Numerical Calculation of Torsional Vibrations During the Interaction of a Circular Cylindrical Shell With a Viscous Liquid

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Abstract. The article is devoted to the study of the propagation of a torsion wave in a cylindrical shell interacting with an internal liquid. In this case, the equations of the torsional oscillations of a cylindrical shell interacting with a viscous liquid, obtained in the authors' earlier works, were used as the equations of oscillations. The problems were solved numerically using an undisclosed scheme. The state of extensibility and deformability of sections of a cylindrical shell is determined. The influence of fluid flow and geometric parameters of the shell on the wave propagation process is investigated.

INTRODUCTION

It is known that questions of hydraulic elasticity are posed on the basis of dynamic and kinematic conditions on the contact surface of a solid and a liquid. In the mathematical formulation of problems on torsional vibrations of elements of engineering structures interacting with a viscous liquid, it is necessary to take into account the equality of torsional stresses on this surface, and also to use the Neve-Stokes equation for the movement of the liquid. However, a general solution to this equation has not yet been found. Therefore, the number of works devoted to the study of the influence of liquid on the torsional vibrations of round cylindrical shells and rods is small. In this sense [1,2] studies have examined the influence of a viscous fluid on the process of propagation of a vortex wave on a circular rod. The problem of flutter and divergence of rotational oscillations of a thin cylindrical shell interacting with a fluid moving in the direction of the axis was considered in [3]. Freely rotating vibrations of a cylindrical shell filled with a viscous fluid were studied in [4].

METHOD OF RESEARCH

Consider a cylindrical shell at rest, with inner radius a, outer radius a_1, length l, acting internally with a viscous fluid in a cylindrical coordinate system (r, θ, z). A kinematic impact, designated as $g(t)$, with an amplitude of z=0 is applied to the free end of the shell. Let the parameters of the stress-strain state of an arbitrary section of a round cylindrical shell in contact with a viscous liquid be determined depending on the spatial coordinate z and time t.

To solve the problem, we will use the system of equations for torsional vibrations of a round cylindrical two-layer shell interacting with internal flows of a viscous fluid, created in work [5]

$$\frac{a_1^2}{4a_2^2}\left[4+h_1\left(2a_1+h_1\right)\lambda_1+\frac{a_1^2h_1}{4}\left(2a_1+h_1\right)\lambda_1^2\right]\cdot\left\{\begin{array}{l}2\sum_{n=0}^{\infty}\frac{(a_2/2)^{2n+2}}{n!(n+2)!}\lambda_0^{2n+1}\tilde{u}_{\theta,0}^{(0)}+\\ \xi\left[\frac{1}{2}\lambda_0-\frac{2}{r_2^2}+\sum_{n=0}^{\infty}\eta_{2,n}(a_2)\frac{(a_2/2)^{2n+2}}{n!(n+2)!}\lambda_0^2\right]\tilde{u}_{\theta,1}^{(0)}\end{array}\right\}=\bar{M}_{\mu_1}^{-1}[f_{r\theta}^{(2)}(k,p)].$$

$$\begin{gathered}2\sum_{n=0}^{\infty}\lambda_0^{n+1}\left[\frac{a}{2}\lambda_0-\left(n+2\right)\frac{\mu' a}{6}M_{\mu_0}^{-1}q\delta_0\right]\frac{\left(a/2\right)^{2n+1}}{n!\left(n+2\right)!}u_{\theta,0}^{(0)}+\\ \xi\left[\frac{1}{2}\lambda_0-\frac{2}{a^2}-\frac{\mu'}{6}\bar{M}_{\mu_0}^{-1}q\delta_0+\sum_{n=0}^{\infty}\left[\frac{a}{2}\lambda_0^2\eta_{2,n}(a)-\left(n+2\right)\frac{\mu' a}{6}\bar{M}_{\mu_0}^{-1}\eta_{1,n}(a)q\delta_0\lambda_0\right]\frac{\left(a/2\right)^{2n+1}}{n!\left(n+2\right)!}\lambda_0^n\right]u_{\theta,1}^{(0)}=0\end{gathered}\tag{1}$$

where $\xi=\frac{r_1+r_2}{2}$ is the radius of the average surface of the shell; μ is the displacement coefficient; ρ is the density of the shell material; ρ_0' is the internal density of the liquid; μ' is the dynamic viscosity coefficient of the internal liquid. h is the thickness.

Let us limit ourselves if in the system of equations (1) only the first terms of the infinite series are equal to n=0 and if we say that a_1=a_2 is equal, then we get the following system of equations

$$\begin{gathered}\left[\frac{a_1^2}{4}\lambda_0 u_{\theta,0}^{(0)}+\xi\left(\frac{1}{2}(\lambda_0-\frac{4}{r_1^2})+\frac{a_1^2}{8}(\ln\frac{a_1}{\xi}-\frac{1}{4})\lambda_0^2\right)u_{\theta,1}^{(0)}\right]=\bar{M}_{\mu_1}^{-1}[f_{r\theta}^{(2)}(k,p)].\\ \left(\frac{a^2}{4}\lambda_0-\frac{\mu' a^2}{6}\hat{M}_{\mu_0}^{-1}q\delta_0\right)u_{\theta,0}^{(0)}+\xi\left(\frac{1}{2}(\lambda_0-\frac{4}{a^2})+\frac{a^2}{8}(\ln\frac{a}{\xi}-\frac{1}{4})\lambda_0^2-\frac{\mu'}{6}\bar{M}_{\mu_0}^{-1}q\delta_0-\frac{\mu' a^2}{12}\bar{M}_{\mu_0}^{-1}\ln\frac{a}{\xi}q\delta_0\lambda_0\right)u_{\theta,1}^{(0)}=0\end{gathered}\tag{2}$$

Assuming that these equations satisfy the conditions depending on the application domain [6], we discard the extremes that involve derivatives of an order of magnitude greater than two in time. We also assume that the crust is thin-walled. λ_m^n and δ_m^n $\left(m=0,1;\ n=1,2,3,...\right)$.

$$\begin{gathered}\lambda_m^n\left(\zeta\right)=\left[\frac{1}{b_m^2}M_m^{-1}\left(\frac{\partial^2\zeta}{\partial t^2}\right)-\frac{\partial^2\zeta}{\partial t^2}\right]^n,\\ q\delta_0=\left(\frac{1}{\nu'}\frac{\partial^2}{\partial t^2}-\frac{\partial^3}{\partial t\partial z^2}\right),m=0,1;\ n=0,1,2...\end{gathered}\tag{3}$$

$b_m=\sqrt{\mu_m/\rho_m}$ - the speed of propagation of a transverse wave in an elastic medium.

As a result, we obtain the following system of partial differential equations

$$\begin{gathered}\left\{\frac{a_1^2}{4}\left(1-N_0\left(\omega\right)\right)\left(\frac{1}{b_0^2}\frac{\partial^2}{\partial t^2}-\left(1-N_0\left(\omega\right)\right)\frac{\partial^2}{\partial z^2}\right)u_{\theta,0}^{(0)}+\xi\left[\frac{1}{2}\left(1-N_0\left(\omega\right)\right)\left(\frac{1}{b_0^2}\frac{\partial^2}{\partial t^2}-\left(1-N_0\left(\omega\right)\right)\frac{\partial^2}{\partial z^2}\right)-\right.\right.\\ \left.\left.-\frac{2}{a_1^2}\left(1-N_0\left(\omega\right)\right)^2+\frac{a_1^2}{8}\left(\ln\frac{a_1}{\xi}-\frac{1}{4}\right)\left(\frac{1}{b_0^2}\frac{\partial^2}{\partial t^2}-\left(1-N_0\left(\omega\right)\right)\frac{\partial^2}{\partial z^2}\right)^2\right]u_{\theta,1}^{(0)}\right\}=0\\ \left[\frac{a^2}{4}\left(1-N_0\left(\omega\right)\right)\left(\frac{1}{b_0^2}\frac{\partial^2}{\partial t^2}-\left(1-N_0\left(\omega\right)\right)\frac{\partial^2}{\partial z^2}\right)-\frac{\mu'}{\mu}\frac{a^2}{6}\left(1-N_0\left(\omega\right)\right)\left(\frac{1}{\nu'}\frac{\partial^2}{\partial t^2}-\frac{\partial^3}{\partial t\partial z^2}\right)\right]u_{\theta,0}^{(0)}+\\ +\xi\left[\frac{1}{2}\left(1-N_0\left(\omega\right)\right)\left(\frac{1}{b_0^2}\frac{\partial^2}{\partial t^2}-\left(1-N_0\left(\omega\right)\right)\frac{\partial^2}{\partial z^2}\right)-\frac{2}{a^2}\left(1-N_0\left(\omega\right)\right)^2+\frac{a^2}{8}\left(\ln\frac{a}{\xi}-\frac{1}{4}\right)\left(\frac{1}{b_0^2}\frac{\partial^2}{\partial t^2}-\left(1-N_0\left(\omega\right)\right)\frac{\partial^2}{\partial z^2}\right)^2-\right.\\ \left.-\frac{\mu'}{6\mu}\left(1-N_0\left(\omega\right)\right)\left(\frac{1}{\nu'}\frac{\partial^2}{\partial t^2}-\frac{\partial^3}{\partial t\partial z^2}\right)-\frac{\mu'}{\mu}\frac{a^2}{12}\ln\frac{a}{\xi}\left(\frac{1}{\nu'}\frac{\partial^2}{\partial t^2}-\frac{\partial^3}{\partial t\partial z^2}\right)\left(\frac{1}{b_0^2}\frac{\partial^2}{\partial t^2}-\left(1-N_0\left(\omega\right)\right)\frac{\partial^2}{\partial z^2}\right)\right]u_{\theta,1}^{(0)}=0\end{gathered}\tag{4}$$

In the problem we are considering, we consider the shell as thin-walled, and also move on to dimensionless variables using the formulas below [7,8]

$$t = t^* \frac{\xi}{b_0};z = z^* \xi; u_{\theta,1}^{(0)} = u_{\theta,1}^{*(0)}; u_{\theta,0}^{(0)} = \xi u_{\theta,0}^{*(0)}; a = \xi a^*;\; a_1 = \xi a_1^*; \mu' = \frac{\mu}{b_0^2}\mu'^* \tag{5}$$

As a result, we obtain the following system of partial differential equations

$$\begin{gathered}
\frac{1}{2}\left(1-N_0(\omega)\right)\left(1+\frac{a_1^2}{2}\right)\frac{\partial^2 u_{\theta,0}^{(0)}}{\partial t^2} - \frac{1}{2}\left(1-N_0(\omega)\right)\left(1+\frac{a_1^2}{2}\left(1-N_0(\omega)\right)\right)\frac{\partial^2 u_{\theta,0}^{(0)}}{\partial z^2} + \\
+\frac{1}{2}\left(1-N_0(\omega)\right)\frac{\partial^2 u_{\theta,1}^{(0)}}{\partial t^2} - \frac{1}{2}\left(1-N_0(\omega)\right)^2\frac{\partial^2 u_{\theta,1}^{(0)}}{\partial z^2} - \frac{2}{r_1^2}\left(1-N_0(\omega)\right)^2 u_{\theta,1}^{(0)} = 0 \\
\frac{a^2}{2}\left(1-N_0(\omega)\right)\left(\frac{1}{2}-\frac{1}{3}\rho_0'\right)\frac{\partial^2 u_{\theta,0}^{(0)}}{\partial t^2} - \frac{a^2}{6}\left(1-N_0(\omega)\right)\mu'\frac{\partial^3 u_{\theta,0}^{(0)}}{\partial t \partial z^2} - \frac{a^2}{4}\left(1-N_0(\omega)\right)^2\frac{\partial^2 u_{\theta,0}^{(0)}}{\partial z^2} + \\
+\left(1-N_0(\omega)\right)\left(\frac{1}{2}-\rho_0'\right)\frac{\partial^2 u_{\theta,1}^{(0)}}{\partial t^2} + \left(1-N_0(\omega)\right)\mu'\frac{\partial^3 u_{\theta,1}^{(0)}}{\partial t \partial z^2} - \frac{a^2}{4}\left(1-N_0(\omega)\right)^2\frac{\partial^2 u_{\theta,1}^{(0)}}{\partial z^2} = 0
\end{gathered} \tag{6}$$

$N_m(\omega)$ - viscoelastic material layer operator [9].

We use the obtained equations (6) to solve the problem

$$\begin{gathered}
a_1\frac{\partial^2 u_{\theta,0}^{(0)}}{\partial t^2} + a_2\frac{\partial^2 u_{\theta,0}^{(0)}}{\partial z^2} + a_3\frac{\partial^2 u_{\theta,1}^{(0)}}{\partial t^2} + a_4\frac{\partial^2 u_{\theta,1}^{(0)}}{\partial z^2} + a_5 u_{\theta,1}^{(0)} = 0 \\
b_1\frac{\partial^2 u_{\theta,0}^{(0)}}{\partial t^2} + b_2\frac{\partial^3 u_{\theta,0}^{(0)}}{\partial t \partial z^2} + b_3\frac{\partial^2 u_{\theta,0}^{(0)}}{\partial z^2} + b_4\frac{\partial^2 u_{\theta,1}^{(0)}}{\partial t^2} + b_5\frac{\partial^3 u_{\theta,1}^{(0)}}{\partial t \partial z^2} + b_6\frac{\partial^2 u_{\theta,1}^{(0)}}{\partial z^2} + b_7 u_{\theta,1}^{(0)} = 0
\end{gathered} \tag{7}$$

We get a system of equations. Here

$$a_1 = \frac{1}{2}(1-N_0(\omega))\left(1+\frac{a_1^2}{2}\right);\; a_2 = -\frac{1}{2}(1-N_0(\omega))\left(1+\frac{a_1^2}{2}(1-N_0(\omega))\right); a_3 = \frac{1}{2}(1-N_0(\omega)); a_4 = -\frac{1}{2}(1-N_0(\omega))^2; a_5 = -\frac{2}{a_1^2}(1-N_0(\omega))^2;$$

$$b_1 = \frac{a^2}{2}(1-N_0(\omega))\left(\frac{1}{2}-\frac{1}{3}\frac{\rho'}{\rho}\right); b_2 = -\frac{a^2}{4}(1-N_0(\omega))^2; b_3 = -\frac{a^2}{4}\frac{\mu'}{b}(1-N_0(\omega))^2; b_4 = (1-N_0(\omega))\left(\frac{1}{2}-\frac{\rho'}{\rho}\right)$$

$$b_5 = -\frac{1}{2}(1-N_0(\omega))^2; b_6 = -\frac{1}{2}\frac{\mu'}{b}(1-N_0(\omega))^2;\; b_7 = -\frac{2}{a^2}(1-N_0(\omega))^2;$$

Let us assume that one end of the shell is kinematically excited and the other end is tightly tightened. Then the boundary conditions [10]

$$\begin{gathered}
z = 0, \quad u_{\theta,0}^{(0)} = f(t, t_1) = \begin{cases} A\sin\left(\dfrac{\pi t}{t_1}\right), & t \le t_1/4; \\ 0, & t > t_1/4, \end{cases} \\
z = l, \quad u_{\theta,0}^{(0)} = 0.
\end{gathered} \tag{8}$$

will be in the field of view, where A is the amplitude, t_1 is the observation time, l is the dimensionless length. Let's assume that the initial conditions are zero, that is,

$$t = 0, \quad u_{\theta,0}^{(0)} = \frac{\partial u_{\theta,0}^{(0)}}{\partial t} = 0 \tag{9}$$

RESULTS AND DISCUSSION

We will determine the numerical solution of equation (7) of the problem under consideration using the mathematical package of the Maple-17 program, in which conditions (8) and (9) are specified. The calculation results showed that without taking into account the influence of the liquid, the equation of oscillations is reduced to the classical wave equation, therefore, for the shell, the thickness and dimensions of the average radius of the surface become less important. Therefore, we will present the results only for those cases where the internal liquid is taken into account. For the calculations, the physical, mechanical and geometric parameters of the shell material were selected, such as ρ = 7850 kg/m3, ν = 0.25, E=2·1011 Pa, l = 10 m. Varnish is obtained in the form of a viscous liquid inside the shell (kerosene, heavy oil). The ratio of the liquid density to the density of the shell material is determined as $\eta = \rho'/\rho$. The constructed graphs of the tortuous migration of points of shell U_θ are presented below in Figure 1 and Figure2.

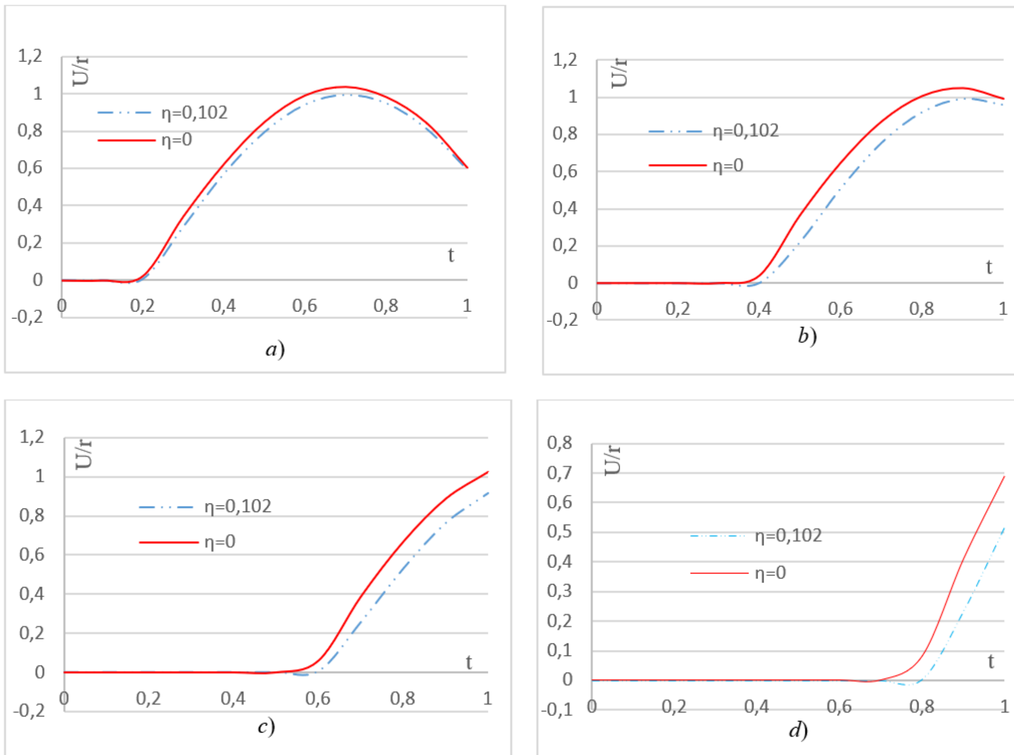

FIGURE 1. Graphs of time-dependent shifts of turns of $\eta = \rho/\rho'$ points (R = 0.02 m) at the edges of different sections of the shell a) z = 0.2, b) z = 0.4, c) z = 0.6, d) z = 0.8, for different values.

Figure 1 shows graphs of time-dependent changes in points of different sections (z = 0.2, 0.4, 0.6, 0.8) of a circular cylindrical shell interacting with a viscous liquid, at $\eta = \rho/\rho'$, i.e., with a viscous liquid, and at U_θ, not counting the viscous liquid. The graphs are sinusoidal in nature, regardless of the section they are in. It follows that the cylindrical is a consequence of the fact that the kinematic impact transmitted from one end of the shell is transmitted through a sinusoidal function. The displacement of the point of a circular cylindrical shell is explained by the fact that the excitations have a sinusoidal wave character, and the kinematic impact is a harmonic function. This means that the migration graph disappears over time. Without taking into account the influence of the viscous liquid, the displacement of the point of the cylindrical shell U_θ to its greatest value of 1.049004 in section z=0.4 (Figure 1b) reaches.

The presented Figure 2 shows graphs of coordinate-dependent changes in the displacements of turns U_θ in the presence of a viscous liquid and in the absence of liquid at time points (t = 0.2, 0.4, 0.6, 0.8). As can be seen, the presence of an external force leads to a significant increase in the torsion amplitude. As can be seen from the graphs (Figure2. a), b), c), d)) the changes in the migration torsion along the longitudinal coordinate are also sinusoidal. Since there is a viscous fluid flow inside the cylindrical shell, the amplitudes of the migration torsion are sharply reduced at any time, i.e. the fluid flow causes the oscillations to disappear.

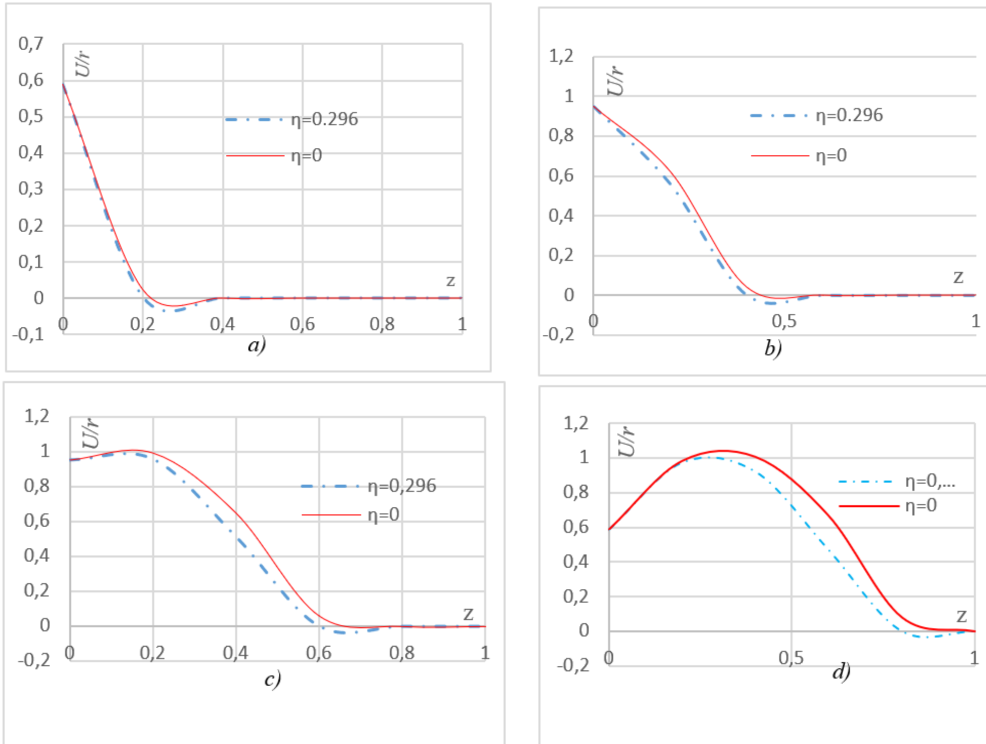

FIGURE 2. Graphs of coordinate-dependent variations of torsional U_θ displacements of points of different sections of the shell a) $t = 0.2$, b) $t = 0.4$, c) $t = 0.6$, d) $t = 0.8$, for different values of $\eta = \rho/\rho'$.

CONCLUSION

Thus, the work proposed an algorithm for a numerical solution using an undisclosed representation of the problem of torsion wave propagation in a circular cylindrical shell, the internal connection of which interacts with a fluid flow, and created a software tool. Migration graphs in the deformation-deformed state of circular cylindrical cross-sections of shells were constructed. The influence of the viscous fluid flow and geometric parameters of the shell on the wave propagation process was studied.

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