3rd International Conference
Advanced Mechanics: Structure, Materials, Tribology

Design of Strengthening Statically Indeterminate Structures

AIPCP25-CF-AMSMT2025-00049 | Article

Design of Strengthening Statically Indeterminate Structures

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Abstract: This article presents the results of studies obtained from the perspective of the engineering theory of bending for calculating the index of the degree of change in the initial load-bearing capacity of a structure and its element, equal to the ratio of the load-bearing capacity after repair or reinforcement to the load-bearing capacity before the occurrence of damage, as applied to statically indeterminate structures.

Keywords: Bearing capacity recovery index, displacement, frame structures, overpasses pile structures, rotation angle, rigidity, stress.

INTRODUCTION

A large number of previously constructed pile-supported overpass structures on foundations that deform over time have suffered significant damage due to the rheological properties of the soil [4, 7, 8]. Therefore, strengthening them is a pressing issue. This raises the question of the load-bearing capacity of the structures after their strengthening.

Strengthening of structures in operation can be carried out with the aim of restoring their original load-bearing capacity or durability, partially lost due to damage or weakening of structural elements, as well as with the aim of increasing their load-bearing capacity or improving other qualities to create the possibility of more intensive operation.

It is known [3, 5] that a generalized characteristic of the effectiveness of the repair or reinforcement performed is the indicator of the ratio of the change in the initial bearing capacity of the structure or its element θ, equal to the ratio of the bearing capacity after repair or reinforcement S_{res}, S_{rein} to the bearing capacity before the occurrence of damage S:

$$\theta = S_{res} / S_{rein} \text{ or } \theta = S_{rein} / S \quad (1)$$

In each specific case, the values S_{res}, S_{rein} and S are understood to mean those forces in the structural elements that they are designed to withstand (for bending elements these are bending moments, for stretched and compressed elements – longitudinal forces, etc.).

The first studies in the field of strengthening mechanics for port structures were undertaken by A. Y. Budin from the perspective of the technical theory of bending. This problem was further developed in the works of M. V. Chekreneva and A. P .Benois.

In most cases, pile overpass structures are, in terms of calculations, repeatedly statically indeterminate structures. The bearing capacity of building structures reinforced without removing them from the stress state, as is known [1, 2, 3, 4, 5], depends on the stresses that act in them during the reinforcement period. Moreover, for statically determinate structures, the reinforcement index θ, equal to the ratio of the bearing capacity of the structure after reinforcement Srein to its bearing capacity before damage S, depends only on the nature and conditions of reinforcement of the damaged element [4, 5]. A fundamentally different picture occurs for statically indeterminate structures, in which the stress distribution always depends on the ratio of the stiffnesses of the bearing elements. Any damage to any of the elements, for example, frame structures, has two consequences: 1) a decrease in the bearing capacity of this element and 2) an increase in stresses in other (undamaged) elements due to the resulting change in the stiffness ratio. Here, to restore the original bearing capacity of the structure, it is not enough to reinforce only the damaged elements; It is also necessary to strengthen the undamaged elements, since, as already noted, they are overloaded.

Thus, in the case of statically indeterminate systems, the θ indicator for a damaged element does not simultaneously serve as an indicator of the degree of restoration of the bearing capacity for the structure as a whole.

CALCULATION METHOD

Let us consider a certain statically indeterminate system consisting of n load-bearing elements (Fig. 1), loaded with arbitrary forces P_i. For simplicity, we will assume the frame beam is completely rigid. Each of its uprights experiences bending stresses, the magnitude of which depends on the displacement of beam Δ and its rotation β (the latter, for a rigid beam, is a function of Δ), as well as the stresses from axial forces. The greatest nodal bending moments in any element are due to the system of displacement

$$M_i = f(\Delta I_i) \tag{2}$$

where I_i is the linear rigidity of the element in question.

Let, after applying external forces P_i, the rigidity parameters of the frame correspond to the diagram shown in Figure 1,a). We will assume that element 1 sustained damage, with an initial stiffness of E_1J_1 along its entire length, where E_1 and J_1 are the elastic modulus of the material and of the element moment of inertia. After the damage occurred, the stiffness of this element in the damaged section l_1^1 (Fig. 1b) decreased to $E_1J_1^n$, while in the remaining section l_1^{11} it remained unchanged.

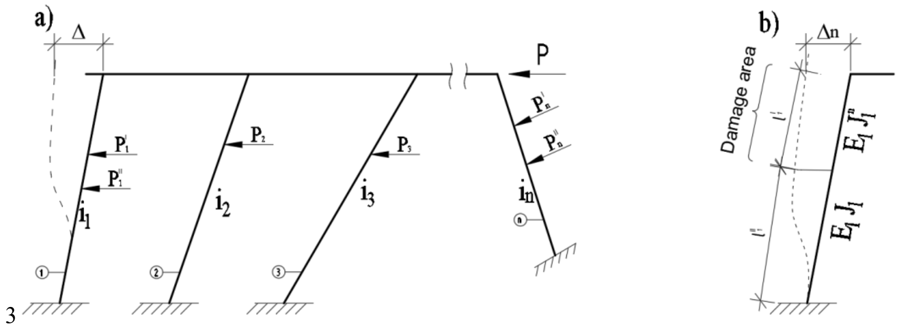

FIGURE. 1. Frame system operation with and without damage
a) - system displacement due to forces P_i in the absence of damage;
b) - system displacement due to forces P_i in the event of damage to element-I.

Since the stiffness of one of the frame's columns has decreased, the beam displacement Δ there is naturally increased to a certain value Δd. However, based on (1), it can then be argued that the bending moments in the undamaged elements have increased. Indeed, the stiffness of these elements remains unchanged, while the system displacements Δ have increased, meaning the nodal moments from the column displacement have also increased, as well as a decrease in its stiffness E_1J_1. Ultimately, this leads to a decrease in the bending moment M in the element. The above reasoning shows that even with full restoration of the original load-bearing capacity of the damaged element ($\theta = 1$), the load-bearing capacity of the structure as a whole is not fully restored. To fully restore it, it is necessary to adequately reinforce the undamaged elements.

It follows from the above that for the design of strengthening of statically indeterminate overpass pile structures it is necessary to have dependencies that allow the calculation of the index θ for both damaged and undamaged elements. The type of these dependencies, as usual [4, 5] should be related to the stress state of the elements in question (bending, compression, tension, eccentric compression and tension). The initial relationships for determining the degree of restoration of the bearing capacity of damaged elements θ^d and θ^u of undamaged elements θ^u differ from the initial relationship for θ in the case of statically determinate structures. Its essence lies in the need to take into account the redistribution of stresses in the structure, which does not occur in statically determinate systems [6]. In accordance with this, it is necessary to write

$$\theta^d = \frac{N^d_{rest}}{N^d - \delta N^d} \quad (3)$$

$$\theta^u = \frac{N^u_{rest}}{N^u + \delta N^u} \quad (4)$$

where N^d_{rest} is the bearing capacity of the damaged element after its restoration; N^u_{rest} is the same, of an undamaged element after its reinforcement; N^d and N^u are the bearing capacity of the damaged element before the damage occurred and of the undamaged element before reinforcement; δN^d is the magnitude of the force release in the damaged element after the damage occurs; δN^u is the magnitude of the increase in force in an undamaged element after damage has occurred.

Below, the use of relations (3) and (4) is demonstrated using the example of elements damaged by bending. For other types of stress state of elements, the approach remains similar.

Degree of restoration of the bearing capacity of a damaged bending element (5)

$$\theta^d = \frac{M^d_{rest}}{M^d - \delta M^d} \quad (5)$$

where M^d_{rest} is the bending moment that the damaged element can withstand after reinforcement; M^d is the same, before damage occurs; δM^d is reduction of the bending moment in the damaged element, which occurs due to the redistribution of the rigidity of the structural elements during damage.

The degree of increase in the bearing capacity of the reinforced intact element

$$\theta^u = \frac{M^u_{rest}}{M^u + \delta M^u} \quad (6)$$

where M^u_{rest} is the bending moment that the damaged element can withstand after reinforcement; M^u is the same, before reinforcement; δM^u is increase in bending moment caused by redistribution of rigidity of structural elements.

To determine θ^d, it is necessary to substitute into (5) the value M^d_{rest}, which consists of the bending moment acting in the element during the period of its strengthening Mdamage and some additional bending moment, the value of which depends on the difference between the calculated resistance of the material R and the value of the repair stresses σ^d_1 [3].

$$M^d_{rest} = W_{damage} \cdot \sigma^d_1 + W^d_r (R - \sigma^d_1) \quad (7)$$

where W_{damage} and W^d_r are the moments of resistance of the element in the presence of damage and after its elimination.

Considering that

$$M^d = R \cdot W^d \quad (8)$$

where W^d is the initial moment of resistance of the element. Substituting (7) and (8) into (5) we can obtain.

$$\overline{W}^d_r = \frac{RW^d - \sigma^d_1 \cdot W_{damage} - \delta M^d}{R - \sigma^d_1} \quad (10)$$

As can be seen from (10), at $\sigma^d_1 = 0$, i.e. when the reinforcement is carried out with the complete removal of the structure from the stress state, (in this case, naturally, and $\delta M^d = 0$) $\overline{W}^d_r = W^d$. To determine θ^u in (6), it is necessary to substitute the value of M^u_{rest}. By analogy with (7) and (8).

$$M^u_{rest} = W^u \cdot \sigma^u_1 + W^u_r (R - \sigma^u_1) \quad (11)$$

and

$$M^u = R \cdot W^u \quad (12)$$

Substituting (11) and (12) into (6) gives

$$\theta^u = \frac{W^u \sigma^u_1 + W^u_r (R - \sigma^u_1)}{W^u R + \delta M^u} \quad (13)$$

The required moment of resistance of an undamaged element $\overline{W}_r^u$ at which its strength ensures the complete restoration of the bearing capacity of the structure as a whole can be found from (13) by setting $\theta^u = 1$.

Then,

$$\overline{W}_r^u = \frac{W^u(R - \sigma_1^u) + \sigma M^u}{R - \sigma_1^u} \tag{14}$$

Thus, in order to completely restore the original load-bearing capacity of the frame system, one of the elements that has been damaged must increase the moment of resistance of the damaged element to the value $\overline{W}_r^d$, and the moment of resistance of any i-th undamaged element to the value $\overline{W}_{ri}^u$.

For a more general case, when the bearing capacity of the reinforced structure may not only be equal to, but also greater or less than the original, expressions (10) and (14) based on (7) and (11) are rewritten as:

$$W_r^d = \frac{\theta^d(R \cdot W^d - \sigma \cdot M^d) - \sigma_1^d W_{damage}}{R - \delta_1^d} \tag{15}$$

$$W_r^u = \frac{\theta^u(R \cdot W^u + \delta \cdot M^u) - \sigma_1^u W^u}{R - \sigma_1^u} \tag{16}$$

CALCULATION RESULTS

The following specific numerical example provides an idea of the degree of force redistribution in real pile structures that have sustained damage.

The object of consideration is an embankment of the "gantry bulwark" type, which is a statically indeterminate frame system with an absolutely rigid beam. The embankment structure is shown in Fig. 2a, the frame design diagram in Fig. 2b, and the bending moment diagrams due to its displacement in Fig. 2c.

The existing calculation method for the structure under consideration [3] gives the value of the system displacement Δ in the direction of the conditional connection "1" equal to

$$\Delta = \frac{R_{sh} \cdot \cos\alpha + R_{inc} + R_{can} \cdot \cos\alpha - (M_{can}/b)\sin\alpha}{6(\frac{2i_2}{l_2^2} + \frac{i_2}{l_2} \cdot \frac{\sin\alpha}{b} + \frac{2i_1}{l_1^2} \cdot \cos^2\alpha - \frac{i_1 \cdot \sin\ \alpha}{2l_1 \cdot b})} \cdot \frac{1}{\frac{M_b^I + M_c^I}{b} \cdot \sin\alpha} \tag{17}$$

where R_{sh}, R_{inc} and R_{can} are the anchor reactions of the vertical and inclined posts and the resultant active soil pressure on the cantilever part of the structure; M_{can} is moment of force relative to the bottom of the beam; i_1 and i_2 are linear rigidities of vertical and inclined frame posts (sheet piles and anchor piles); l_1 and l_2 calculated lengths of vertical and inclined posts; M_b^I and M_c^I - nodal moments at points "B" and "C" from a single displacement of the system in the direction of conditional connection "1" equal to:

$$M_b^I = 6i_1\left(\frac{\cos\alpha}{l_1} + \frac{3\sin\alpha}{3 \cdot b}\right) \tag{18}$$

$$M_c^I = 6i_2\left(\frac{1}{l_2} + \frac{2}{3} \cdot \frac{\sin\alpha}{b}\right) \tag{19}$$

where R_{sh}, R_{inc} and R_{can} are the anchor reactions of the vertical and inclined posts and the resultant active soil pressure on the cantilever part of the structure;

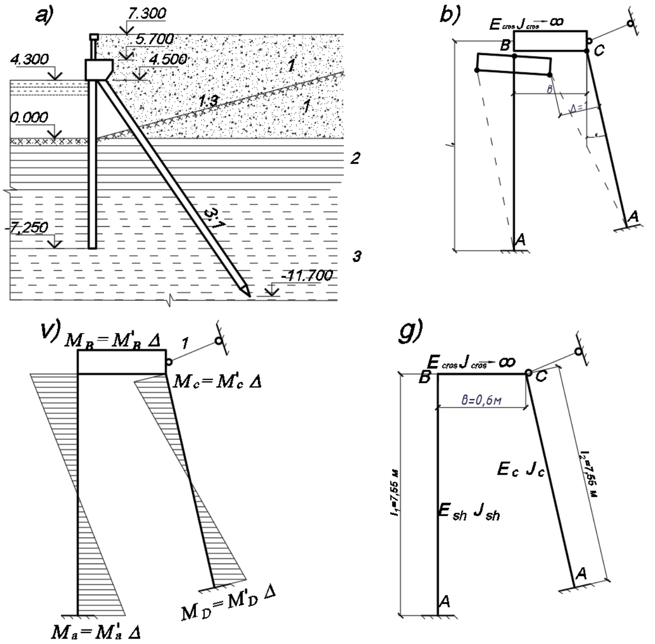

FIGURE 2. Embankment in the form of a gantry bulwark and its design scheme.
a) - structure; b) - design scheme; v) - bending moment diagrams due to displacement; g) - geometric dimensions.

Let's assume that a vertical frame post (a sheet pile wall) has been damaged, resulting in a uniform decrease in rigidity along its entire length. (It would also be possible to assume that the post is damaged only along a portion of its length; this is not essential, since in this case, the system displacement would be equal to the displacement due to damage along the entire post's length, multiplied by some constant coefficient.).

Let us present a numerical analysis applicable to a real structure, the calculation of which is given in [2]. The geometric dimensions are shown in Fig. 2.2, g.

The modulus of elasticity of the structure material $E_{sh} = E_c = 2,1\cdot 10^4\ MPa$; the rigidity of the sheet piles and piles per 1 running meter along the length of the structure $E_{sh}J_c = 4,7\cdot 10^9\ MPa\cdot sm^4$; $E_cJ_c = 2,98\cdot 10^9\ MPa\cdot sm^4$; the values included in (17) of the trigonometric functions of the angle α : $\sin\alpha = 0,316$; $\cos\alpha = 0,948$; $\cos^2\alpha = 0,898$; $\sin 2\alpha = 0,6$. The numerator in formula (17) is equal to 144.00 kN [2]. The linear rigidity of the CD column is $i_2 = 5,03\cdot 10^6\ MPa\cdot sm^6$, and that of the AB column in the absence of damage is $i_1 = 6,26\cdot 10^6\ MPa\cdot sm^6$.

In order to trace the change in the stress state of the structure depending on the magnitude of the damage, we will assume that the thickness of the vertical post Δ_{sh} is successively reduced from the initial value of 30sm to 26, 24, 22 and 20 sm. The results of the calculations performed using formulas (17), (18) and (19), taking into account that the total value of the nodal moments $M_b = M_b^/\cdot\Delta$ and $M_c = M_c^/\cdot\Delta$ are given in Table 1.

TABLE 1. Change in the stress state of a structure

Δ_{sh} (sm)	$E_{sh}J_c$ (MPa·sм⁴)	i_1 (MPa·sm³)	Δ (sm)	M_b (N·m)	M_c (N·m)
30	$4,7\cdot 10^9$	$6,26\cdot 10^6$	0,56	$1,0\cdot 10^5$	$0,88\cdot 10^5$
26	$3,7\cdot 10^9$	$4,9\cdot 10^6$	0,63	$0,9\cdot 10^5$	$0,99\cdot 10^5$
24	$2,9\cdot 10^9$	$3,84\cdot 10^6$	0,71	$0,77\cdot 10^5$	$1,15\cdot 10^5$
22	$2,2\cdot 10^9$	$2,9\cdot 10^6$	0,77	$0,64\cdot 10^5$	$1,21\cdot 10^5$
20	$1,6\cdot 10^9$	$2,1\cdot 10^6$	0,85	$0,51\cdot 10^5$	$1,33\cdot 10^5$

As can be seen from the data provided, as the stiffness of the AB column decreases, the bending moments in it decrease, and the bending moments in the CD column increase accordingly. The overall displacement of the system also increases.

The degree of change in the specified quantities can be conveniently seen from Table 2, which shows their relative values and the graphs constructed from it (Fig. 3). In the table, M_b / M_b^0 Mb / M°b and M_c / M_c^0 are the ratios of the bending moments at nodes "B" and "C" in the presence of damage (M_b and M_c) to their initial values in the absence of damage (M_b and M_c), and a Δ / Δ^0 are the relative displacements of the system in the direction of the conditional connection.

TABLE 2. Extent of change in the specified values

Δ_{sh} (sm)	Δ / Δ^0	$M_b / M^0{}_b$	$M_c / M^0{}_c$
30	1	1	1
26	1,12	0,9	1,12
24	1,25	0,77	1,25
22	1,37	0,64	1,37
20	1,51	0,51	1,51

We will also calculate how much the structure must be reinforced to fully restore its original load-bearing capacity if the plaster wall thickness is reduced from 30 to 24 sm due to damage. We will only consider bending stresses, since the axial forces in the studs change only slightly during damage. The calculation is performed using formulas (10) and (14). The values included in them are: R = 30,0 MPa; W^{II} = 15000 sm³; W^u = 7110 sm³; W_{damage} = 9600 sm³; $M_d = R \cdot W^d$ = 4,5·10⁵ N·m; $M_u = R \cdot W^u$ = 2,13 · 10⁵ N·m; δM^d = 0,22·10⁵ N·m; δM_u = 0,22 · 10⁵ N·m; σ_1^d = 5,13 MPa; σ_1^u = 15,49 MPa.

For a damaged element according to formula (11)

$$\overline{W}_r^d = \frac{300 \cdot 15000 - 5,13 \cdot 9600 - 220000}{300 - 51,3} = 15230 sm^3$$

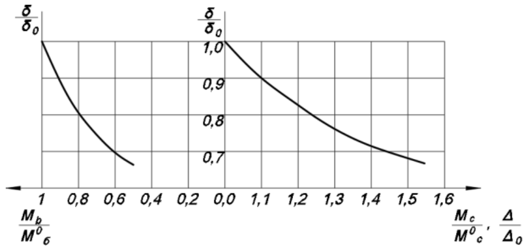

FIGURE 3. Graphs of the dependence of δ / δ_0 on M / M^0 and Δ / Δ^0 .

The obtained value $\overline{W}_r^d$ corresponds to the required thickness of the element Δ_{sh} =30,2 sm. For an undamaged element according to formula (14).

$$\overline{W}_r^u = \frac{7110(300 - 154,9) + 220000}{300 - 154,9} = 8626 sm^3$$

The found value $\overline{W}_r^u$ corresponds to the required thickness of the element Δ_{sh} =44 sm.

Thus, the calculation shows that the undamaged structural element requires more significant reinforcement than the damaged one.

CONCLUSIONS

1) To restore the original load-bearing capacity of damaged statically indeterminate structures, not only the damaged but also the undamaged elements must be reinforced. The fallacy of the prevailing notion that it is sufficient to reinforce only the damaged elements is obvious in light of the above.

2) The required degree of reinforcement of damaged elements of statically indeterminate systems is in all cases less than that of statically determinate structures.

3) The design of reinforcement of a statically indeterminate structure should be carried out in the following sequence:

- determination of forces in the undamaged structure;
- determination of forces in the damaged structure;

4) - determination of the magnitude of the increase in force release in the damaged element and the magnitude of forces in the undamaged elements;

5) - determination of the required values W_r^d and W_r^u corresponding to the specified values of θ^d and θ^u .

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Article Title ("Work"):
Design of Strengthening Statically Indeterminate Structures

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Title of Conference: AMSMT2025

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