Nonlinear Thermoelastic Deformation of a Circular Plate with Ribs

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Abstract. The article presents research focused on studying the stress-strain state of a circular plate reinforced with ribs under the influence of an external short-term dynamic thermal load.

INTRODUCTION

Nowadays, special attention is paid to ensuring the strength of thin elastic structures in engineering and construction that are subjected to various short-term dynamic loads and high-temperature effects, which cause deformation processes to occur at high rates. In this research, the problems of identifying structurally vulnerable regions of such structures and enhancing their strength by reinforcing these areas with ribs to redistribute accumulated stresses were addressed. A numerical solution algorithm was developed to determine the dynamic response of annular ribs with constant cross-sections under non-stationary vibrations and thermoelastic deformation states of a circular plate, taking into account the nonlinear geometric relationships between deformations and displacements.

METHOD

We consider a circular plate reinforced with ribs in a ring-like pattern. The ribs have a rectangular cross-sectional shape and are firmly attached to the inner surface of the plate in a circular arrangement. It is assumed that the outer surface of the plate is subjected to a short-term, instantaneously increasing load that varies with time and temperature

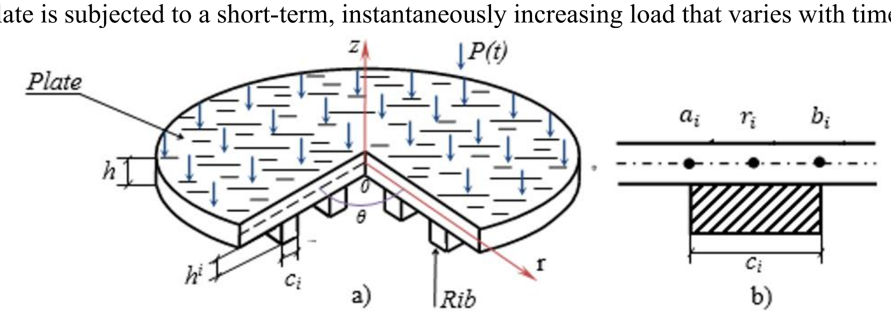

FIGURE 1. Schematic view of the ribbed plate

Mathematical Model of the Problem. Taking into account the axial symmetry of the external dynamic load, the differential equation of motion for the ribbed plate can be written in the following form [1,2]:

$$\left(N_1 r\right)' - N_2 = r\rho\left[\frac{\partial^2 u}{\partial t^2}\left(h+F\right)+\frac{\partial^2 \psi}{\partial t^2}S\right]$$

$$\frac{\partial(rQ)}{\partial r}+\left[N_1 r(\frac{\partial w}{\partial r})\right]' = r\rho\frac{\partial^2 w}{\partial t^2}(h+F)-rP \quad (1)$$

$$\frac{\partial\left(rM_1\right)}{\partial r}-M_2-rQ = r\rho\left[\frac{\partial^2 \psi}{\partial t^2}\left(h^3/12+J\right)+\frac{\partial^2 u}{\partial t^2}S\right],$$

The boundary conditions for the structure are defined as follows:

a) The edges of the plate are clamped: $u=w=\psi=0$;

b) From the condition of symmetry, at the central point $r=0$ of the shell: $u=\frac{\partial w}{\partial r}=\psi=0$,

The initial conditions are given at time $t=0$ as follows: $u=w=0$

The expressions for the internal stress resultants including normal and shear forces, as well as moments corresponding to a unit section of the ribbed plate are determined as follows:

$$Q=Q^0+Q^R;\ N_i=N_i^0+N_i^R;\ M_i=M_i^0+M_i^R,\ (i=1,2).$$

For the case without ribs, the internal stress resultants are taken into account as follows:

$$N_1^0 = Eh[\varepsilon_1^0+\mu\varepsilon_2^0-(1+\mu)\beta T]/(1-\mu^2);$$

$$N_2^0 = Eh[\varepsilon_2^0+\mu\varepsilon_1^0-(1+\mu)\beta T]/(1-\mu^2);$$

$$M_1^0 = D[\varepsilon_1^1+\mu\varepsilon_2^1-Eh(1+\mu)\beta T]/(1-\mu^2); \quad (2)$$

$$M_2^0 = D[\varepsilon_2^1+\mu\varepsilon_1^1-Eh(1+\mu)\beta T]/(1-\mu^2);$$

$$Q^0 = \frac{k^2 Eh}{2(1+\mu)}\varepsilon_{13}^0;$$

Here, E is the modulus of elasticity and μ is the Poisson's coefficient.; $D=\mathrm{Eh}^3(1+\mu)^{-1}/12$; The mid-surface deformation can be expressed as follows.

$$\varepsilon_1 = \frac{\partial u}{\partial r}-\frac{w}{R}+\frac{1}{2}(\frac{\partial w}{\partial r})^2;\quad \varepsilon_2 = u/r-w/R. \quad (3)$$

The deformation of an arbitrary point within the plate layer is as follows:

$$\varepsilon_1^z=\varepsilon_1+z\frac{\partial\psi}{\partial r};\ \varepsilon_2^z=\varepsilon_2+z\frac{\psi}{r};\ \varepsilon_{13}^z=f(z)\left(\frac{\partial w}{\partial r}+\psi\right); \quad (4)$$

$f\left(z\right)$ is a function. $f\left(z\right)=f_0\left(z\right)$ represents the distribution law of the σ_{13}^O stress through the thickness of the plate, while $f\left(z\right)=f_1\left(z\right)$ indicates the distribution law of the σ_{13}^O stress at points located in the ring [1];

$$f_o(z)=6[0{,}25-(zh)^2],\qquad k^2=56;\ f_1(z)=\frac{3h(h+2H)}{2(h+H)^2}\cdot(1+2zh)[1-2z(h+2H)],$$

Here, h is the height of the ring.

The stress forces generated in the cross-sections of ribs installed on a circular plate are determined as follows [3]:

$$N_1^R = \int_{h/2}^{h/2+H} G[\varepsilon_1+\mu\varepsilon_2-(1+\mu)\beta T]/(1-\mu^2)dz;$$

$$N_2^R = \int_{h/2}^{h/2+H} G[\varepsilon_2+\mu\varepsilon_1-(1+\mu)\beta T]/(1-\mu^2)dz;$$

$$M_1^R = \int_{h/2}^{h/2+H} G[\varepsilon_1+\mu\varepsilon_2-(1+\mu)\beta T]/(1-\mu^2)zdz; \quad (5)$$

$$M_2^R = \int_{h/2}^{h/2+H} G[\varepsilon_2+\mu\varepsilon_1-(1+\mu)\beta T]/(1-\mu^2)zdz;$$

$$Q^R = D_{13}\left(\frac{\partial w}{\partial r}+\psi\right)G_{13}dz.$$

Here, $G=\frac{E}{1-\mu^2}$, $G_{13}=\frac{5}{6}\cdot\frac{E}{2(1+\mu)}$, $D_{13}=G_{13}H\left(r\right)$ elastic constants for ribs.

Thus, the dynamic deformation problem of the plate consists of the joint solution of the system of equations (1)-(5) given with the complete mathematical formulation and the boundary and initial conditions.

Solving the problem. The obtained system of equations (1), written using finite difference expressions, is as follows [4-6].

$$u_i^{n+1} = 2u_i^n - u_i^{n-1} + \tau^2 \left\{ U_i^n / a - b \times (\Psi_i^n - bU_i^n / a) / (ac - b^2) \right\};$$
$$w_i^{n+1} = 2w_i^n - w_i^{n-1} + \tau^2 \, W_i^n / a; \quad (6)$$
$$\psi_i^{n+1} = 2\psi_i^n - \psi_i^{n-1} + \tau^2 \times (\Psi_i^n - bU_i^n) / a / (c - b^2 / a)$$

Here, $a = \rho(h + F)$, $b = \rho S$, $c = \rho(h^3 / 12 + J)$.

$$U_i^n = \frac{(rN_1)_{(i+1/2)}^n - (rN_1)_{(i-1/2)}^n}{\Delta r_{i+1/2} r_{i+1/2}} + \frac{(N_2)_{(i+1/2)}^n - (N_2)_{(i-1/2)}^n}{2r_{i+1/2}};$$

$$W_i^n = \frac{(rQ)_{(i+1/2)}^n - (rQ)_{(i+1/2)}^n}{\Delta r_{i+1/2} r_{i+1/2}} + \left(\frac{w_i^n - w \; -1}{\Delta r_i} \right) \frac{1}{\Delta r_{i+1/2}} \frac{(rN_1)_{(i+1/2)}^n - (rN_1)_{(i-1/2)}^n}{\Delta r_{i+1/2}} + z$$
$$+ \frac{w_{i+1}^n - 2w \quad + w \; -1}{(\Delta r_i)^2} \frac{(N_1)_{(i+1/2)}^n + (N_1)_{(i-1/2)}^n}{2} + P_{i+1/2}^n;$$

$$\Psi_i^n = \frac{(rM_1)_{i+1/2}^n - (rM_1)_{i-1/2}^n}{\Delta r_{i+1/2} r_{i+1/2}} - \frac{(M_2)_{i+1/2}^n + (M_2)_{i-1/2}^n}{2r_{i+1/2}} - \frac{Q_{i+1/2}^n + Q_{i-1/2}^n}{2};$$

Using the given equations, the u_i^n, w_i^n, ψ_i^n values at any point of the finite difference grid at any time can be determined.

Thus, the solution of differential equation (1) leads to calculations based on the recurrence formula (6).

EXPERIMENTS

Using the obtained equations (6), an instantaneously increasing $P(t) = P_0 \cdot \begin{cases} 1 - t/\theta, & 0 \le t \le \theta, \\ 0, & t > \theta. \end{cases}$ load is applied to a ribbed circular plate with clamped edges. Here, $P_0 = 5\,MPa$, $\alpha = 10^{-3} s$. The stress-strain state of the circular plate under the loading described above is analyzed. The geometric and physical characteristics of the circular plate are as follows: $R = 0.5\,m$; $h = 0.01\,m$; $E_0 = 75600\,MPa$; $\nu = 0{,}3$; $\rho = 2640\ kg/m^3$. The coefficient of linear thermal expansion is $\beta = (\beta_0 + \beta_1 T)10^{-6} s^{-1}$. The temperature field is given by the function $T = T(\rho)$. $E = (E_0 + E_1 T)10^3\ MPa$, $\beta_0 = 11.5$; $T_\rho = 0.9\rho + 20$; $\rho = r + k\Delta\rho$, $k = 0.1 \ldots\ldots 31$; $\Delta\rho = 0{,}0005\,m$.

The following cases of the stress-strain state of a circular plate reinforced with ribs, associated with variations in the cross-sections of the ribs attached to the plate, are studied.

Let a circular plate be equipped with four ribs spaced equally at 10 centimeters apart. We consider the following cases where the thickness and width of the ribs are changed without altering their cross-sectional area:

Variant 1: The ribs have a thickness of $h^p = 0.02\ m$ and a width of $c = 0.067\,m$;

Variant 2: The ribs have a thickness of $h^p = 0.04\ m$ and a width of $c = 0.034\,m$;

Variant 3: The ribs have a thickness of $h^p = 0.067\ m$ and a width of $c = 0.02\,m$;

The results obtained from the calculations for these variants are depicted in the graphs:

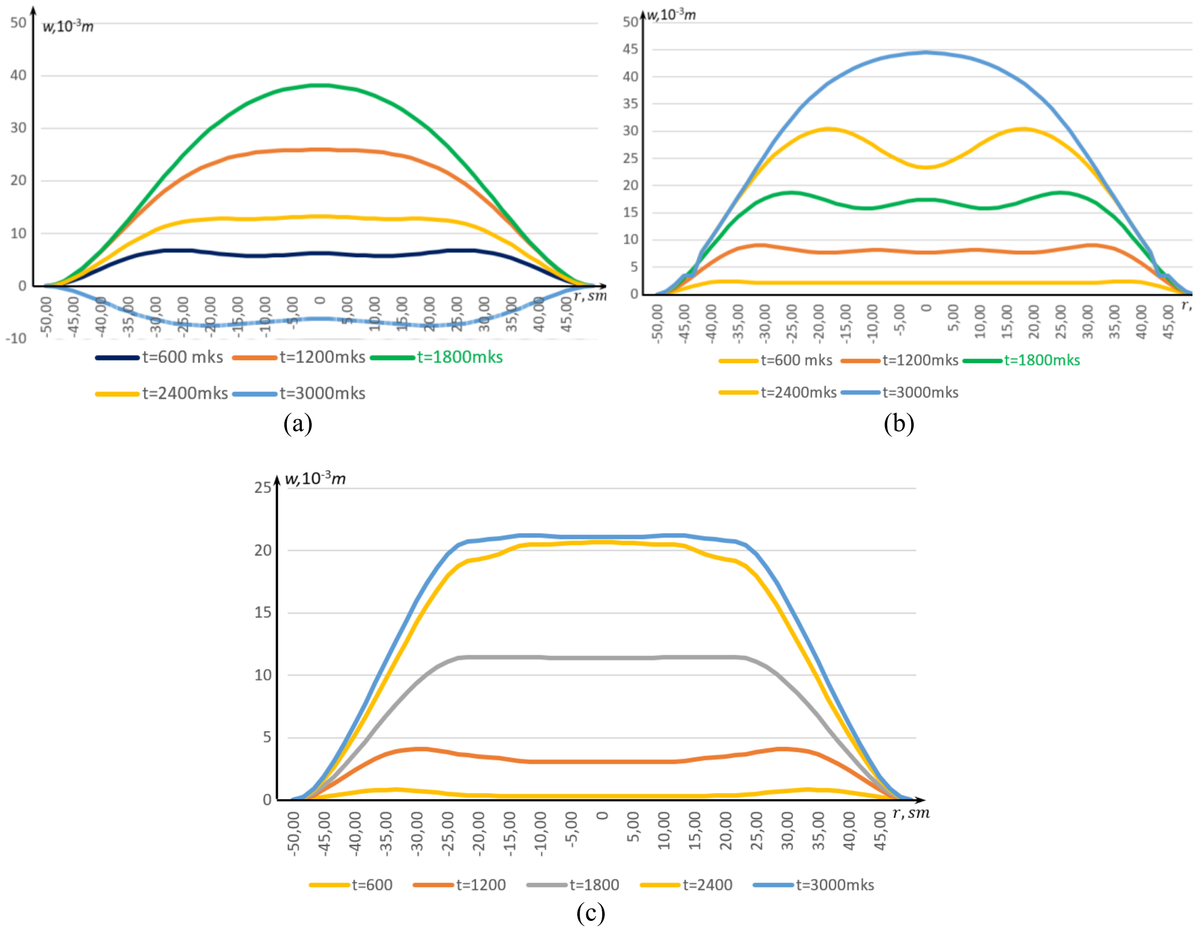

FIGURE 2. Bending shapes of the plates at different time moments for cases with rib geometric dimensions (a) $h^p = 0.02\ m$, $c = 0.067\,m$; (b) $h^p = 0.04\ m$, $c = 0.034\,m$; (c) $h^p = 0.067\ m$, $c = 0.02\,m$.

As can be seen from Fig. 2, the smallest maximum displacement at the center of the circular plate is observed in Variant 3, where ribs with smaller width and greater height are used. This leads to the conclusion that, without changing the volume of rib material and while maintaining the same cross-sectional area, increasing the height of the ribs can enhance the structural stiffness.

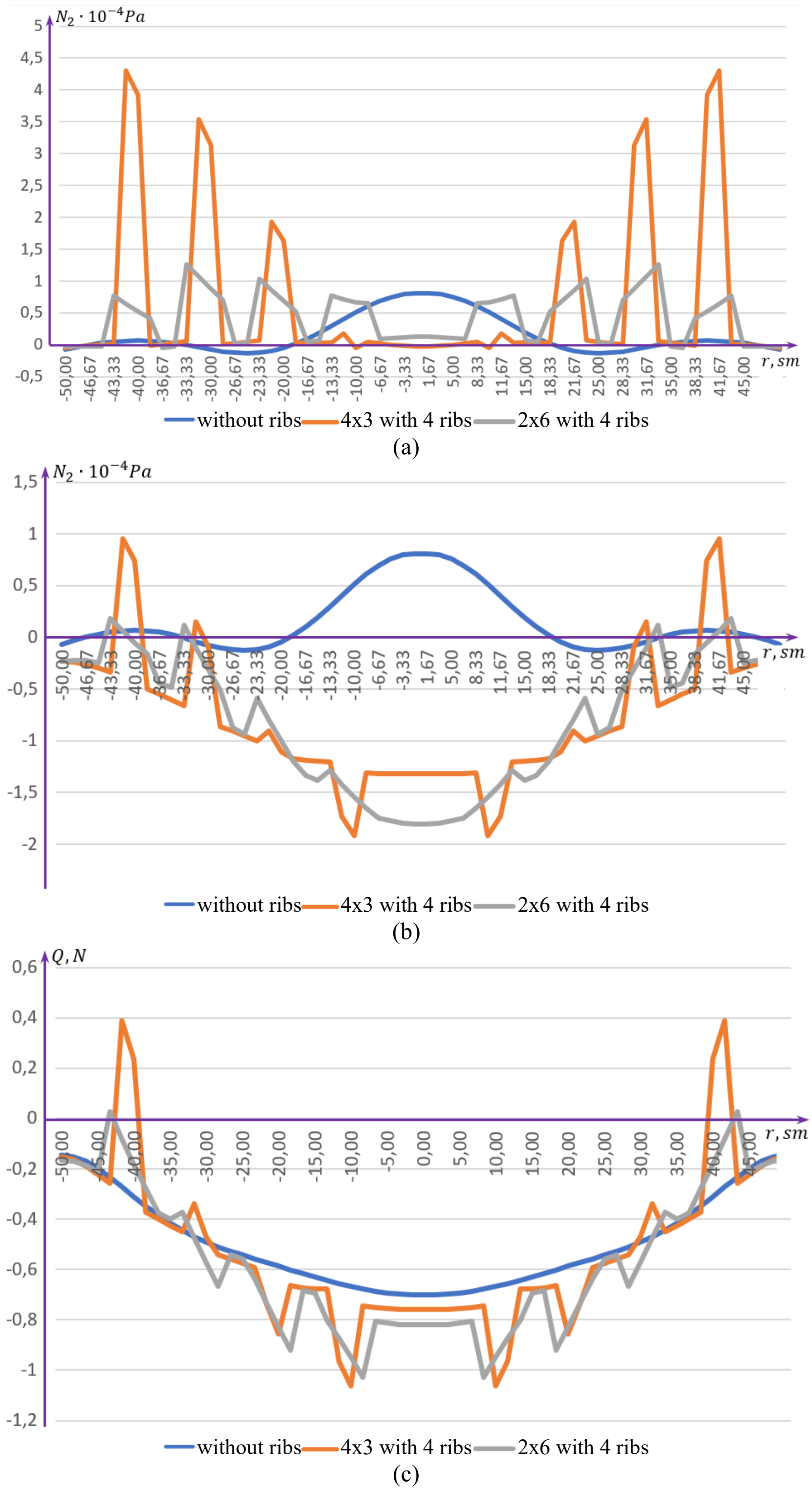

FIGURE 3. At the time moment t = 0.6 μs, the distribution of internal stress forces along the radius: (a) radial moments, (b) normal forces, and (c) shear forces.

CONCLUSION

Three different variants were considered in which the width and height of the cross-section of the ribs attached to the circular plate were changed without altering the cross-sectional area. In these variants, the displacements of the plate points, internal stress forces, and moments are compared using graphs. As seen from Fig. 2, the smallest maximum displacement at the center point of the circular plate is observed in Variant 3, where ribs with smaller width and greater height are used. This leads to the conclusion that it is possible to increase the structural strength by increasing the height of the ribs without changing the volume of the rib material while keeping the cross-sectional area constant.

Figure 3 illustrates the processes characterizing the stress forces and moments within the structure. It can be observed that the variation of the N_1 normal stress along the radius is uniform across all three variants at different time moments. In contrast, the N_2 normal force and the M_2 bending moment exhibit non-uniform variations, with radial stresses increasing in a stepwise manner at the locations where ribs are attached.

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