

# **3rd International Conference Advanced Mechanics: Structure, Materials, Tribology**

---

**Numerical solution of the bending equation for a three-layered plate with two parallel sides rigidly fixed and the other two parallel sides simply supported**

AIPCP25-CF-AMSMT2025-00034 | Article

PDF auto-generated using **ReView**



# Numerical solution of the bending equation for a three-layered plate with two parallel sides rigidly fixed and the other two parallel sides simply supported

Zokir Khudoyberdiyev<sup>1, a)</sup>, Burkhan Yalgashev<sup>2, b)</sup>, Sobir Xoljigitov<sup>3, c)</sup>, Zulfira Suyunova<sup>4, d)</sup>, Sherzod Omonov<sup>5, e)</sup>

<sup>1</sup>*Samarkand State University, Samarkand, Uzbekistan*

<sup>2</sup>*Samarkand State University Urgut Branch, Samarkand, Uzbekistan*

<sup>3</sup>*Jizzakh Branch of Mirzo Ulugbek National University of Uzbekistan Jizzakh, Uzbekistan,*

<sup>4</sup>*Samarkand State Architectural and Civil Engineering University, Samarkand, Uzbekistan*

<sup>5</sup>*Chirchik State Pedagogical University, Toshkent, Uzbekistan*

<sup>a)</sup> Corresponding author: xudoyberdiyevz@mail.ru

<sup>b)</sup> burkhon@gmail.com

<sup>c)</sup> sobirjon2020@inbox.ru

<sup>d)</sup> zulfira@gmail.com

<sup>e)</sup> sherkakrem@gmail.com

**Abstract.** The longitudinal vibration equation of a three-layer plate, with two parallel sides rigidly fixed and the other two parallel sides simply supported, is numerically solved using the finite difference method. First, the longitudinal vibration equations of the three-layer plate and the corresponding boundary conditions are expressed in finite-difference form. As a result, a system of algebraic equations is obtained. This system of algebraic equations is solved in Maple software for different step sizes. Based on the obtained solutions, the displacements and stresses in the layers of the three-layer elastic plate are determined. The calculated displacements and stresses are illustrated in graphs, and final conclusions are drawn.

**Keywords.** finite difference, layer, plate, external force, displacement

## INTRODUCTION

Nowadays, three-layer plate-type structures are widely used in all fields of construction and engineering. Such plates are considered integral elements of various constructions and mechanisms. In these cases, determining the stresses that arise within them is one of the important problems of modern mechanics. Therefore, since the first half of the last century, many researchers have studied such problems and proposed various methods for their solutions. For example, the “Zig-Zag” theory [2], the generalized theory [3], the E. Reissner theory [4], and others.

However, to this day, the problem of determining the displacements and stresses in plates with rigidly clamped edges has not been completely solved. Studies devoted to the longitudinal vibrations of three-layer plates with two parallel edges clamped and the other two parallel edges free under dynamic stresses are relatively few. Research works [5–8] can be cited as examples of such studies. In all these studies, the vibration equations are solved using exact analytical methods. In works [9–12], the longitudinal and transverse vibration equations of three-layer plates have been derived. In the present work, the longitudinal and transverse vibration equations of a three-layer plate derived in [9–12] are numerically solved using the finite difference method.

## FORMULATION OF THE PROBLEM

Let us consider a three-layer rectangular elastic plate located in the Cartesian coordinate system (Fig. 1), where one side has a length of  $l$  and the other side has a length of  $b$ . The parallel sides  $x=0$  and  $x=l$  of the rectangular three-layer elastic plate are rigidly clamped, while the other two parallel sides  $y=0$  and  $y=l$  are simply supported. The upper layer of the rectangular three-layer elastic plate has a thickness of  $h_1$ , the middle layer a thickness of  $2h_0$ , and the lower layer a thickness of  $h_2$ .

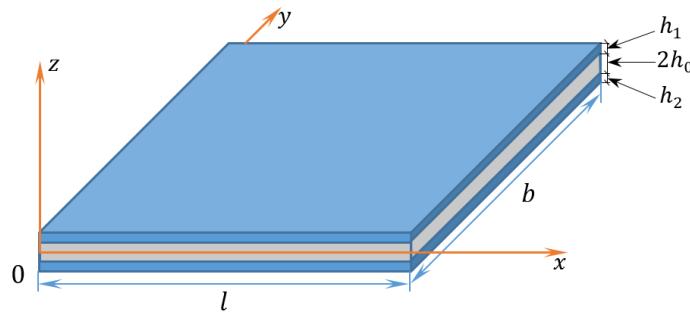


FIGURE 1. Rectangular three-layer elastic plate

In works [13–15], a system of longitudinal vibration equations for a rectangular three-layer elastic plate was derived. In the present study, we make use of this system of longitudinal vibration equations for the three-layer plate.

$$\begin{aligned} N_1 \left[ \frac{\partial}{\partial x} W_0^{(0)}(x, t) \right] + K_1 [U_0^{(0)}(x, t)] &= F_1 [f_x^{(1)}(x, t)], \\ N_2 [W_0^{(0)}(x, t)] + K_2 \left[ \frac{\partial}{\partial x} U_0^{(0)}(x, t) \right] &= F_2 [f_z^{(2)}(x, t)]. \end{aligned} \quad (1)$$

In this system of equations, the coefficients  $N_1, N_2, K_1, K_2, F_1$  and  $F_2$  are differential operators. Their forms are presented in detail in works [13–15]. The expressions of these differential operators include coefficients that depend on the geometric and physical properties of the rectangular three-layer elastic plate.

The side  $x=0$  of the three-layer rectangular plate under investigation is rigidly clamped. Therefore, for this side, we have the following:

$$W(0, t) = 0, \quad \frac{\partial}{\partial x} W(0, t) = 0 \quad \text{va} \quad U(0, t) = 0, \quad \frac{\partial}{\partial x} U(0, t) = 0 \quad (2)$$

The  $x=a$ -side is also rigidly clamped:

$$W(a, t) = 0 \quad \text{va} \quad \frac{\partial}{\partial x} W(a, t) = 0; \quad U(a, t) = 0 \quad \text{va} \quad \frac{\partial}{\partial x} U(a, t) = 0 \quad (3)$$

At time  $t=0$  the three-layer rectangular elastic plate was in a state of rest. Therefore, the initial conditions are as follows:

$$U(x, 0) = \frac{\partial}{\partial x} U(x, 0) = \frac{\partial^2}{\partial x^2} U(x, 0) = 0; \quad W(x, 0) = \frac{\partial}{\partial x} W(x, 0) = \frac{\partial^2}{\partial x^2} W(x, 0) = 0. \quad (4)$$

The longitudinal vibration problem of a three-layer elastic rectangular plate with two parallel edges hinged and fixed, and the other two parallel edges simply supported, was reduced to solving the system of differential equations (1) using the boundary conditions (2)–(4).

## RESEARCH RESULTS

To solve the problem of longitudinal vibration of a three-layer elastic rectangular plate with two opposite sides rigidly hinged and the other two opposite sides freely supported, we express (1) the system of equations, as well as (2)–(3) boundary and (4) initial conditions in finite difference form. By expressing the system of equations (1) in finite difference form, we obtain the following:

$$\begin{aligned}
& n_{11}W_{i+1}^{j+2} + n_{12}W_{i-1}^{j+2} + n_{13}W_{i+1}^{j+1} + n_{14}W_{i-1}^{j+1} + n_{15}W_{i+1}^{j-1} + n_{16}W_{i-1}^{j-1} + n_{17}W_{i+1}^j + n_{18}W_{i-1}^j + n_{19}W_{i+1}^{j-2} + \\
& + n_{10}W_{i-1}^{j-2} + n_{11}W_{i+2}^{j+1} + n_{12}W_{i+2}^j + n_{13}W_{i+2}^{j-1} + n_{14}W_{i-2}^{j+1} + n_{15}W_{i-2}^j + n_{16}W_{i-2}^{j-1} + n_{17}W_{i+3}^j + \\
& + n_{18}W_{i-3}^j + k_{11}U_i^{j+2} + k_{12}U_i^{j+1} + k_{13}U_i^{j-1} + k_{14}U_i^j + k_{15}U_i^{j-2} + k_{16}U_{i+1}^{j+1} + k_{17}U_{i-1}^{j+1} + k_{18}U_{i+1}^j + \\
& + k_{19}U_{i-1}^j + k_{110}U_{i+1}^{j-1} + k_{111}U_{i-1}^{j-1} + k_{112}U_{i+2}^j + k_{113}U_{i-2}^j = d_{11}\tilde{f}_i^{j+2} + d_{12}\tilde{f}_i^{j+1} + d_{13}\tilde{f}_i^{j-1} + d_{14}\tilde{f}_i^j + \\
& + d_{15}\tilde{f}_i^{j-2} + d_{16}\tilde{f}_{i+1}^{j+1} + d_{17}\tilde{f}_{i-1}^j + d_{18}\tilde{f}_{i+1}^j + d_{19}\tilde{f}_{i-1}^j + d_{110}\tilde{f}_{i+1}^{j-1} + d_{111}\tilde{f}_{i-1}^{j-1} + d_{112}\tilde{f}_{i+2}^j + d_{113}\tilde{f}_{i-2}^j; \quad (5) \\
& n_{21}W_i^{j+2} + n_{22}W_i^{j+1} + n_{23}W_i^{j-1} + n_{24}W_i^j + n_{25}W_i^{j-2} + n_{26}W_{i+1}^{j+1} + n_{27}W_{i-1}^{j+1} + n_{28}W_{i+1}^j + \\
& + n_{29}W_{i-1}^j + n_{210}W_{i+1}^{j-1} + n_{211}W_{i-1}^{j-1} + n_{212}W_{i+2}^j + n_{213}W_{i-2}^j + k_{21}U_{i+1}^{j+2} + k_{22}U_{i-1}^{j+2} + k_{23}U_{i+1}^{j+1} + \\
& + k_{24}U_{i-1}^{j+1} + k_{25}U_{i+1}^{j-1} + k_{26}U_{i-1}^{j-1} + k_{27}U_{i+1}^j + k_{28}U_{i-1}^j + k_{29}U_{i+1}^{j-2} + k_{210}U_{i-1}^{j-2} + k_{211}U_{i+2}^{j+1} + k_{212}U_{i+2}^{j-1} + \\
& + k_{213}U_{i+2}^{j-1} + k_{214}U_{i-2}^{j+1} + k_{215}U_{i-2}^j + k_{216}U_{i-2}^{j-1} = d_{21}\tilde{f}_i^{j+2} + d_{22}\tilde{f}_i^{j+1} + d_{23}\tilde{f}_i^{j-1} + d_{24}\tilde{f}_i^j + d_{25}\tilde{f}_i^{j-2} + \\
& + d_{26}\tilde{f}_{i+1}^{j+1} + d_{27}\tilde{f}_{i-1}^{j+1} + d_{28}\tilde{f}_{i+1}^j + d_{29}\tilde{f}_{i-1}^j + d_{210}\tilde{f}_{i+1}^{j-1} + d_{211}\tilde{f}_{i-1}^{j-1} + d_{212}\tilde{f}_{i+2}^j + d_{213}\tilde{f}_{i-2}^j;
\end{aligned}$$

We take the following initial conditions:

$$U_i^0 = 0; U_i^1 = U_i^0; U_i^{-1} = -U_i^1; W_i^0 = 0; W_i^1 = W_i^0; W_i^{-1} = -W_i^1. \quad (6)$$

From the boundary conditions, we take the following:

$$U_0^j = 0; U_1^j = U_0^j; W_0^j = 0; W_1^j = W_0^j. \quad (7)$$

Thus, to solve the problem of longitudinal vibration of a three-layer elastic rectangular plate with two opposite sides rigidly hinged and the other two opposite sides freely supported, it is necessary to solve equation (5) taking into account conditions (6) and (7).

**Practical issue.** Now, to solve the practical problem, we specify the dimensions of the three-layer elastic rectangular plate with two opposite sides rigidly hinged and the other two opposite sides freely supported. The plate length is  $1.2m$ , and the thicknesses of the first and second outer layers are both  $0.006m$ . The middle layer thickness is  $0.04m$ . We consider the middle layer of the three-layer plate as polymer and the outer layers as aluminum. For the polymer material, the modulus of elasticity is  $0.165 \cdot 10^9 Pa$  and the Poisson's ratio is  $0.03125$ . For aluminum, the modulus of elasticity is  $69 \cdot 10^9 Pa$  and the Poisson's ratio is  $0.33$ . The density of the polymer material is  $30 kg/m^3$ , and the density of aluminum is  $2700 kg/m^3$ . A longitudinal force of  $60 N$  is applied to the surface of the three-layer plate with two opposite sides rigidly hinged and the other two opposite sides freely supported.

Using the given values in the practical problem and specifying the number of steps, we determine the coefficients of equation (5). We divide the time and coordinate intervals into 20 segments each. As a result, 400 equations are formed. These 400 equations contain 400 unknowns. We solve the resulting system of equations using the Maple software. From the obtained solutions, we plot the graphs of longitudinal and transverse displacements occurring at the points of the three-layer elastic rectangular plate with two opposite sides rigidly hinged and the other two opposite sides freely supported (see Figure 2).

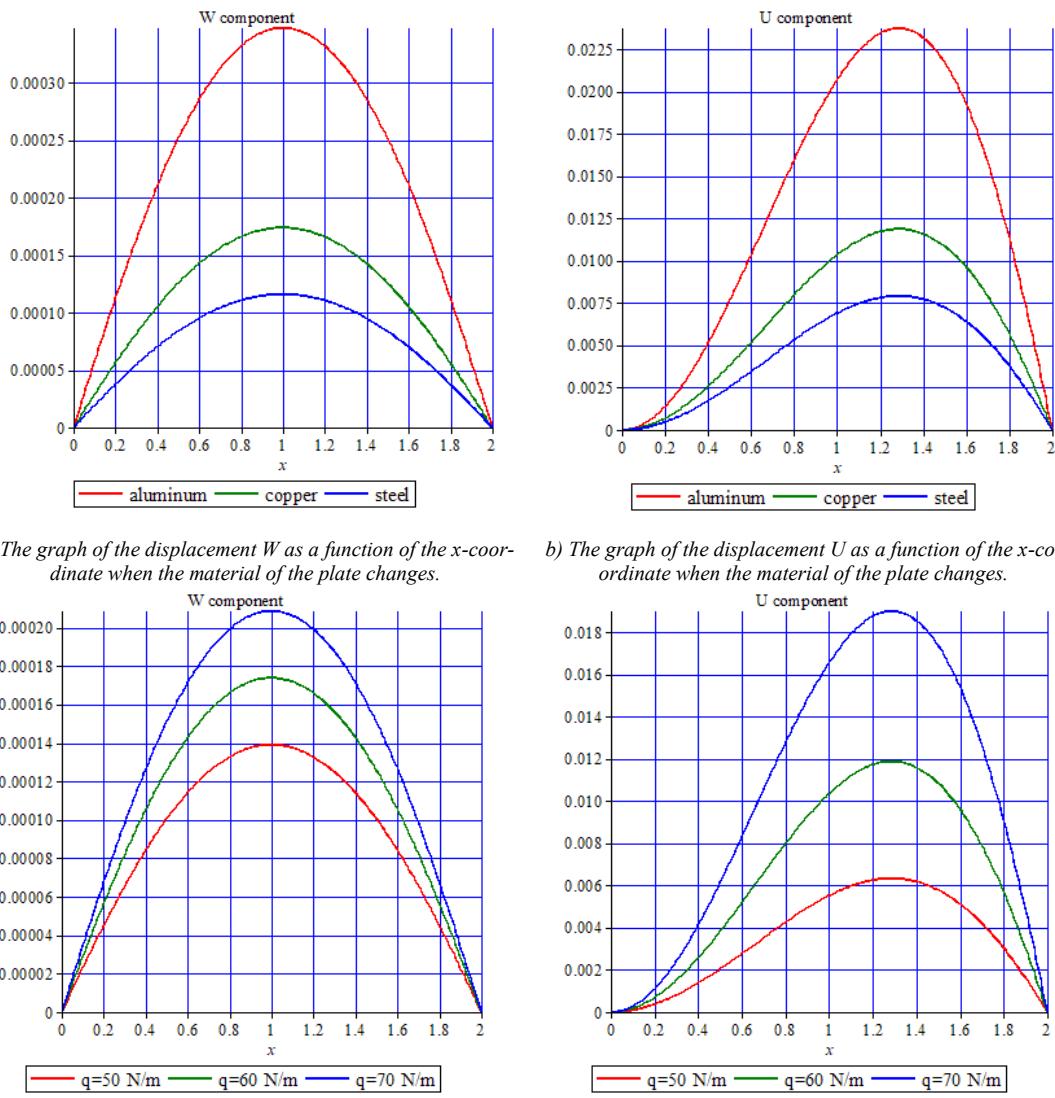


FIGURE 2. Transverse and longitudinal displacements of a three-layer plate.

In Figure 2, the displacement vector components  $U$  and  $W$  are shown. In Figure 2a, the graph of the  $W$  component of the displacement vector is presented as a function of the  $x$ -coordinate for cases where the materials of the outer layers of the plate are aluminum, copper, and steel, respectively. From the graph, it is evident that as the material of the outer layer of the three-layer plate becomes stiffer, the displacements decrease. In Figure 2b, the graph of the  $U$  displacement component versus the  $x$ -coordinate is shown for the same plate with outer layers made of aluminum, copper, and steel. The graph indicates that the longitudinal displacements are smaller at the beginning of the plate and increase towards the end. Figures 2b and 2c present graphs of the variation of the displacement vector components  $W$  and  $U$  along the  $x$ -coordinate for a three-layer plate with copper outer layers under distributed forces of 50, 60, and 70 N/m applied to its surface.

## CONCLUSION

In Figure 2a, the value of the  $W$  component of the displacement vector for the three-layer plate with aluminum outer layers differs by 65.7% from that of the plate with steel outer layers. In Figure 2b, the value of the  $U$  component of the displacement vector for the plate with aluminum outer layers differs by 69% from that of the plate with steel outer layers, and by 52% from that of the plate with copper outer layers. This result is consistent with the physical nature of the problem. The reliability of the obtained solutions can also be observed from the graphs presented in Figures 2c and 2d. It can be seen from these figures that as the magnitude of the external load applied to the surface of the three-layer elastic rectangular plate—with two opposite sides rigidly hinged and the other two opposite sides freely supported—increases, the values of the  $W$  and  $U$  components of the displacement vector also increase. This fact further confirms the correctness of the obtained solution.

## REFERENCES

1. I. G. Filippov, S. I. Filippov, Oscillatory and wave processes in continuous compressible media, (M.: Publishing house MGSU, 2007), 430 p.
2. Kh. Khudoynazarov, Longitudinal-radial vibrations of a viscoelastic cylindrical three-layer structure // Facta universitatis, Series: Mechanical Engineering, (2024). <https://doi.org/10.22190/FUME231219010K>
3. Kh. Khudoynazarov, B. Yalgashov, Longitudinal vibrations of a cylindrical shell filled with a viscous compressible liquid. E3S Web of Conferences **264**, 02017, (2021). <https://doi.org/10.1051/e3sconf/202126402017>
4. Kh. Kh. Khudoynazarov, R. I. Khalmuradov, B. F. Yalgashov, Longitudinal-radial vibrations of a elastic cylindrical shell filled with a viscous compressible liquid. *Tomsk State University. Journal of Mathematics and Mechanics.* **69**, 139-154, (2021). DOI: 10.17223/19988621/69/11.
5. Kh. Kh. Khudoynazarov and Sh. R. Yaxshiboyev, The Mathematical Model of Transverse Vibrations of the Three-Layer Plate, *IOP Conf. Ser.: Earth Environ. Sci.* **614** 012062, (2020). DOI: [10.1088/1755-1315/614/1/012062](https://doi.org/10.1088/1755-1315/614/1/012062).
6. Kh. Khudoynazarov and Z.B. Khudoyberdiyev, Unsteady vibrations of a three-layer plate with an asymmetric structure *IOP Conf. Ser.: Earth Environ. Sci.* **614** 012061 (2020). DOI:[10.1088/1755-1315/614/1/012061](https://doi.org/10.1088/1755-1315/614/1/012061)
7. R. I. Khalmuradov, Kh. Khudoynazarov, U.A. Nishanov, Elastic-plastic deformation of a round plate reinforced with stiffeners, Magazine of Civil Engineering **116** (8), Pp. 49-63. (2022). <https://doi.org/10.18721/JPM>
8. B. F. Yalgashov, E. A. Ismoilov, Z. B. Khudoyberdiyev, Torsional Vibrations of Layered Cylindrical Viscoelastic Shells and Rods, AIP Conference Proceedings 2637, 030023 (2022); <https://doi.org/10.1063/5.0118588>. Published Online: 20 October 2022
9. Z. B. Khudoyberdiyev, Sh. R. Yaxshiboyev, Bending of a Cantilever Beam Under the Influence of a Force Applied to its Tip. AIP Conference Proceedings (2024). 060014, 3244(1); <https://doi.org/10.1063/5.0241681>
10. M. U. Khodjabekov, Kh. M. Buranov, A. E. Qudratov, Modal Mass and Stiffness of Hysteresis Type Elastic Dissipative Characteristic Plate AIP Conference Proceedings 2637, 050004 (2022); <https://doi.org/10.1063/5.0118292> Published Online: 20 October 2022
11. R. I. Khalmuradov, and B. F. Yalgashov, “Frequency analysis of longitudinal-radial vibrations of a cylindrical shell” IOP Conf. Series: Earth and Environmental Science 614 (2020) 012087 doi:[10.1088/1755-1315/614/1/012087](https://doi.org/10.1088/1755-1315/614/1/012087)
12. R I Khalmuradov, K Khudoynazarov and S B Omonov, Model for calculation of anchor parameters fixings for vertical exploration works. IOP Conf. Series: Earth and Environmental Science 937 042092 (2021) doi:[10.1088/1755-1315/937/4/042092](https://doi.org/10.1088/1755-1315/937/4/042092)
13. Z. Khudoyberdiyev, Sh. Khudayberdiyeva, Sh. Yakhshiboyev, A. Begjanov, AIP Conf. Proc. 3177, 050010 (2025) <https://doi.org/10.1063/5.02944897>
14. R. Khalmuradov, Kh. Ismoilov, Z. Khudoyberdiyev, B. Babajanov, AIP Conf. Proc. 3177, 050011 (2025) <https://doi.org/10.1063/5.0295160>
15. Z. Khudoyberdiyev, Z. Suyunova, A. Begjanov, J. Khasanov, AIP Conf. Proc. 3177, 050012 (2025) <https://doi.org/10.1063/5.029489>



## LICENSE TO PUBLISH AGREEMENT FOR CONFERENCE PROCEEDINGS

This License to Publish must be signed and returned to the Proceedings Editor before the manuscript can be published. If you have questions about how to submit the form, please contact the AIP Publishing Conference Proceedings office (confproc@aip.org). For questions regarding the copyright terms and conditions of this License, please contact AIP Publishing's Office of Rights and Permissions, 1305 Walt Whitman Road, Suite 300, Melville, NY 11747-4300 USA; Phone 516-576-2268; Email: [rights@aip.org](mailto:rights@aip.org).

Article Title ("Work"):

### Numerical solution of the bending equation for a three-layered plate with two parallel sides rigidly fixed and the other two parallel sides simply supported

All Author(s):

Zokir Khudoyberdiyev, Burkhan Yalgashev,  
Sobir Xoljigitov, Zulfira Suyunova, Sherzod Omonov

AMSMT2025

Title of Conference:

Valentin L. Popov

All Copyright Owner(s), if not Author(s):

(Please list all copyright owner(s) by name. In the case of a Work Made for Hire, the employer(s) or commissioning party(es) are the copyright owner(s). For large groups of copyright owners, attach a separate list to this form.)

### Copyright Ownership and Grant of Rights

For the purposes of this License, the "Work" consists of all content within the article itself and made available as part of the article, including but not limited to the abstract, tables, figures, graphs, images, and multimedia files, as well as any subsequent errata. "Supplementary Material" consists of material that is associated with the article but linked to or accessed separately (available electronically only), including but not limited to data sets and any additional files.

This Agreement is an Exclusive License to Publish not a Transfer of Copyright. Copyright to the Work remains with the Author(s) or, in the case of a Work Made for Hire, with the Author(s)' employer(s). AIP Publishing LLC shall own and have the right to register in its name the copyright to the proceedings issue or any other collective work in which the Work is included. Any rights granted under this License are contingent upon acceptance of the Work for publication by AIP Publishing. If for any reason and at its own discretion AIP Publishing decides not to publish the Work, this License is considered void.

Each Copyright Owner hereby grants to AIP Publishing LLC the following irrevocable rights for the full term of United States and foreign copyrights (including any extensions):

1. The exclusive right and license to publish, reproduce, distribute, transmit, display, store, translate, edit, adapt, and create derivative works from the Work (in whole or in part) throughout the world in all formats and media whether now known or later developed, and the nonexclusive right and license to do the same with the Supplementary Material.
2. The right for AIP Publishing to freely transfer and/or sublicense any or all of the exclusive rights listed in #1 above. Sublicensing includes the right to authorize requests for reuse of the Work by third parties.
3. The right for AIP Publishing to take whatever steps it considers necessary to protect and enforce, at its own expense, the exclusive rights granted herein against third parties.

### Author Rights and Permitted Uses

Subject to the rights herein granted to AIP Publishing, each Copyright Owner retains ownership of copyright and all other proprietary rights such as patent rights in the Work.

Each Copyright Owner retains the following nonexclusive rights to use the Work, without obtaining permission from AIP Publishing, in keeping with professional publication ethics and provided clear credit is given to its first publication in an AIP Publishing proceeding. Any reuse must include a full credit line acknowledging AIP Publishing's publication and a link to the Version of Record (VOR) on AIP Publishing's site.

Each Copyright Owner may:

1. Reprint portions of the Work (excerpts, figures, tables) in future works created by the Author, in keeping with professional publication ethics.
2. Post the Accepted Manuscript (AM) to their personal web page or their employer's web page immediately after acceptance by AIP Publishing.
3. Deposit the AM in an institutional or funder-designated repository immediately after acceptance by AIP Publishing.

4. Use the AM for posting within scientific collaboration networks (SCNs). For a detailed description of our policy on posting to SCNs, please see our Web Posting Guidelines (<https://publishing.aip.org/authors/web-posting-guidelines>).
5. Reprint the Version of Record (VOR) in print collections written by the Author, or in the Author's thesis or dissertation. It is understood and agreed that the thesis or dissertation may be made available electronically on the university's site or in its repository and that copies may be offered for sale on demand.
6. Reproduce copies of the VOR for courses taught by the Author or offered at the institution where the Author is employed, provided no fee is charged for access to the Work.
7. Use the VOR for internal training and noncommercial business purposes by the Author's employer.
8. Use the VOR in oral presentations made by the Author, such as at conferences, meetings, seminars, etc., provided those receiving copies are informed that they may not further copy or distribute the Work.
9. Distribute the VOR to colleagues for noncommercial scholarly use, provided those receiving copies are informed that they may not further copy or distribute the Work.
10. Post the VOR to their personal web page or their employer's web page 12 months after publication by AIP Publishing.
11. Deposit the VOR in an institutional or funder-designated repository 12 months after publication by AIP Publishing.
12. Update a prior posting with the VOR on a noncommercial server such as arXiv, 12 months after publication by AIP Publishing.

### Author Warranties

Each Author and Copyright Owner represents and warrants to AIP Publishing the following:

1. The Work is the original independent creation of each Author and does not infringe any copyright or violate any other right of any third party.
2. The Work has not been previously published and is not being considered for publication elsewhere in any form, except as a preprint on a noncommercial server such as arXiv, or in a thesis or dissertation.
3. Written permission has been obtained for any material used from other sources and copies of the permission grants have been supplied to AIP Publishing to be included in the manuscript file.
4. All third-party material for which permission has been obtained has been properly credited within the manuscript.
5. In the event that the Author is subject to university open access policies or other institutional restrictions that conflict with any of the rights or provisions of this License, such Author has obtained the necessary waiver from his or her university or institution.

This License must be signed by the Author(s) and, in the case of a Work Made for Hire, also by the Copyright Owners. One Author/Copyright Owner may sign on behalf of all the contributors/owners only if they all have authorized the signing, approved of the License, and agreed to be bound by it. The signing Author and, in the case of a Work Made for Hire, the signing Copyright Owner warrants that he/she/it has full authority to enter into this License and to make the grants this License contains.

1. The Author must please sign here (except if an Author is a U.S. Government employee, then please sign under #3 below):

Zokir Khudoyberdiyev

24.11.2025

Author(s) Signature

Print Name

Date

2. The Copyright Owner (if different from the Author) must please sign here:

Name of Copyright Owner

Authorized Signature and Title

Date

3. If an Author is a U.S. Government employee, such Author must please sign below. The signing Author certifies that the Work was written as part of his/her official duties and is therefore not eligible for copyright protection in the United States.

Name of U.S. Government Institution (e.g., Naval Research Laboratory, NIST)

Author Signature

Print Name

Date

PLEASE NOTE: NATIONAL LABORATORIES THAT ARE SPONSORED BY U.S. GOVERNMENT AGENCIES BUT ARE INDEPENDENTLY RUN ARE NOT CONSIDERED GOVERNMENT INSTITUTIONS. (For example, Argonne, Brookhaven, Lawrence Livermore, Sandia, and others.) Authors at these types of institutions should sign under #1 or #2 above.

If the Work was authored under a U.S. Government contract, and the U.S. Government wishes to retain for itself and others acting on its behalf, a paid-up, nonexclusive, irrevocable, worldwide license in the Work to reproduce, prepare derivative works from, distribute copies to the public, perform publicly, and display publicly, by or on behalf of the Government, please check the box below and add the relevant Contract numbers.

Contract #(s) \_\_\_\_\_

[1.16.1]

## LICENSE TERMS DEFINED

**Accepted Manuscript (AM):** The final version of an author's manuscript that has been accepted for publication and incorporates all the editorial changes made to the manuscript after submission and peer review. The AM does not yet reflect any of the publisher's enhancements to the work such as copyediting, pagination, and other standard formatting.

**arXiv:** An electronic archive and distribution server for research article preprints in the fields of physics, mathematics, computer science, quantitative biology, quantitative finance, and statistics, which is owned and operated by Cornell University, <http://arxiv.org/>.

**Commercial and noncommercial scholarly use:** *Noncommercial* scholarly uses are those that further the research process for authors and researchers on an individual basis for their own personal purposes. They are author-to-author interactions meant for the exchange of ideas. *Commercial* uses fall outside the author-to-author exchange and include but are not limited to the copying or distribution of an article, either in hard copy form or electronically, for resale or licensing to a third party; posting of the AM or VOR of an article by a site or service where an access fee is charged or which is supported by commercial paid advertising or sponsorship; use by a for-profit entity for any type of promotional purpose. Commercial uses require the permission of AIP Publishing.

**Embargo period:** The period of time during which free access to the full text of an article is delayed.

**Employer's web page:** A web page on an employer's site that highlights the accomplishments and research interests of the company's employees, which usually includes their publications. (See also: Personal web page and Scholarly Collaboration Network).

**Exclusive License to Publish:** An exclusive license to publish is a written agreement in which the copyright owner gives the publisher exclusivity over certain inherent rights associated with the copyright in the work. Those rights include the right to reproduce the work, to distribute copies of the work, to perform and display the work publicly, and to authorize others to do the same. The publisher does not hold the copyright to the work, which continues to reside with the author. The terms of the AIP Publishing License to Publish encourage authors to make full use of their work and help them to comply with requirements imposed by employers, institutions, and funders.

**Full Credit Line:** AIP Publishing's preferred format for a credit line is as follows (you will need to insert the specific citation information in place of the capital letters): "Reproduced from [FULL CITATION], with the permission of AIP Publishing." A FULL CITATION would appear as: Journal abbreviation, volume number, article ID number or page number (year). For example: Appl. Phys. Lett. 107, 021102 (2015).

**Institutional repository:** A university or research institution's digital collection of articles that have been authored by its staff and which are usually made publicly accessible. As authors are encouraged and sometimes required to include their published articles in their institution's repository, the majority of publishers allow for deposit of the Accepted Manuscript for this purpose. AIP Publishing also allows for the VOR to be deposited 12 months after publication of the Work.

**Journal editorial office:** The contact point for authors concerning matters related to the publication of their manuscripts. Contact information for the journal editorial offices may be found on the journal websites under the "About" tab.

**Linking to the Version of Record (VOR):** To create a link to your article in an AIP Publishing journal or proceedings, you need to know the CrossRef digital object identifier (doi). You can find the doi on the article's abstract page. For instructions on linking, please refer to our Web Posting Guidelines at <https://publishing.aip.org/authors/web-posting-guidelines>.

**National Laboratories:** National laboratories are sponsored and funded by the U.S. Government but have independent nonprofit affiliations and employ private sector resources. These institutions are classified as Federally Funded Research and Development Centers (FFRDCs). Authors working at FFRDCs are not

considered U.S. Government employees for the purposes of copyright. The Master Government List of FFRDCs may be found at <http://www.nsf.gov/statistics/ffrclist/>.

**Personal web page:** A web page that is hosted by the author or the author's institution and is dedicated to the author's personal research interests and publication history. An author's profile page on a social media site or scholarly collaboration network site is *not* considered a personal web page. (See also: Scholarly Collaboration Network; Employer's web page).

**Peer X-Press:** A web-based manuscript submission system by which authors submit their manuscripts to AIP Publishing for publication, communicate with the editorial offices, and track the status of their submissions. The Peer X-Press system provides a fully electronic means of completing the License to Publish. A hard copy of the Agreement will be supplied by the editorial office if the author is unable to complete the electronic version of the form. (Conference Proceedings authors will continue to submit their manuscripts and forms directly to the Conference Editors.)

**Preprint:** A version of an author's manuscript intended for publication but that has not been peer reviewed and does not reflect any editorial input or publisher enhancements.

**Professional Publication Ethics:** AIP Publishing provides information on what it expects from authors in its "Statement of ethics and responsibilities of authors submitting to AIP Publishing journals" (<http://publishing.aip.org/authors/ethics>). AIP Publishing is also a member of the Committee on Publication Ethics (COPE) (<http://publicationethics.org/>), which provides numerous resources and guidelines for authors, editors, and publishers with regard to ethical standards and accepted practices in scientific publishing.

**Scholarly Collaboration Network (SCN):** Professional networking sites that facilitate collaboration among researchers as well as the sharing of data, results, and publications. SCNs include sites such as Academia.edu, ResearchGate, and Mendeley, among others.

**Supplementary Material:** Related material that has been judged by peer review as being relevant to the understanding of the article but that may be too lengthy or of too limited interest for inclusion in the article itself. Supplementary Material may include data tables or sets, appendixes, movie or audio clips, or other multimedia files.

**U.S. Government employees:** Authors working at Government organizations who author works as part of their official duties and who are not able to license rights to the Work, since no copyright exists. Government works are in the public domain within the United States.

**Version of Record (VOR):** The final published version of the article as it appears in the printed journal/proceedings or on the Scitation website. It incorporates all editorial input, is formatted in the publisher's standard style, and is usually viewed in PDF form.

**Waiver:** A request made to a university or institution to exempt an article from its open-access policy requirements. For example, a conflict will exist with any policy that requires the author to grant a nonexclusive license to the university or institution that enables it to license the Work to others. In all such cases, the Author must obtain a waiver, which shall be included in the manuscript file.

**Work:** The "Work" is considered all the material that comprises the article, including but not limited to the abstract, tables, figures, images, multimedia files that are directly embedded within the text, and the text itself. The Work does not include the Supplementary Material (see Supplementary Material above).

**Work Made for Hire:** Under copyright law, a work prepared by an employee within the scope of employment, or a work that has been specially ordered or commissioned for which the parties have agreed in writing to consider as a Work Made for Hire. The hiring party or employer is considered the author and owner of the copyright, not the person who creates the work.