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Solving The Bending Equation of A Rectangular Plate Using The Finite Difference Method

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Solving the Bending Equation of a Rectangular Plate Using the Finite Difference Method

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Abstract: This work addresses the problem of solving the bending equation of a rectangular plate using the finite difference method. Initially, the correctness of the finite difference method is verified. For this, the bending equation of a plate with all four sides clamped is solved using the finite difference method. The obtained solution is compared with the exact solution of the bending equation of a plate with all four sides clamped. Then, the bending equation of a plate with all four sides simply supported is solved using the finite difference method. In the third case, the bending equation of a plate with two parallel sides clamped and the other two parallel sides simply supported is solved using the finite difference method. In the final section, the solutions of the bending equation obtained for all three boundary conditions are compared. Conclusions are drawn based on the comparison results.

Keywords: plate, boundary condition, bending equation, finite difference, bending, thickness

INTRODUCTION

Currently, in various fields of technology, devices in the form of rectangular plates are widely used in a horizontal position. The edges of these plates can be clamped in different ways. Depending on how the edges are clamped, the plates exhibit different types of bending. In many cases, the calculation of such bending in plates is carried out based on classical theory [1]. Many researchers have worked on the development of classical theory, and this work continues to this day. The bending equation of a plate was derived by Sofi-German through classical theory. Only a few solutions to this bending equation have been obtained to date. With the advancement of technology, it has become easier to obtain approximate solutions to the bending equation using electronic computers [2-6], which ensures that greater attention is paid to these equations [6-11].

In this article, the bending equation of a plate is solved numerically using the finite difference method with the Maple mathematical software. The obtained solutions are compared with the analytical solution.

STATEMENT OF THE PROBLEMS (ISSUE)

We consider an elastic plate in three-dimensional space with one side of length a and the other side of length b. The thickness of the plate is h. The plate under consideration is a three-dimensional elastic body. As in [12–15], a rectangular Cartesian coordinate system 0xyz is introduced for the plate (see Fig. 1). The 0x and 0y coordinate axes are directed along the mid-surface that divides the plate's thickness into two equal parts.

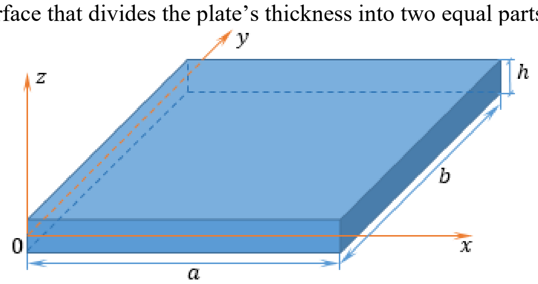

FIGURE 1. Schematic of the plate

We direct the Oz-axis upward, perpendicular to the Oxy-plane, along the height of the plate [6–11]. For the plate material, let the modulus of elasticity (Young's modulus) be E, and let the Poisson's ratio be ν. Since the rectangular plate under consideration is subjected only to bending, we use the Sophie–Germain bending equation derived in the classical theory. The deflection of the plate is denoted by $f(x,y)$.

The bending equation for the plate in the classical case is written as follows:

$$\frac{\partial^4 f(x,y)}{\partial x^4}+2\frac{\partial^4 f(x,y)}{\partial x^2 \partial y^2}+\frac{\partial^4 f(x,y)}{\partial y^4}=\frac{q(x,y)}{D} \quad (1)$$

Here, $f(x,y)$- is the bending function, $q(x,y)$ -is the distributed load applied to the outer surface of the plate, and D- is the flexural rigidity of the plate material.

$$D=\frac{Eh^3}{12(1-\nu^2)}$$

The edges of the plate can be hinged, rigidly fixed, or free. In solving practical problems, we consider a plate with edges connected as follows:

a) The plate has all four edges hinged:

In $x=0$ and $x=a$

$$f(0,y)=f(a,y)=0\,,\ \frac{\partial^2 f(0,y)}{\partial x^2}=\frac{\partial^2 f(a,y)}{\partial x^2}=0\,, \quad (2)$$

In $y=0$ and $y=b$

$$f(x,0)=f(x,b)=0\,,\ \frac{\partial^2 f(x,0)}{\partial y^2}=\frac{\partial^2 f(x,b)}{\partial y^2}=0\,. \quad (3)$$

b) The plate has all four edges rigidly clamped:

In $x=0$ and $x=a$

$$f(0,y)=f(a,y)=0\,,\ \frac{\partial f(0,y)}{\partial x}=\frac{\partial f(a,y)}{\partial x}=0\,, \quad (4)$$

In $y=0$ and $y=b$

$$f(x,0)=f(x,b)=0\,,\ \frac{\partial f(x,0)}{\partial y}=\frac{\partial f(x,b)}{\partial y}=0\,. \quad (5)$$

c) The plate has two parallel edges hinged and the other two parallel edges rigidly clamped:

In $x=0$ and $x=a$

$$f(0,y)=f(a,y)=0\,,\ \frac{\partial^2 f(0,y)}{\partial x^2}=\frac{\partial^2 f(a,y)}{\partial x^2}=0\,, \quad (6)$$

In $y=0$ and $y=b$

$$,\ \frac{\partial f(x,0)}{\partial y}=\frac{\partial f(x,b)}{\partial y}=0\,. \quad (7)$$

Thus, to solve the problem of plate bending, it is necessary to solve the bending equation (1) under one of the given boundary conditions.

SOLUTION OF THE PROBLEM

To solve the given problem, we use the finite difference method. To construct the finite difference scheme, $0 \le x \le a$, $0 \le y \le b$ the domain is covered by a rectangular grid with step sizes $h = \frac{1}{N}$ and $\tau = \frac{1}{M}$ along the x and y -coordinates, respectively. The points (x_i, t_j) ($i = 0,1,...,N$, $j = 0,1,...,M$) are referred to as the nodes of the grid. (x_i, t_0), (x_0, t_j), (x_N, t_j) va (x_i, t_M) the points located on the boundary of the grid. Now, at point (1), we approximate each term in the plate's bending equation using finite differences at point (x_i, t_j) :

$$\left(\frac{\partial^4 f(x,y)}{\partial x^4}\right)_{(x_i,t_j)} = \frac{f_{i-2,j} - 4f_{i-1,j} + 6f_{i,j} - 4f_{i+1,j} + f_{i+2,j}}{h^4}; \tag{8}$$

$$\left(\frac{\partial^4 f(x,y)}{\partial y^4}\right)_{(x_i,t_j)} = \frac{f_{i,j-2} - 4f_{i,j-1} + 6f_{i,j} - 4f_{i,j+1} + f_{i,j+2}}{\tau^4}; \tag{9}$$

$$\left(\frac{\partial^4 f(x,y)}{\partial x^2 \partial y^2}\right)_{(x_i,t_j)} = \frac{f_{i-1,j-1} - 2f_{i-1,j} + f_{i-1,j+1} - 2f_{i,j-1} + 4f_{i,j} - 2f_{i,j+1}}{\tau^2 h^2} + \\ + \frac{f_{i+1,j-1} - 2f_{i+1,j} + f_{i+1,j+1}}{\tau^2 h^2}; \tag{10}$$

Substituting expressions (8), (9), and (10) into the plate bending equation (1), we obtain the following:

$$f_{i-2,j} - \left(4 + \frac{4h^2}{\tau^2}\right) f_{i-1,j} + \left(6 + \frac{8h^2}{\tau^2} + \frac{6h^4}{\tau^4}\right) f_{i,j} - \left(4 + \frac{4h^2}{\tau^2}\right) f_{i+1,j} + f_{i+2,j} + \\ + \frac{2h^2}{\tau^2} f_{i+1,j+1} + \frac{2h^2}{\tau^2} f_{i+1,j-1} - \left(\frac{4h^2}{\tau^2} + \frac{4h^4}{\tau^4}\right) f_{i,j+1} - \left(\frac{4h^2}{\tau^2} + \frac{4h^4}{\tau^4}\right) f_{i,j-1} + \\ + \frac{2h^2}{\tau^2} f_{i-1,j+1} + \frac{2h^2}{\tau^2} f_{i-1,j-1} + \frac{h^4}{\tau^4} f_{i,j-2} + \frac{h^4}{\tau^4} f_{i,j+2} = \frac{q_{i,j} h^4}{D}; \tag{11}$$

First, we solve the bending equation for a plate with all four edges hinged, having geometric properties $a = 2m$, $b = 2m$, and $h = 0.05m$. Therefore, the boundary conditions (2) and (3) are replaced with finite difference equations

$$f(x,y) = f_{i,j}, \quad \frac{\partial^2 f(x,y)}{\partial x^2} = \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{h^2}; \tag{12}$$

$$f(x,y) = f_{i,j}, \quad \frac{\partial^2 f(x,y)}{\partial y^2} = \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{\tau^2}; \tag{13}$$

The plate material is aluminum. For aluminum, the modulus of elasticity $E = 7 \cdot 10^{10}\, Pa$ and the Poisson's ratio $\nu = 0.34$.

Thus, to solve the problem of bending for a plate with all four edges hinged, the system of equations (11), (12), and (13) is formulated to be solved simultaneously. We solve this system numerically using the "Maple" mathematical software package. To perform the practical calculations, we first cover the 0xy domain of the plate with a 4x4 step grid. In this case, the boundary conditions (2) and (3) are written as follows:

$$f_{0,1} = f_{0,2} = f_{0,3} = f_{0,4} = f_{4,0} = f_{4,1} = f_{4,2} = f_{4,3} = f_{4,4} = 0; \\ f_{1,0} = -f_{-1,0}, f_{1,1} = -f_{-1,1}, f_{1,2} = -f_{-1,2}, f_{1,3} = -f_{-1,3}, f_{1,4} = -f_{-1,4}; \\ f_{5,0} = -f_{3,0}, f_{5,1} = -f_{3,1}, f_{5,2} = -f_{3,2}, f_{5,3} = -f_{3,3}, f_{5,4} = -f_{5,3}. \tag{14}$$

$$f_{1,0} = f_{2,0} = f_{3,0} = f_{4,0} = f_{0,4} = f_{1,4} = f_{2,4} = f_{3,4} = f_{4,4} = 0;$$
$$f_{0,1} = -f_{0,-1}, f_{1,1} = -f_{1,-1}, f_{2,1} = -f_{2,-1}, f_{3,1} = -f_{3,-1}, f_{4,1} = -f_{4,-1}; \quad (15)$$
$$f_{0,5} = -f_{0,3}, f_{1,5} = -f_{1,3}, f_{2,5} = -f_{2,3}, f_{3,5} = -f_{3,3}, f_{4,5} = -f_{4,3}.$$

In equation (12), the indices i and j take values from 1 to 3 . As a result, equation (11) transforms into a system of nine equations [12]. The resulting system of equations, together with the boundary conditions (14) and (15), is solved using the Maple mathematical software package, yielding the following solutions:

TABLE 1. Deflections occurring at the nodes

$f_{0,4} = 0$	$f_{1,4} = 0$	$f_{2,4} = 0$	$f_{3,4} = 0$	$f_{4,4} = 0$
$f_{0,3} = 0$	$f_{1,3} = 87 \cdot 10^{-7}$	$f_{2,3} = 139 \cdot 10^{-7}$	$f_{3,3} = 87 \cdot 10^{-7}$	$f_{4,3} = 0$
$f_{0,2} = 0$	$f_{1,2} = 139 \cdot 10^{-7}$	$f_{2,2} = 243 \cdot 10^{-7}$	$f_{3,2} = 139 \cdot 10^{-7}$	$f_{4,2} = 0$
$f_{0,1} = 0$	$f_{1,1} = 87 \cdot 10^{-7}$	$f_{2,1} = 139 \cdot 10^{-7}$	$f_{3,1} = 87 \cdot 10^{-7}$	$f_{4,1} = 0$
$f_{0,0} = 0$	$f_{1,0} = 0$	$f_{2,0} = 0$	$f_{3,0} = 0$	$f_{4,0} = 0$

Similarly, by covering the plate domain Oxy with a six by six, eight by eight, fourteen by fourteen, and twenty by twenty steps we obtain the solutions. The solution obtained with the twenty by twenty grid is compared with the exact solution (Fig. 2).

As seen in Fig. 2, the approximate solution obtained using the finite difference method differs $0.15 \cdot 10^{-7}$ from the exact solution obtained by the analytical method for a plate with all four edges hinged. This shows that the approximate solution obtained through the finite difference method can be considered a reliable solution.

Now, we solve the problem of plate bending with all four edges rigidly clamped (noting that an analytical solution for such a problem does not exist). Similarly, we cover the plate's 0xy domain with four by four, six by six, eight by eight, fourteen by fourteen, and twenty by twenty step grids, and obtain the solutions. These solutions are represented in the following graphs (Fig. 3).

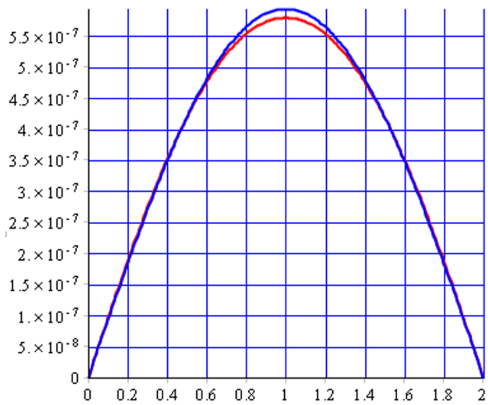

FIGURE 2. The difference between the exact solution and the approximate solution

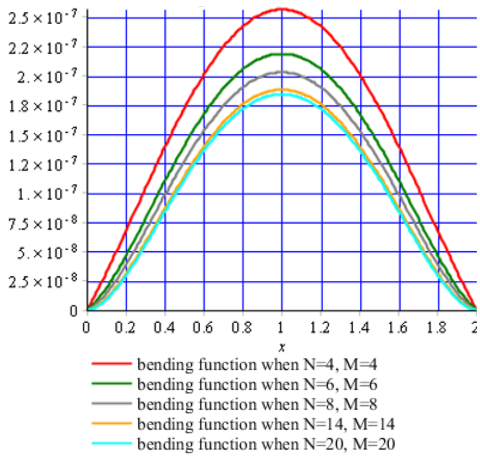

FIGURE 3. Bending of a plate with clamped edges

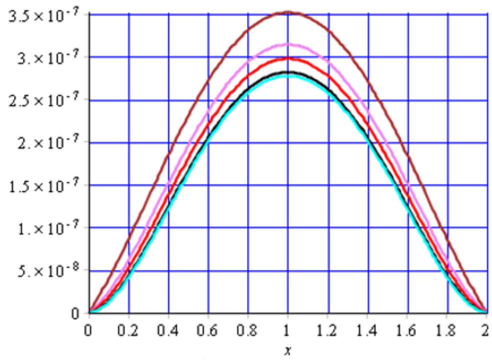

FIGURE 4. Bending of a plate with two parallel edges simply supported and the other two parallel edges rigidly clamped

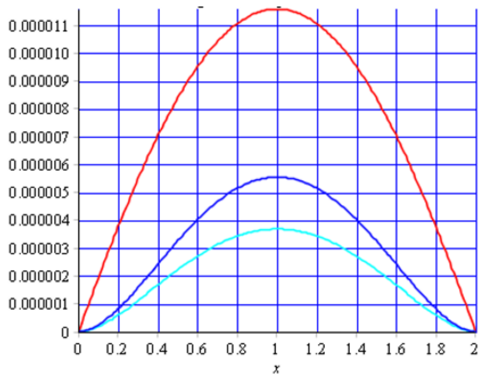

FIGURE 5. Graph of the comparison of bending in plates with differently clamped edges

As can be seen from Fig. 3, the solutions obtained by covering the plate domain Oxy with fourteen by fourteen and twenty by twenty step grids differ by $0.04 \cdot 10^{-7}$. As the number of grid steps increases, this difference becomes infinitesimally small.

When solving the problem of the bending of a plate with two parallel edges clamped and the other two parallel edges simply supported, we obtain the following solutions (Fig. 4):

Here too, as can be seen from Fig. 4, the solutions obtained by covering the plate domain Oxy with fourteen by fourteen and twenty by twenty step grids differ by $0.03 \cdot 10^{-7}$.

To verify the reliability of the solutions obtained using the finite difference method, we plot the solutions obtained by covering the Oxy domain with a twenty by twenty step grid for all three problems on a single graph (Fig. 5).

As seen in Fig. 5, when the dimensions and materials of the plate are the same but the boundary conditions differ, the bending behavior of the plate varies. Specifically, when the edges of the plate are hinged, the bending is larger, whereas when the edges are rigidly clamped, the bending is smaller. For a plate with two hinged edges and two rigidly clamped edges, the bending is between the two cases described above. This behavior is consistent with the physical meaning of the problem from a theoretical perspective.

CONCLUSION

The presented graphs allow the following conclusions to be drawn: the more rigidly the edges of the plate are clamped, the smaller the plate's bending will be. In the problems solved above, where the material and dimensions of the plate are unchanged, the bending of the plate with all four edges rigidly clamped differs from the bending of the plate with all four edges simply supported by $82 \cdot 10^{-7}$. This indicates that, for minimizing bending when using plates, it is advisable to clamp the edges rigidly.

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