9) into (10), while taking into account the orthogonality of the Chebyshev polynomials, we determine the first unknown coefficients of series (12) in the form:

$$A_0=\frac{2(q+P)}{\pi l}\ . \quad (11)$$

Substituting (9) into (2), we obtain the following expression for determining the base settlement:

$$V=lD_0\left[\text{-}A_0 ln2+\sum_{n=1}^{\infty}\frac{A_{2n}}{2n}T_{2n}(x)\right]. \quad (12)$$

Here the following formulas were used**:**

$$\int_{\text{-}1}^{1}(1\text{-}s^2)^{\text{-}\frac{1}{2}}T_0(s)ln\frac{1}{|x\text{-}s|}ds=\text{-}\pi ln2;\ \int_{\text{-}1}^{1}(1\text{-}s^2)^{\text{-}\frac{1}{2}}T_{2k}(s)ln\frac{1}{|x\text{-}s|}ds=\frac{\pi}{2k}T_{2k}(x).$$

The deflections of the beam plate (7) and (8), taking into account (12), will take the following form:

$$y_1=\frac{l^4}{2D}\left\{\sum_{i=1}^{4} C_i x^{4\text{-}i}+f_q(x)\text{-}\frac{D}{l^4}\left[\sum_{i=1}^{4} B_i u_i(\alpha x)+\varphi_q(x)\right]\text{-}\frac{D}{l^4}\sum_{n=0}^{\infty} A_{2n}\left[\frac{l^4}{D}f_{p,2n}(x)+\varphi_{p,2n}(x)\right]\right\}; \quad (13)$$
$$y_2=\frac{l^4}{2D}\left\{\sum_{i=1}^{4} C_i x^{4\text{-}i}+f_q(x)+\frac{D}{l^4}\left[\sum_{i=1}^{4} B_i u_i(\alpha x)+\varphi_q(x)\right]\text{-}\frac{D}{l^4}\sum_{n=0}^{\infty} A_{2n}\left[\frac{l^4}{D}f_{p,2n}(x)\text{-}\varphi_{p,2n}(x)\right]\right\}. \quad (14)$$

Here,

$$\varphi_{p,2n}(x)=\frac{1}{4\alpha^4}\int_0^x u_4[\alpha(x\text{-}z)](1\text{-}z^2)^{\text{-}\frac{1}{2}}T_{2n}(z)dz; \quad (15)$$
$$f_{p,2n}(x)=\frac{1}{32n(2n\text{-}1)(2n\text{-}2)(2n\text{-}3)}(1\text{-}x^2)^{\frac{7}{2}}P_{2n\text{-}4}^{(\frac{7}{2},\frac{7}{2})}(x),\ n>2, \quad (16)$$

where $P_j^{(\beta_1,\beta_2)}(x)$ – Jacobi polynomials [7,16]. The form of the function $f_{p,n}(x)$ for $n\leq 3$ is determined based on the explicit form of Chebyshev polynomials [23], i.e. for the case when n=0,1 it has the form:

$$f_{p,0}(x)=\frac{1}{12}(3x+2x^3)arcsinx+\frac{1}{36}(4+11x^2)(1\text{-}x^2)^{\frac{1}{2}};$$
$$f_{p,2}(x)=\frac{1}{8}xarcsinx+\frac{1}{120}(8+9x^2\text{-}2x^4)(1\text{-}x^2)^{\frac{1}{2}}.$$

Based on the above formulas (13) and (14), the factors of internal forces of three-layer beam plates, i.e. the angle of rotation, bending moments and cutting forces, can be represented as:

$$\varphi_i(x)=\frac{D}{l}y_i'(x);\ M_i(x)=-\frac{D}{l^2}y_i''(x);\ Q_i(x)=-\frac{D}{l^3}y_i''(x),\ i=1,2. \quad (17)$$

The draft of the base (12) and the deflections of the beam plate (13) and (14) are expressed in terms of unknown coefficients A_{2n}. To determine the coefficients A_{2n}, the contact condition (3) is used.

RESULTS AND DISCUSSION

Expressions (16) and (17) defining the deflections of the beam plate, taking into account (21), for each section separately, are presented in the following form:

$$y_1^I=\frac{l^4}{2D}\left[\sum_{i=1}^{4}C_i^I x^{4-i}+\frac{qx^4}{24}\right]-\frac{1}{2}\left[\sum_{i=1}^{4}B_i^I u_i(\alpha x)-\frac{q}{4\alpha^4}\left(1-u_1(\alpha x)\right)\right]-$$
$$-\frac{1}{2}\sum_{n=0}^{\infty}A_{2n}\left[\frac{l^4}{D}f_{p,2n}(x)+\varphi_{p,2n}(x)\right],\ x\in[-a;a]; \quad (18)$$

$$y_1^{II}=\frac{l^4}{2D}\sum_{i=1}^{4}C_i^{II}x^{4-i}-\frac{1}{2}\sum_{i=1}^{4}B_i^{II}u_i(\alpha x)-\frac{1}{2}\sum_{n=0}^{\infty}A_{2n}\left[\frac{l^4}{D}f_{p,2n}(x)+\varphi_{p,2n}(x)\right],\ x\in[-1;-a]\cup[a;1]; \quad (19)$$

$$y_2^I=\frac{l^4}{2D}\left[\sum_{i=1}^{4}C_i^I x^{4-i}+\frac{qx^4}{24}\right]+\frac{1}{2}\left[\sum_{i=1}^{4}B_i^I u_i(\alpha x)-\frac{q}{4\alpha^4}\left(1-u_1(\alpha x)\right)\right]-$$
$$-\frac{1}{2}\sum_{n=0}^{\infty}A_{2n}\left[\frac{l^4}{D}f_{p,2n}(x)-\varphi_{p,2n}(x)\right],\ x\in[-a;a]; \quad (20)$$

$$y_2^{II}=\frac{l^4}{2D}\sum_{i=1}^{4}C_i^{II}x^{4-i}+\frac{1}{2}\sum_{i=1}^{4}B_i^{II}u_i(\alpha x)-\frac{1}{2}\sum_{n=0}^{\infty}A_{2n}\left[\frac{l^4}{D}f_{p,2n}(x)-\varphi_{p,2n}(x)\right],\ x\in[-1;-a]\cup[a;1]. \quad (21)$$

Here, the symbols I and II indicate the values corresponding (Fig.2b) to the sections $-a\le x\le a$ and $-1\le x\le -a,\ a\le x\le 1$. In this case, arbitrary constants $C_i^I, C_i^{II}, B_i^I, B_i^{II}, (i=1,2,3,4)$ are determined from the following boundary conditions:

1. When $x=0$,

$$Q_1^I(0)=0;\ Q_2^I(0)=0;\ \varphi_1^I(0)=0;\ \varphi_2^I(0)=0.$$

2. When $x=\pm 1$,

$$Q_1^I(\pm1)=0;\ Q_2^I(\pm1)=0;\ M_1^I(\pm1)=0;\ \varphi_2^I(\pm1)=0.$$

3. When $x=\pm a$ (conditions for connecting sections) internal force factors satisfy the following equality:

$$Q_1^I(\pm a)-Q_1^{II}(\pm a)=P;\ \ Q_2^I(\pm a)=Q_2^{II}(\pm a);$$
$$M_1^I(\pm a)-M_1^{II}(\pm a)=M;\ \ M_2^I(\pm a)=M_2^{II}(\pm a);$$
$$\varphi_1^I(\pm a)=\varphi_1^{II}(\pm a);\ \varphi_2^I(\pm a)=\varphi_2^{II}(\pm a)$$
$$y_1^I(\pm a)=y_1^{II}(\pm a);\ \ y_2^I(\pm a)=y_2^{II}(\pm a).$$

The relative deflections of the beam plate satisfying the boundary conditions are represented in the following form:

$$y_1(x)=F_1(x)+\sum_{n=0}^{\infty}A_{2n}F_{1,2n}(x); \quad (22)$$

$$y_2(x)=F_2(x)+\sum_{n=0}^{\infty}A_{2n}F_{2,2n}(x). \quad (23)$$

The following designations are introduced here:

$$\left.\begin{aligned}F_1(x)&=\frac{l^4}{2D}\left(C_2^I x^2+\frac{qx^4}{24}\right)-\frac{1}{2}\left[B_1^I u_1(\alpha x)+B_3^I u_3(\alpha x)-\frac{q}{4\alpha^4}\left(1-u_1(\alpha x)\right)\right],\ when\ x\in[-a;a],\\ F_1(x)&=\frac{l^4}{2D}C_2^{II}x^2-\frac{1}{2}\left[B_1^{II}u_1(\alpha x)+B_3^{II}u_3(\alpha x)\right],\ when\ x\in[-1;-a]\cup[a;1].\end{aligned}\right\} \quad (24)$$

$$\left.\begin{aligned} F_2(x)&=\frac{l^4}{2D}\left(C_2^I x^2+\frac{qx^4}{24}\right)+\frac{1}{2}\left[B_1^I u_1(\alpha x)+B_3^I u_3(\alpha x)-\frac{q}{4\alpha^4}\left(1-u_1(\alpha x)\right)\right],\ when x\in[-a;a],\\ F_2(x)&=\frac{l^4}{2D}C_2^{II}x^2+\frac{1}{2}[B_1^{II}u_1(\alpha x)+B_3^{II}u_3(\alpha x)],\ when\ x\in[-1;-a]\cup[a;1]. \end{aligned}\right\} \quad (25)$$

$$F_{1,2n}(x)=-\frac{1}{2}\left[\frac{l^4}{D}f_{p,2n}(x)+\varphi_{p,2n}(x)\right]; \quad (26)$$

$$F_{2,2n}(x)=-\frac{1}{2}\left[\frac{l^4}{D}f_{p,2n}(x)-\varphi_{p,2n}(x)\right]; \quad (27)$$

$$C_2^{II}=\frac{\pi}{4}A_0;\ C_2^I=\frac{1}{1+2\alpha^2}\left(\frac{\pi}{4}A_0\right)-\frac{qa^2\alpha^2}{2};$$

$$B_1^{II}=\frac{A_0}{2\alpha^3 b_1}[u_1(\alpha)\varphi_0'''(1)+\alpha u_4(\alpha)\varphi_0''(1)];\ B_3^{II}=\frac{A_0}{2\alpha^3 b_1}[u_3(\alpha)\varphi_0'''(1)-\alpha u_2(\alpha)\varphi_0''(1)];$$

$$B_1^I=\frac{1}{2\alpha^3 b_2}\left[\left(-2P+\frac{l^4}{D}qa\right)u_1(\alpha a)+\left(-2M+\frac{l^4}{D}2\alpha^3\left(2C_2^I+\frac{q\alpha^2}{2}\right)u_3(\alpha a)\right)\right]-\frac{q}{4\alpha^4}+$$
$$+\frac{A_0}{2\alpha^3 b_1}[u_1(\alpha)\varphi_0'''(1)+\alpha u_4(\alpha)\varphi_0''(1)];$$

$$B_3^I=\frac{1}{2\alpha^3 b_2}\left[\left(-2P+\frac{l^4}{D}qa\right)u_3(\alpha a)+\left(-2M+\frac{l^4}{D}2\alpha^3\left(2C_2^I+\frac{q\alpha^2}{2}\right)u_1(\alpha a)\right)\right]+$$
$$+\frac{A_0}{2\alpha^3 b_1}[u_3(\alpha)\varphi_0'''(1)-\alpha u_2(\alpha)\varphi_0''(1)];$$

$$b_1=u_1(\alpha)u_2(\alpha)+u_3(\alpha)u_4(\alpha);\quad b_2=u_1(\alpha a)u_2(\alpha a)+u_3(\alpha a)u_4(\alpha a).$$

As noted above, the contact conditions (3) are used to determine the unknown coefficients A_{2n}. To do this, we substitute expressions (12) and (22) into equality (3), then multiply both parts of the equalities by expression $(1-x^2)^{-1/2}T_{2j}(x)$ and integrate in the range from -1 to 1.

When integrating, we take into account the orthogonality of the polynomials and obtain an infinite system of linear algebraic equations with an infinite number of unknown relatively unknown coefficients A_{2n}, in the form:

$$a_{2j}+\sum_{n=0}^{\infty}a_{2n,2j}A_{2n}=c_{2j}A_{2j},\ j=1,2,3,4,\dots \quad (28)$$

Where

$$c_{2j}=\frac{\pi(1-\nu_0^2)l}{E_0 j};\ a_{2j}=\frac{l^4}{2D}\int_{-1}^{1}F_1(x)(1-x^2)^{-\frac{1}{2}}T_{2j}(x)dx; \quad (29)$$

$$a_{2n,2j}=-\frac{l^4}{2D}\int_{-1}^{1}F_{1,2n}(x)\,(1-x^2)^{-\frac{1}{2}}T_{2j}(x)dx. \quad (30)$$

By integrating integrals (29) and (30) in parts, we can get rid of the singularity:

$$a_{2j}=\frac{l^4}{2D}\left(-\frac{1}{4j}\right)\int_{-1}^{1}F_1'(x)(1-x^2)^{\frac{1}{2}}P_{2j-1}^{(\frac{1}{2},\frac{1}{2})}(x)dx,$$

$$a_{2n,2j}=-\frac{l^4}{2D}\left(-\frac{1}{4j}\right)\int_{-1}^{1}F_{1,2n}'(x)(1-x^2)^{\frac{1}{2}}P_{2j-1}^{(\frac{1}{2},\frac{1}{2})}(x)dx.$$

$$\left.\begin{aligned} F_1'(x)&=\frac{l^4}{2D}\left(2C_2^I x+\frac{qx^3}{6}\right)-\frac{1}{2}\left[-B_1^I\alpha u_4(\alpha x)+B_3^I\alpha u_2(\alpha x)-\frac{q}{4\alpha^3}u_4(\alpha x)\right],\ when\ x\in[-a;a],\\ F_1'(x)&=\frac{l^4}{2D}C_2^{II}2x-\frac{1}{2}[-B_1^{II}\alpha u_4(\alpha x)+B_3^{II}\alpha u_2(\alpha x)],\ when\ x\in[-1;-a]\cup[a;1]. \end{aligned}\right\}$$

$$F_{2n}'(x)=-\frac{1}{2}\left[\frac{l^4}{D}f_{p,2n}'(x)+\varphi_{p,2n}'(x)\right];$$

$$f_{p,2n}'(x)=\frac{1}{16n(2n-1)(2n-2)}(1-x^2)^{\frac{5}{2}}P_{2n-3}^{(\frac{5}{2},\frac{5}{2})}(x),\ n>1;$$

$$\varphi_{p,2n}'(x)=\frac{1}{2\alpha^3}\int_0^x u_3\,[\alpha(x-z)](1-z^2)^{-\frac{1}{2}}T_{2n}(z)dz.$$

The resulting formulas have a convenient form and it becomes possible to implement calculations using computer technology.

The solution of the system (28) is determined by the reduction method. Based on the principle of the reduction method, we limit ourselves to a few first-order unknowns $A_2, A_4, \ldots, A_{2r}$ with corresponding r equations, systems of equations (28). We determine the solutions $A_2, A_4, \ldots, A_{2r}$ from the compiled system and, substituting them in (22) and (23), we find the deflections of the beam plate. Then, using deflection formulas, it is possible to calculate the internal forces of the beam plate based on formulas (17). Here it can be easily seen that the constraints, respectively, by r equations with r unknowns in system (28), uniquely correspond to those taken in place of the infinite series (9) in the form of a finite series consisting of r terms.

TEST CASE

Let's consider a numerical example to illustrate the presented methodology. The calculation is carried out in the following mechanical and geometric parameters:

for soil - $E_0 = 5 \cdot 10^2 \frac{kg}{sm^2}$; $\nu_0 = 3 \cdot 10^{-1}$.

for plates - $E = 1{,}25 \cdot 10^5 \frac{kg}{sm^2}$; $\nu = 1{,}67 \cdot 10^{-1}$; $l = 4 \cdot 10^2 sm$; $h = 2{,}5 \cdot 10\ sm$.

for the stiffness coefficient of the filler k, having the dimension $\frac{kg}{sm^3}$ -

$1 \cdot 10^{-1}; 1{,}5 \cdot 10^{-1}; \ 2 \cdot 10^{-1}; \ 2{,}5 \cdot 10^{-1}; \ 3 \cdot 10^{-1}; \ 3{,}5 \cdot 10^{-1}; \ 4 \cdot 10^{-1}; 4{,}5 \cdot 10^{-1}; \ 5 \cdot 10^{-1}$.

To carry out calculations in series (18), we take the first four terms with unknown coefficients. The numerical values of the unknown coefficients A_0, A_2, A_4, A_6 corresponding to different values of the filler stiffness coefficient k are given in Table 1.

TABLE 1. Numerical values for solving algebraic equations

k	$A_0 l(a(q+P))^{-1}$	$A_2 l(a(q+P))^{-1}$	$A_4 l(a(q+P))^{-1}$	$A_6 l(a(q+P))^{-1}$
0.10	0.636619734	-0.087965346	-0.006879273	0.000589234
0.15	0.636619734	-0.088674923	-0.007182671	0.000577423
0.20	0.636619734	-0.089776138	-0.007326346	0.000576461
0.25	0.636619734	-0.091682197	-0.007581275	0.000577862
0.30	0.636619734	-0.093138624	-0.007813469	0.000571349
0.35	0.636619734	-0.094386526	-0.008095364	0.000569784
0.40	0.636619734	-0.095163543	-0.008274618	0.000568126
0.45	0.636619734	-0.096857319	-0.008337421	0.000561237
0.50	0.636619734	-0.098976734	-0.008419263	0.000557329

According to Table 1, it can be noted that:

1. Changing the stiffness value of the aggregate does not lead to a noticeable change in the solution of the algebraic equations and, similarly, does not lead to a change in the distribution of the base pressure.
2. To ensure the necessary calculation accuracy with a uniform distribution of base pressure in the Chebyshev polynomial, it is sufficient to limit ourselves to the first four terms of the series.

Based on the calculation results, Table 2 shows the maximum values of bending moments in beam plates with comparisons with the results [23], - $M_1(ql^2)^{-1}$, $M_2(ql^2)^{-1}$.

TABLE 2. Maximum values of bending moments in beam plates

k	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$M_1(aql^2)^{-1}$	0.076583	0.077361	0.079714	0.080675	0.082305	0.082576	0.082971	0.082987	0.083271
$M_1(a(q+P)l^2)^{-1}$	0.079321	0.080638	0.081275	0.082621	0.084019	0.085219	0.086233	0.087683	0.088624
$M_2(aql^2)^{-1}$	0.114672	0.112318	0.110623	0.108937	0.098601	0.092461	0.098119	0.098362	0.097876
$M_2(a(q+P)l^2)^{-1}$	0.119864	0.118638	0.115563	0.110695	0.109487	0.106228	0.101278	0.099264	0.098967

Analyzing the data in Table 2, it can be noted that:

1. An increase in the rigidity of the filler leads to the maximum values of bending moments in the plates approaching each other.
2. Bending moments will increase to 10%.

CONCLUSION

Analyzing the results of the study, we can conclude:

1. A mathematical model and an analytical method have been developed to evaluate the bending of three-layer plates interacting with elastic bases loaded with generalized symmetrical external loads.
2. An improved algorithm for calculating three-layer plates lying freely on an elastic base is presented.
3. The nature of the influence of the filler layer on the internal forces in three-layer plates lying on an elastic base is determined.
4. Limits of numerical limitations have been established to ensure the accuracy of calculating practical calculations of three-layer plates having continuous contact with elastic half-spaces.

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