Formulation of a Boundary Value Problem for Longitudinal-Radial Vibrations of a Transversely Isotropic Cylindrical Shell Interacting With a Viscous Fluid

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Abstract. This article investigates longitudinal-radial vibrations of a transversely isotropic cylindrical shell subjected to unsteady viscous fluid action. The problem is considered in a cylindrical coordinate system, and the motion of the shell and fluid is modelled based on the corresponding differential equations. The vibration process is described dynamically, taking into account the physical and mechanical properties of the shell and fluid materials. The stress-strain state of the shell is determined by equations based on boundary and contact conditions. The results obtained serve as a theoretical basis that can be used in solving practical problems.

Keywords: boundary conditions, contact conditions, cylindrical shell, deformation, displacement and stress, longitudinal-radial vibration, transverse-isotropic, viscous fluid.

INTRODUCTION

Within the framework of elasticity theory, numerous studies have been conducted on the study of the deformed and vibration states of cylindrical shells. The deformation of transversely isotropic cylindrical shells under normal pressure was analyzed using the theory of anisotropic shells [1]. These researches utilized the equations of three-dimensional elasticity theory, identified some errors compared to classical theory, and proposed asymptotic solutions describing the stress-strain state of anisotropic shells.

The longitudinal vibrations of a cylindrical elastic shell filled with a viscous compressible fluid were studied in [2] and the general equations of longitudinal-radial vibrations were derived taking into account the dynamic interactions between the internal fluid and the shell medium. These results allow us to analyze the coordinated motions between the cylindrical shell and the fluid.

Bango and Guz [3] studied the propagation of waves in the interaction of a compressible viscous fluid with a pre-deformed elastic layer and proposed general solutions based on the three-dimensional linear theory of elasticity and the Navier-Stokes equations. Their work is of great importance in the analysis of wave processes in hydroelastic systems.

Also, [4–6] consider the problems of determining the flow velocity, pressure distribution, and waveforms propagating along the surface of elastic cylindrical shells filled with viscous fluid. In [7,8], based on the theory of potential flow, an analysis of the pressure of a viscous fluid is carried out, and using the Galerkin method and the contour method, the influence of fluid viscosity and rotational inertia on the stability of the shell is evaluated.

Research on torsional and longitudinal-radial vibrations of viscoelastic shells is presented in [9–12], where the dynamic properties of multilayer shells are analyzed based on the Boltzmann–Volterra integral model. Within the framework of these works, general solutions have been developed that lead to refined equations of the Timoshenko type. The results of the study show that the parameters of viscoelastic material have a significant effect on vibration frequencies and energy dissipation. Changes in the thickness and elasticity modules of the layers are also evaluated as the main factor determining the dynamic priority of the structure and the nature of the vibrations. These scientific results provide a theoretical basis for improving the vibration resistance of shell systems and preventing resonance phenomena.

In the special case of cylindrical shells, the interaction of conical shells with the environment and axisymmetric vibrations have been analyzed [13,14]. This paper investigates the influence of the geometric parameters of the shell and the elastic properties of the external environment on the natural frequencies. The results obtained serve as an important scientific basis for assessing the priority of cylindrical structures and their effective design.

The paper considers mathematical modeling of mechanical problems [15]. In this case, it can be used to study the interaction of a cylindrical shell with an elastic base. In general, the literature analyses the issues of vibration, deformation, and priority of cylindrical shell systems filled with viscous fluid based on various theoretical approaches, and the results obtained provide a scientific basis for a more in-depth study of longitudinal-radial vibrations in the interaction of a cylindrical shell with a viscous fluid.

Cylindrical shells interacting with liquids are widely used in many areas of modern technology, including aerospace structures, energy systems, hydraulic structures, oil and gas pipelines, and biotechnical structures. During the operation of such systems, it is very important to study their dynamic properties in detail, especially to determine oscillatory processes and ensure their stability and rigidity. Therefore, the analysis of the oscillatory states of shells interacting with liquids is one of the most relevant areas of research in the mechanics of deformable solids.

Previous studies have often examined the interaction of shells made of isotropic materials with ideal or incompressible fluids. However, many real technical materials, such as composite or layered structures, have transversely isotropic properties. Since their elastic parameters vary with direction, an approach based on the theory of anisotropic materials is required for accurate modeling of oscillatory processes in this case. At the same time, the viscosity of the fluid also has a significant effect on the dynamics of the system, since viscous forces cause energy dissipation and changes in oscillation frequencies..

This paper presents a mathematical model of a boundary value problem for longitudinal-radial vibrations of a transversely isotropic cylindrical shell interacting with a viscous fluid. The structure of the problem is based on a joint consideration of the equations of elastic dynamics of the shell and the Navier–Stokes equations of the fluid. Conditions of continuity of surface motion and forces between the shell and the fluid are introduced, and complete boundary value problems are formulated. This approach allows for a more realistic modelling of the physical properties of the fluid-shell system vibrations and provides a theoretical basis for the analysis of complex problems in applied mechanics.

RESEARCH METHOD

The problems of unsteady vibrations of transversely isotropic cylindrical layers and shells interacting with an internal viscous fluid are formulated. For this purpose, the types of vibrations of cylindrical transversely isotropic layers and shells are given. For these, the basic equations of elasticity theory and expressions for internal and external forces acting on the surface of isotropic cylindrical layers and shells are given. Problems concerning longitudinal-radial vibrations of circular cylindrical transversely isotropic layers and shells are formulated.

In the case of longitudinal-radial vibrations of a circular transversely isotropic cylindrical shell, if the components of the displacement vector U_r and U_z are non-zero, then the components of the stress tensor $\sigma_{rr}, \sigma_{zz}, \sigma_{\theta\theta}, \tau_{rz}$ are also non-zero.

Consequently, the equations of motion for a circular transversally isotropic cylindrical shell are as follows:

$$\begin{cases} \dfrac{\partial \sigma_{rr}}{\partial r} + \dfrac{\partial \sigma_{rz}}{\partial z} + \dfrac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = \rho \dfrac{\partial^2 U_r}{\partial t^2}; \\ \dfrac{\partial \sigma_{rz}}{\partial r} + \dfrac{\partial \sigma_{zz}}{\partial z} + \dfrac{\sigma_{rz}}{r} = \rho \dfrac{\partial^2 U_z}{\partial t^2}; \end{cases} \quad (r_1 \le r \le r_2). \tag{1}$$

The components of the strain tensor are written as follows, using the non-zero components of the displacement vector:

$$\varepsilon_{rr} = \frac{\partial U}{\partial r}; \ \varepsilon_{\theta\theta} = \frac{U}{r}; \ \varepsilon_{zz} = \frac{\partial W}{\partial z}; \ \gamma_{zr} = \frac{\partial W}{\partial r} + \frac{\partial U}{\partial z} \tag{2}$$

Longitudinal-radial vibrations of a transversely isotropic cylindrical shell arise under the action of forces $f_r(z,t)$ and $f_z(z,t)$ applied to its inner and outer surfaces. In this case, the boundary conditions are as follows:

a) when $r = r_1$

$$\sigma_{zz}(r,z,t)\Big|_{r=r_1} = -p_{zz}(r,z,t)\Big|_{r=r_1}; \quad \sigma_{rr}(r,z,t)\Big|_{r=r_1} = -p_{rr}(r,z,t)\Big|_{r=r_1}. \tag{3}$$

b) when $r = r_2$

$$\sigma_{zz}(r,z,t)\Big|_{r=r_2} = f_z(z,t)\Big|_{r=r_2}; \quad \sigma_{rz}(r,z,t)\Big|_{r=r_2} = f_z(z,t)\Big|_{r=r_2}. \tag{4}$$

c) the initial conditions are assumed to be zero.

In the case of longitudinal-radial vibrations of a circular transversely isotropic cylindrical shell, the equations of motion are solved together with the boundary conditions, and a system of vibration equations is compiled.

Using the above boundary and initial conditions, we derive the general equation for longitudinal-radial vibrations of a circular cylindrical transversely isotropic shell. The components of the stress tensor of a cylindrical body in an axisymmetric coordinate system can be written as.

$$\sigma_{rr} = C_{11}\frac{\partial U_r}{\partial r} + C_{12}\frac{U_r}{r} + C_{13}\frac{\partial U_z}{\partial z};\ \sigma_{\theta\theta} = C_{12}\frac{\partial U_r}{\partial r} + C_{11}\frac{U_r}{r} + C_{13}\frac{\partial U_z}{\partial z};\ \sigma_{zz} = C_{13}\left(\frac{\partial U_r}{\partial r} + \frac{U_r}{r}\right) + C_{33}\frac{\partial U_z}{\partial z}; \tag{5}$$

here

$$C_{ij}(\zeta) = a_{ij}\left[\zeta(t) - \int_0^t f_{ij}(t-\xi)\zeta(\xi)d\xi\right];$$

$f_{ij}(t)$ Kernels of elastic-flexible operators satisfying the condition of integration over t time; a_{ij} is elastic constants of the material.

Let us substitute the expressions for stresses (5) into the equations of motion (1) given above

$$C_{11}\frac{\partial^2 U_r}{\partial r^2} - \frac{C_{12}}{r^2}U_r + \frac{C_{12}}{r}\frac{\partial U_r}{\partial r} + C_{13}\frac{\partial^2 U_z}{\partial r\partial z} + C_{44}\left(\frac{\partial^2 U_r}{\partial^2 z} + \frac{\partial^2 U_z}{\partial r\partial z}\right) + \frac{C_{11}-C_{12}}{r}\frac{\partial U_r}{\partial r} + \frac{C_{12}-C_{11}}{r^2}U_r = \rho\frac{\partial^2 U_r}{\partial t^2};$$
$$C_{44}\left(\frac{\partial^2 U_r}{\partial r\partial z} + \frac{\partial^2 U_z}{\partial r^2}\right) + C_{13}\left(\frac{\partial^2 U_r}{\partial z\partial r} + \frac{1}{r}\frac{\partial U_r}{\partial r}\right) + C_{33}\frac{\partial^2 U_z}{\partial z^2} + C_{44}\left(\frac{1}{r}\frac{\partial U_r}{\partial z} + \frac{1}{r}\frac{\partial U_z}{\partial r}\right) = \rho\frac{\partial^2 U_z}{\partial t^2}; \tag{6}$$

Describe the components of the displacement vector as follows:

$$U_r = \int_0^\infty \begin{Bmatrix}\sin kz\\ -\cos kz\end{Bmatrix} dk \int_{(l)} \tilde{U}_r e^{pt} dp; \quad U_z = \int_0^\infty \begin{Bmatrix}\cos kz\\ \sin kz\end{Bmatrix} dk \int_{(l)} \tilde{U}_z e^{pt} dp. \tag{7}$$

Let us substitute expressions (7) into the system of differential equations of motion (6).

$$\tilde{C}_{11}\left(\frac{\partial^2 \tilde{U}_r}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde{U}_r}{\partial r} - \frac{1}{r^2}\tilde{U}_r\right) + \tilde{C}_{44}k^2\tilde{U} - k\left(\tilde{C}_{13} + \tilde{C}_{44}\right)\frac{\partial^2 \tilde{U}_z}{\partial r} = \rho p^2 \tilde{U}_r;$$
$$\tilde{C}_{44}\left(\frac{\partial^2 \tilde{U}_z}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde{U}_z}{\partial r}\right) - k^2\tilde{C}_{33}\tilde{U}_z - k\left(\tilde{C}_{44} + \tilde{C}_{13}\right)\left(\frac{\partial \tilde{U}_r}{\partial r} + \frac{\tilde{U}}{r}\right) = \rho p^2 \tilde{U}_z; \tag{8}$$

The second member of the system of equations (8) can be written as follows

$$\tilde{\Delta}_0\tilde{U}_r + \left(\tilde{C}_{44}\tilde{C}_{11}^{-1}k^2 - \rho p^2\tilde{C}_{11}^{-1}\right)\tilde{U}_r - k\tilde{B}_1\frac{\partial \tilde{U}_z}{\partial r} = 0;$$
$$\tilde{\Delta}_0\frac{\partial \tilde{U}_z}{\partial r} - \left(k^2\tilde{C}_{33}\tilde{C}_{44}^{-1} + \rho p^2\tilde{C}_{44}^{-1}\right)\frac{\partial \tilde{U}_z}{\partial r} - k\tilde{B}_2\tilde{\Delta}_0\tilde{U}_r = 0; \tag{9}$$

Here

$$\tilde{\Delta}_0 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}; \quad \tilde{B}_1 = \left(\tilde{C}_{13} + \tilde{C}_{44}\right)\tilde{C}_{11}^{-1}; \quad \tilde{B}_2 = \left(\tilde{C}_{13} + \tilde{C}_{44}\right)\tilde{C}_{44}^{-1};$$

As above, we can form expressions for the interchangeable migrations of $\tilde{U}_r$ and $\tilde{U}_z$ as follows:

$$\tilde{U}_z=\frac{1}{\alpha^2 k\tilde{B}_1}\sum_{n=0}^{\infty}\left\{k\tilde{B}_1\alpha_1^2\alpha_2^2\left(\tilde{P}_n-\alpha^2\tilde{P}_{n-1}\right)\tilde{U}_{z,0}-\left(\alpha_1^2-\alpha^2\right)\left(\alpha_2^2-\alpha^2\right)\tilde{P}_n\tilde{U}_{r,0}\right\}\frac{\left(r/2\right)^{2n}}{\left(n!\right)^2}+$$
$$+\frac{\xi}{k\tilde{B}_1}\sum_{n=0}^{\infty}\eta_{6,n}\left(r\right)\left[k\tilde{B}_1\left(\tilde{P}_{n+1}-\alpha^2\tilde{P}_n\right)\tilde{U}_{z,1}\right]-\left(\alpha_1^2-\alpha^2\right)\left(\alpha_2^2-\alpha^2\right)P_nU_{r,1}\left]\frac{\left(r/2\right)^{2n}}{\left(n!\right)^2}\right.; \quad (10)$$
$$\tilde{U}_r=\frac{\xi}{r}\tilde{U}_{r,1}+\frac{1}{\alpha^2}\sum_{n=0}^{\infty}\left[\left(\alpha^2\tilde{P}_{n+1}-\alpha_1^2\alpha_2^2\tilde{P}_n\right)\tilde{U}_{r,0}+k\tilde{B}_1\alpha_1^2\alpha_2^2\tilde{P}_n\tilde{U}_{z,0}\right]\frac{\left(r/2\right)^{2n+1}}{\left(n!\right)\left(n+1\right)!}+$$
$$+\xi\sum_{n=0}^{\infty}\eta_{7,n}\left(r\right)\left[\left(\alpha^2\tilde{P}_{n+1}-\alpha_1^2\alpha_2^2\tilde{P}_n\right)\tilde{U}_{r,1}+k\tilde{B}_1\tilde{P}_{n+1}\tilde{U}_{z,1}\right]\frac{\left(r/2\right)^{2n+1}}{n!\left(n+1\right)!}.$$

Here

$$\tilde{P}_n=\sum_{i=0}^{n-1}\alpha_2^{2(n-i-1)}\alpha_1^{2i};\ \ \tilde{P}_0\equiv 0;\ \ \tilde{P}_1\equiv 1;\ \ \tilde{P}_2=\alpha_1^2+\alpha_2^2;\ \ \alpha_1^2\alpha_2^2P_{-1}^2=-1.$$

Let us express the stresses through the components $\tilde{U}_z$ and $\tilde{U}_r$ of this displacement vector, for which we apply operators $\lambda,\ \lambda_1^2,\ \lambda_2$ to (8) and (10):

$$\lambda=C_{11}^{-1}\left(\rho\frac{\partial^2}{\partial t^2}-C_{44}\frac{\partial^2}{\partial z^2}\right);$$
$$\lambda_1^2=C_{11}^{-1}\left(\rho^2C_{44}^{-1}\frac{\partial^4}{\partial t^4}-\rho\left(1+C_{33}C_{44}^{-1}\right)\frac{\partial^4}{\partial t^2\partial z^2}+C_{33}\frac{\partial^4}{\partial z^4}\right); \quad (11)$$
$$\lambda_2=\rho\left(C_{11}^{-1}+C_{44}^{-1}\right)\frac{\partial^2}{\partial t^2}-\left(C_{44}C_{11}^{-1}+C_{44}^{-1}C_{33}+C_{44}^{-1}C_{11}^{-1}\left(C_{13}+C_{44}\right)^2\right)\frac{\partial^2}{\partial z^2};$$

(11) performing mathematical simplifications and introducing definitions using action operators, we write it in the following form,

$$A_{11}\frac{\partial U_{z,0}}{\partial z}+B_{11}U_{r,0}+N_{11}\frac{\partial U_{z,1}}{\partial z}+M_{11}U_{r,1}=S_{11}f_r(z,t);\ A_{21}\frac{\partial U_{z,0}}{\partial z}+B_{21}U_{r,0}+N_{21}\frac{\partial U_{z,1}}{\partial z}+M_{21}U_{r,1}=S_{21}f_{rz}(z,t);$$
$$A_{31}\frac{\partial U_{z,0}}{\partial z}+B_{31}U_{r,0}+N_{31}\frac{\partial U_{z,1}}{\partial z}+M_{31}U_{r,1}=S_{31}p_s;\ \ A_{41}\frac{\partial U_{z,0}}{\partial z}+B_{41}U_{r,0}+N_{41}\frac{\partial U_{z,1}}{\partial z}+U_{r,1}=0; \quad (12)$$

the following definitions are included here

$$A_{11}=a_{11}\frac{\partial^4}{\partial t^4}+a_{12}\frac{\partial^4}{\partial t^2\partial z^2}+a_{13}\frac{\partial^4}{\partial z^4}+a_{14}\frac{\partial^2}{\partial t^2}+a_{15}\frac{\partial^2}{\partial z^2},\ \ B_{11}=b_{11}\frac{\partial^4}{\partial t^4}+b_{12}\frac{\partial^4}{\partial t^2\partial z^2}+b_{13}\frac{\partial^4}{\partial z^4}+b_{14}\frac{\partial^2}{\partial t^2}+b_{15}\frac{\partial^2}{\partial z^2},$$
$$N_{11}=n_{11}\frac{\partial^4}{\partial t^4}+n_{12}\frac{\partial^4}{\partial t^2\partial z^2}+n_{13}\frac{\partial^4}{\partial z^4}+n_{14}\frac{\partial^2}{\partial t^2}+n_{15}\frac{\partial^2}{\partial z^2},\ \ M_{11}=m_{11}\frac{\partial^4}{\partial t^4}+m_{12}\frac{\partial^4}{\partial t^2\partial z^2}+m_{13}\frac{\partial^4}{\partial z^4}+m_{14}\frac{\partial^2}{\partial t^2}+m_{15}\frac{\partial^2}{\partial z^2},$$
$$S_{11}=s_{11}\frac{\partial^2}{\partial t^2}+s_{12}\frac{\partial^2}{\partial z^2},\ A_{21}=a_{21}\frac{\partial^4}{\partial t^4}+a_{22}\frac{\partial^4}{\partial t^2\partial z^2}+a_{23}\frac{\partial^4}{\partial z^4},\ \ B_{21}=b_{21}\frac{\partial^4}{\partial t^2\partial z^2}+b_{22}\frac{\partial^4}{\partial z^4},$$
$$N_{21}=n_{21}\frac{\partial^4}{\partial t^4}+n_{22}\frac{\partial^4}{\partial t^2\partial z^2}+n_{23}\frac{\partial^4}{\partial z^4},\ M_{21}=m_{21}\frac{\partial^4}{\partial t^4}+m_{22}\frac{\partial^4}{\partial t^2\partial z^2}+m_{23}\frac{\partial^4}{\partial z^4},\ S_{21}=s_{21}\frac{\partial^3}{\partial z\partial t^2}+s_{22}\frac{\partial^3}{\partial z^3},$$
$$A_{31}=a_{31}\frac{\partial^4}{\partial t^4}+a_{32}\frac{\partial^4}{\partial t^2\partial z^2}+a_{33}\frac{\partial^4}{\partial z^4}+a_{34}\frac{\partial^3}{\partial t^3}+a_{35}\frac{\partial^3}{\partial t\partial z^2}+a_{36}\frac{\partial^2}{\partial t^2}+a_{37}\frac{\partial^2}{\partial z^2},$$
$$B_{31}=b_{31}\frac{\partial^4}{\partial t^4}+b_{32}\frac{\partial^4}{\partial t^2\partial z^2}+b_{33}\frac{\partial^4}{\partial z^4}+b_{34}\frac{\partial^2}{\partial t^2}+b_{35}\frac{\partial^2}{\partial z^2},$$
$$N_{31}=n_{31}\frac{\partial^4}{\partial t^4}+n_{32}\frac{\partial^4}{\partial t^2\partial z^2}+n_{33}\frac{\partial^4}{\partial z^4}+n_{34}\frac{\partial^3}{\partial t^3}+n_{35}\frac{\partial^3}{\partial t\partial z^2}+n_{36}\frac{\partial^2}{\partial t^2}+n_{37}\frac{\partial^2}{\partial z^2},$$
$$M_{31}=m_{31}\frac{\partial^4}{\partial t^4}+m_{32}\frac{\partial^4}{\partial t^2\partial z^2}+m_{33}\frac{\partial^4}{\partial z^4}+m_{34}\frac{\partial^2}{\partial t^2}+m_{35}\frac{\partial^2}{\partial z^2},\ \ S_{31}=s_{31}\frac{\partial^2}{\partial t^2}+s_{32}\frac{\partial^2}{\partial z^2},$$

$$A_{41} = a_{41}\frac{\partial^4}{\partial t^4} + a_{42}\frac{\partial^4}{\partial t^2 \partial z^2} + a_{43}\frac{\partial^4}{\partial z^4},\quad B_{41} = b_{41}\frac{\partial^4}{\partial t^2 \partial z^2} + b_{42}\frac{\partial^4}{\partial z^4},\quad N_{41} = n_{41}\frac{\partial^4}{\partial t^4} + n_{42}\frac{\partial^4}{\partial t^2 \partial z^2} + n_{43}\frac{\partial^4}{\partial z^4},$$

$$M_{41} = m_{41}\frac{\partial^4}{\partial t^4} + m_{42}\frac{\partial^4}{\partial t^2 \partial z^2} + m_{43}\frac{\partial^4}{\partial z^4}.$$

In these definitions, $a_{ij}, b_{ij}, n_{ij}, m_{ij}$ depends on quantities that depend on the geometric and physical parameters of the cylindrical shell under consideration [16]. By solving this system of equations (12) using the necessary functions of type $U_{r,0}, U_{z,0}, U_{r,1}$ and $U_{z,1}$, it is possible to find the displacements and stresses arising at the points of their cross-sections during unsteady oscillations of circular transversally isotropic cylindrical layers and shells interacting with an internal viscous fluid.

RESULTS ANALYSIS

To solve the system of equations (12), use the finite difference method. The geometric dimensions of the circular cylindrical shell for solving the system of equations using the finite difference method in Maple are as follows: $l = 1m,\ r_1 = 0.2\ m,\ r_1 = 0.128\ m.$ Let us compare the displacements that occur in them under the action of torque when the material of a transversely isotropic cylindrical shell interacting with a viscous fluid (zinc $\rho = 7140\ kg/m^3$, $C_{11} = 1.583 \cdot 10^{11}\ N/m^2$, $C_{12} = 0.315 \cdot 10^{11}\ N/m^2$, $C_{13} = 0.474 \cdot 10^{11}\ N/m^2$, $C_{33} = 0.616 \cdot 10^{11}\ N/m^2$, $C_{44} = 0.40 \cdot 10^{11}\ N/m^2$, graphite epoxy $\rho = 1700\ kg/m^3$, $C_{11} = 0.139 \cdot 10^{11}\ N/m^2$, $C_{12} = 0.064 \cdot 10^{11}\ N/m^2$, $C_{13} = 0.064 \cdot 10^{11}\ N/m^2$, $C_{33} = 1.160 \cdot 10^{11}\ N/m^2$, $C_{44} = 0.070 \cdot 10^{11}\ N/m^2$) changes (Fig. 1 and Fig. 2). We compare the displacements arising in them under the action of torque when changing the material of a transversely isotropic cylindrical shell interacting with a viscous fluid (Fig. 1 and Fig. 2). Using the solutions of the obtained system of equations for longitudinal-radial vibrations of a circular cylindrical shell, we construct graphs of the change in the components of the displacement vector U_z and U_r as a function of the coordinate z.

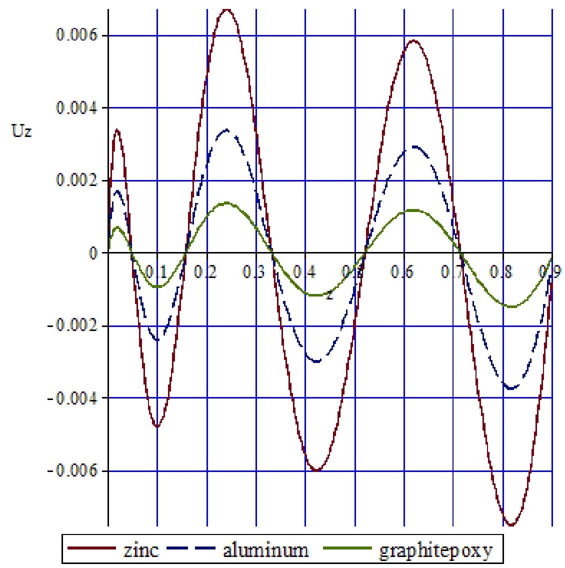

FIGURE 1. Graph showing the change in the displacement vector component U_z as a function of the *z* coordinate when the material of the cylindrical shell changes.

Figure 1 shows a graph of the change in the displacement vector component U_z as a function of coordinate z for shell materials – zinc, aluminium and graphite epoxy resin – under a torque $10 \cdot 10^3\ N \cdot m$ applied to the end face of a circular cylindrical shell interacting with an internal viscous fluid.

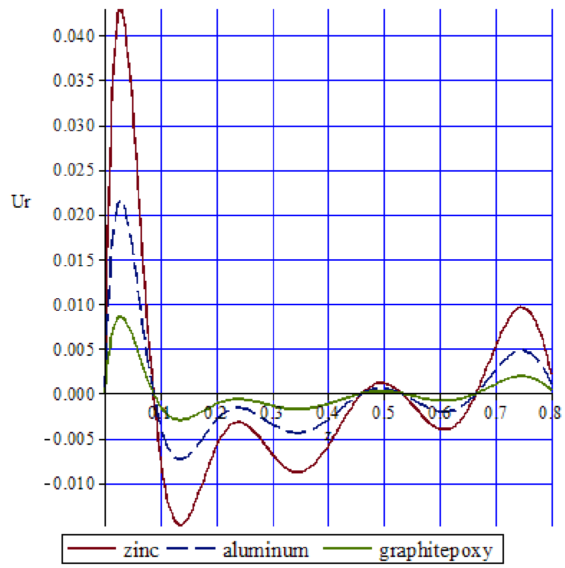

FIGURE 2. Graph showing the change in the displacement vector component U_r as a function of the *z* coordinate when the material of the cylindrical shell changes.

Figure 2 shows a graph of the change in the displacement vector component as a function of coordinate z when torque U_r is applied to the end face of a circular cylindrical shell interacting with an internal viscous fluid $10 \cdot 10^3 \, N \cdot m$ for shell materials: zinc, aluminium, graphite epoxy resin.

CONCLUSION

This paper develops a mathematical model of the boundary value problem for longitudinal-radial vibrations of a transversely isotropic cylindrical shell interacting with a viscous fluid. The problem was considered in a cylindrical coordinate system and modeled based on the elastic theory of shell and fluid motion and the Navier-Stokes equations. A complete boundary value problem is formulated, taking into account the conditions of surface adhesion between the shell and the fluid. By solving the system of equations (12), the displacements and stress states of the shell are determined, and the dynamic properties of a circular cylindrical layer under the influence of an internal viscous fluid are analyzed. The calculations used the finite difference method and the Maple mathematical package, and a comparison was made of the amplitudes of vibrations of shells made of different materials – zinc, aluminum and graphite epoxy.

The results show that the shell material and fluid viscosity have a significant effect on the dynamic properties of the system. In particular, as the elastic modulus of the material increases, the displacement decreases, while viscosity weakens the amplitude of oscillations and increases energy dissipation. The results of the study serve as an important theoretical basis for determining the vibration processes of cylindrical shells interacting with viscous fluids, assessing their priority, and effectively designing structures used in aerospace, hydraulic, and energy engineering.

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