

# Elimination Transverse Vibrations on an Elastic Beam Using Absorbers

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**Abstract.** This article examines the optimization of the parameters of an elastic beam with dynamic vibration absorbers during transverse vibrations. Methods for damping transverse vibrations in the elastic beam under consideration were analytically found. In particular, the change in the optimal parameters of the system under consideration is analyzed depending on the mass ratio and changes in the installation locations of dynamic vibration absorbers. The article discusses the optimization of the parameters of a system of elastic beams under stationary vibrations with two dynamic vibration absorbers. The solution to the problem of transverse vibrations of a beam with two parallel installed dynamic vibration absorbers is considered, using the method of series expansion in vibration modes. This method is more convenient for optimizing the parameters of dynamic vibration absorbers for various types of beam vibrations with boundary conditions, when it is necessary to repeatedly calculate the amplitude-frequency characteristics of the system.

**Keywords.** amplitude-frequency response, bending moment, dynamic vibration absorber, elastic beam, Laplace operator, transverse vibrations.

## INTRODUCTION

Modern development of engineering and technology requires the development of elastic beams with the most economical and less material-intensive design. In this case, problems associated with transverse vibrations on elastic beams often arise. Insufficient elaboration of issues to solve problems of transverse vibrations leads to the fact that during the design, construction and commissioning of such elements there is a need for additional changes in the design, which leads to an increase in development time or to changes in the main characteristics of the product. These disadvantages reduce the consumer properties of elastic beams [1, 5].

Many scientific articles are devoted to the problems of damping oscillations of systems with distributed parameters using dynamic oscillation absorbers. It is shown in [2,6] that when a dynamic vibration absorber is attached to a beam, a new natural frequency of the system appears, close to the partial frequency of the absorber, which, depending on the parameters of the system, can take values less than, greater than, or equal to the partial frequency of the absorber.

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In experimental studies [2,8], a comparative analysis of the vibrations of a beam with two dynamic vibration absorbers, symmetrically located relative to the ends of the beam, is carried out. Differential equations of motion are nonlinear and require the use of appropriate methods to solve.

In works [3, 4, 9], problems of nonlinear vibrations of a beam with a dynamic vibration absorber are considered, taking into account elastic-damping properties of the hysteresis type under harmonic influences. A solution to the system was obtained in the form of transfer functions.

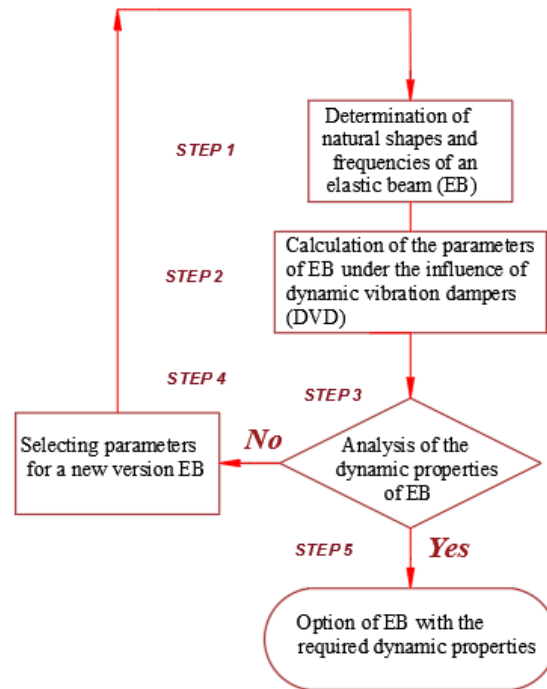
The problems of dynamics [5, 6, 10, 12] of nonlinear oscillations, as well as their stability [7, 8, 11, 16, 18], were studied. Based on the above, it follows that the study of vibrations and vibration damping of beams remains an urgent task of modern science. The article discusses the optimization of system parameters during stationary vibrations of a beam with two DVD.

A device is presented [5, 6, 12, 17] consisting of compression and tension springs, working together to withstand both vertical and horizontal loads resulting from permanent, temporary and seismic influences. In addition to the main task of absorbing vertical and horizontal loads, the device is capable of returning the span structure to its original position after exposure to seismic influences. It also eliminates resonance without increasing costs, both for the span and for supports and foundations, and does not complicate the installation conditions of the structure.

## PROPOSED METHODOLOGY, EXPERIMENTS AND RESULTS

In the presented research, the task was set to dampen transverse vibrations on an elastic beam using dynamic vibration absorbers (DVD). The algorithm of the sequence of work with the required properties is obtained by a sequence of a number of operations. In Fig. 1 presents the algorithm of operations and stages of the tactical process.

Let us consider the solution to the problem of transverse vibrations of a beam with two parallel installed DVD using the method of series expansion according to vibration modes. This method is more convenient for optimizing the parameters of the DVD for various types of beam vibrations with boundary conditions, when it is necessary to repeatedly calculate the amplitude-frequency characteristics (AFC) of the system.



**FIGURE 1.** An algorithm of operations and stages of the tactical process.

The results of the above works confirm that with a sufficiently large decrement of vibrations of the material of the elastic-damping element of the DVD, the nonlinearity of the internal resistance characteristics of the beam material has little effect on the vibrations of the beam and the determination of the optimal parameters of the DVD. A beam of length  $l$ , width  $b$ , height  $h$ , is fixed on a vibrating base; its movement is specified along the  $Oz$  axis. At points of the beam with coordinates  $X_1, X_2$ , DVDs are installed [13,14,15] (Fig. 2.).

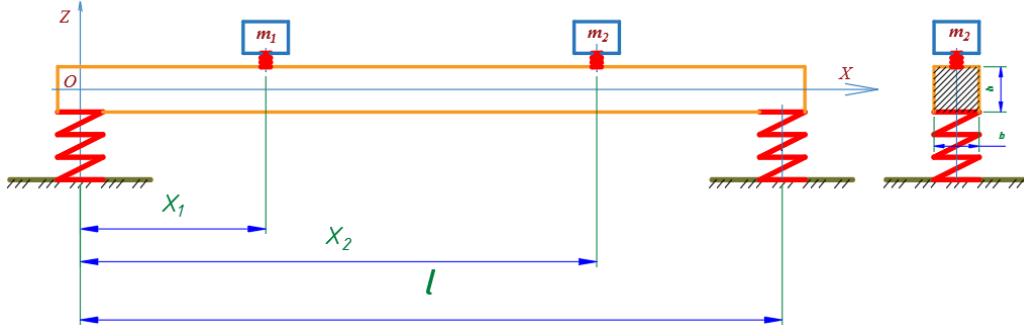


FIGURE 2. Design diagram of an elastic beam (EB) with two dynamic vibration absorbers (DVD).

## ANALYSIS OF EXPERIMENTS AND RESEARCH RESULTS

We write the differential equations of an elastic beam with two DVD under kinematic excitation in the following form:

$$\begin{aligned} \frac{\partial^2 M}{\partial x^2} + \rho F \frac{\partial^2 w}{\partial t^2} - c_1 R_1 \delta_1(x - x_1) \zeta_1 - c_2 R_2 \delta_2(x - x_2) \zeta_2 &= -\rho F \frac{\partial^2 w_0}{\partial t^2}; \\ m_1 \frac{\partial^2 w(x_1)}{\partial t^2} + m_1 \frac{\partial^2 \zeta_1}{\partial t^2} + c_1 R_1 \zeta_1 &= -m_1 \frac{\partial^2 w_0}{\partial t^2}; \\ m_2 \frac{\partial^2 w(x_2)}{\partial t^2} + m_2 \frac{\partial^2 \zeta_2}{\partial t^2} + c_2 R_2 \zeta_2 &= -m_2 \frac{\partial^2 w_0}{\partial t^2}, \end{aligned}$$

where  $M$  is the bending moment;  $\rho$  is material density;  $F$  is the cross-sectional area of the beam;  $w$  is beam deflection function;  $w_0$  is moving the base;  $w(x_1), w(x_2)$  are movement of the point of the beam in which the DVDs are installed;  $c_1, c_2$  are stiffness coefficients of the elastic damping elements of the DVD;  $m_1, m_2$  are mass of DVD;  $\zeta_1, \zeta_2$  are displacements of the DVD relative to the beam;  $\delta_1 \cdot (x - x_1), \delta_2 \cdot (x - x_2)$  are Dirac delta functions;  $x_1, x_2$  are the coordinates of the DVD installation;

To solve this system of equations, we used the method of separation of variables:

$$w(x, t) = \sum_{i=1}^{\infty} u_i(x) q_i(t),$$

After some calculations, the system of equations is reduced to the form (1):

$$\begin{aligned} \ddot{q}_i + R p_i^2 q_i - \mu_1 \mu_{0i} n_1^2 u_{i1} \zeta_1 - \mu_2 \mu_{0i} n_2^2 u_{i2} \zeta_2 &= -d_i W_0; \\ u_{i1} \ddot{q}_i + \ddot{\zeta}_1 + n_1^2 \zeta_1 &= -W_0; \\ u_{i2} \ddot{q}_i + \ddot{\zeta}_2 + n_2^2 \zeta_2 &= -W_0, \end{aligned} \quad (1)$$

where  $p_i$  is the natural frequency of the beam;

$$\mu_1 = \frac{m_1}{m_s}; \mu_2 = \frac{m_2}{m_s}; \mu_{0i} = \frac{l}{d_{2i}}; d_i = \frac{d_{1i}}{d_{1i}}; d_{1i} = \int_0^l u_i dx; d_{2i} = \int_0^l u_i^2 dx,$$

$m_s = \rho F l$  is beam mass;  $m_1, m_2$  are masses of dynamic vibration absorbers;  $u_i(x)$  is natural vibration modes of the beam;  $W_0$  is base acceleration,  $u_{k1} = u_k(x_1)$ ;  $u_{k2} = u_k(x_2)$ ; is  $x_1, x_2$  - coordinates of the DVD installation;

$n_1 = \sqrt{\frac{c_1}{m_1}}$ ;  $n_2 = \sqrt{\frac{c_2}{m_2}}$ ,  $\zeta_1, \zeta_2$  - oscillation frequencies; stiffness coefficients of elastic elements and relative movements of the DVD.

Acceleration of the base during harmonic vibrations

$$W_0 = w_0 \cos \omega t,$$

where  $w_0$  is the amplitude value of acceleration;  $\omega$  is frequency.

We look for solutions to the system in the form:

$$\begin{aligned} q_i &= a_i \cos(\omega t + \alpha_i); \\ \zeta_1 &= b_1 \cos(\omega t + \beta_1); \\ \zeta_2 &= b_2 \cos(\omega t + \beta_2); \end{aligned} \quad (2)$$

Substituting these expressions into the differential equations of motion and assuming that the coefficients vary slowly, we obtain the following normal equations for the system under consideration:

$$\begin{aligned} \dot{a}_i &= (2\omega)^{-1} [d_i w_0 \sin \alpha_i + l_1 n_1^2 b_1 \sin \phi_1 + l_2 n_2^2 b_2 \sin \phi_2]; \\ \dot{\alpha}_i &= (2a_i \omega)^{-1} [d_i w_0 \cos \alpha_i - a_i \omega^2 - l_1 n_1^2 b_1 \cos \phi_1 - l_2 n_2^2 b_2 \cos \phi_2]; \\ \dot{b}_1 &= (2\omega)^{-1} [(1 - d_i u_{i1}) w_0 \sin \beta_1 - l_2 n_2^2 u_{i1} b_2 \sin \phi_3 - u_{i1} p_i^2 a_i \sin \phi_1]; \end{aligned} \quad (3)$$

$$\begin{aligned} \dot{\beta}_1 &= (2b_1 \omega)^{-1} [(1 - d_i u_{i1}) w_0 \cos \beta_1 + b_1 n_1^2 T_6 - b_1 \omega^2 + l_2 n_2^2 u_{i1} b_2 \cos \phi_3 - u_{i1} p_i^2 a_i \cos \phi_1]; \\ \dot{b}_2 &= (2\omega)^{-1} [(1 - d_i u_{i2}) w_0 \sin \beta_2 - l_1 n_1^2 u_{i2} b_1 \sin \phi_3 - u_{i2} p_i^2 a_i \sin \phi_2]; \end{aligned} \quad (4)$$

$$\dot{\beta}_2 = (2b_2 \omega)^{-1} [(1 - d_i u_{i2}) w_0 \cos \beta_2 + b_2 n_2^2 T_7 - b_2 \omega^2 + l_1 n_1^2 u_{i2} b_1 \cos \phi_3 - u_{i2} p_i^2 a_i \cos \phi_2];$$

Where,  $\phi_1 = \beta_1 - \alpha_i$ ;  $\phi_2 = \beta_2 - \alpha_i$ ;  $\phi_3 = \beta_2 - \beta_1$ ;  $l_1 = \mu_1 \mu_{0i} u_{i1}$ ;  $l_2 = \mu_2 \mu_{0i} u_{i2}$ .

From the system of equations (4), putting zeros instead of derivatives on the left side, we obtain the required stationary solutions in the following form:

$$\begin{aligned} |q_{ik}| = |a_i| &= \left| \frac{d_i \omega^4 - A_1 \omega^2 + A_2}{-\omega^6 + A_3 \omega^4 - A_4 \omega^2 + A_5} \right|; \\ |\zeta_1| = |b_1| &= \left| \frac{(1 - d_i u_{i1}) \omega^4 - A_6 \omega^2 + A_7}{-\omega^6 + A_3 \omega^4 - A_4 \omega^2 + A_5} \right|; \\ |\zeta_2| = |b_2| &= \left| \frac{(1 - d_i u_{i2}) \omega^4 - A_8 \omega^2 + A_9}{-\omega^6 + A_3 \omega^4 - A_4 \omega^2 + A_5} \right|. \end{aligned} \quad (5)$$

where  $n_1 = \sqrt{\frac{c_1}{m_1}}$ ;  $n_2 = \sqrt{\frac{c_2}{m_2}}$  natural mode of vibration  $u_i(x) = \sin \frac{i\pi}{l} x$ , in this case, in a particular case for we find;

$$u_{i1} = 0.8660254037; \quad u_{i2} = 0.8660254035; \quad \mu_{0i} = \frac{l}{d_{2i}} = 2; \quad \text{as well as coefficients:}$$

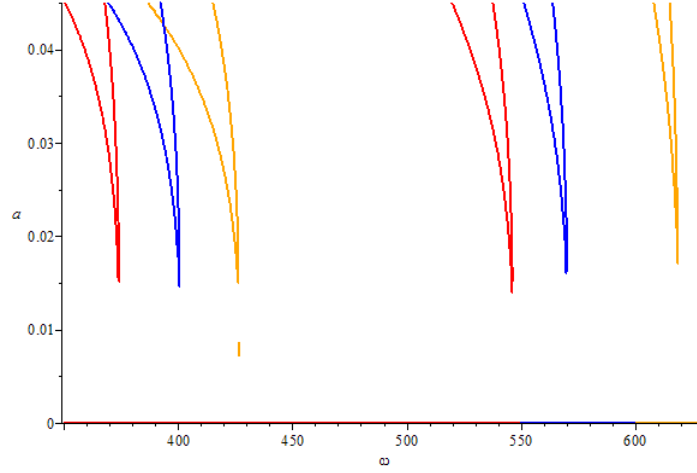
$$\begin{aligned} A_1 &= n_1^2 T_1 + n_2^2 T_2; \quad A_2 = n_1^2 n_2^2 T_3; \quad A_3 = n_1 T_6 + n_2^2 T_7 + p_i^2; \quad A_4 = n_1 p_i^2 + n_2^2 p_i^2 + n_1^2 n_2^2 T_8; \\ A_5 &= n_1 n_2^2 p_i^2; \quad A_6 = p_i^2 + n_2^2 T_4; \quad A_7 = p_i^2 n_2^2; \quad A_8 = p_i^2 + n_1^2 T_5; \quad A_9 = p_i^2 n_1^2; \\ T_1 &= d_i + \mu_{0i} \mu_1 u_{i1}; \quad T_2 = d_i + \mu_{0i} \mu_2 u_{i2}; \quad T_3 = d_i + \mu_{0i} (\mu_1 u_{i1} + \mu_2 u_{i2}); \\ T_4 &= 1 + \mu_{0i} \mu_2 u_{i2} (u_{i2} - u_{i1}) - d_i u_{i1}; \quad T_5 = 1 + \mu_{0i} \mu_1 u_{i1} (u_{i1} - u_{i2}) - d_i u_{i2}; \\ T_6 &= 1 + \mu_{0i} \mu_1 u_{i1}^2; \quad T_7 = 1 + \mu_{0i} \mu_2 u_{i2}^2; \quad T_8 = 1 + \mu_{0i} (\mu_1 u_{i1}^2 + \mu_2 u_{i2}^2); \end{aligned}$$

## RESULTS OF NUMERICAL STUDIES

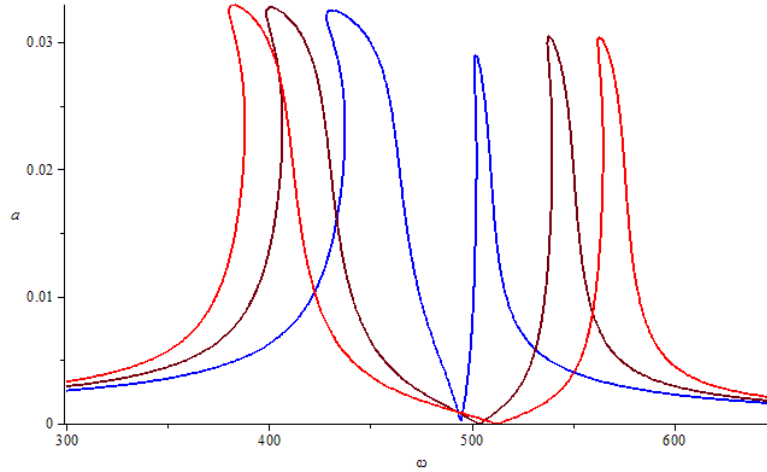
Numerical analysis is carried out to determine the first eigenshape from two separate cases: 1) first we carry out a numerical analysis with changes in the mass ratios  $\mu_1$  and  $\mu_2$ , the ratios of the masses of the DVDs to the mass of the beam; 2) from the obtained relationships, we construct graphs of the amplitude-frequency characteristics (AFC) of the system and find approximate locations for installing a DVD.

During the initial experiment, AFCs were constructed for the ratio of the absorber masses to the beam mass from 0.04 to 0.1. In this case, the approximate mass ratio for the system under consideration can be taken as 0.06 (Fig. 3).

In the red curve, the dynamic damper masses correspond to 0.01 of the rod mass; in the blue curve, 0.02; in the green curve, 0.03; and in the yellow curve, 0.04. It follows that, as the masses of the dynamic dampers increase, the dominant region shifts as a function of frequency.



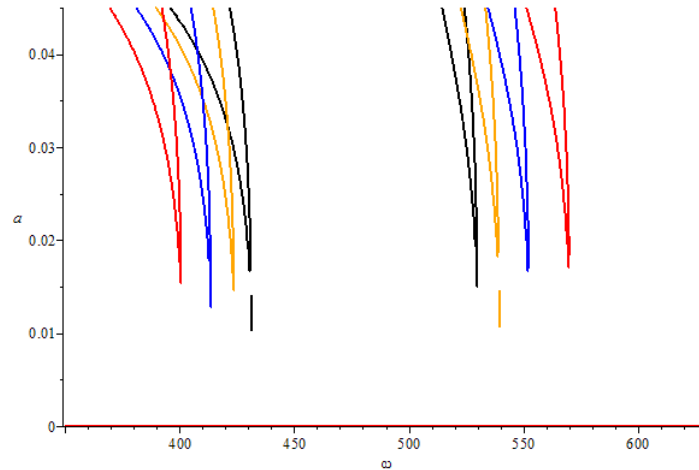
**FIGURE 3.** Stability areas when changing mass ratios 0.05; 0.04; 0.03; (blue line corresponds to 0.04).



**FIGURE 4.** AFCs when changing the mass ratio of 0.04 and the installation location of DVDs  $l/3, 2l/3$  (red),  $l/5, 4l/5$  (black),  $l/12, 11l/12$  (blue).

In the Fig. 4 it was shown AFCs of considered system. When we move the points where the dynamic dampers are installed on the rod symmetrically along the length of the rod without changing the masses, the stability area also changes depending on the frequency. In the first case, the red line is installed at points  $l/3$  and  $2l/3$ , in the second case, the blue line is installed at points  $l/4$  and  $3l/4$ , in the third case, the yellow line is installed at points  $l/5$  and  $4l/5$ , and in the fourth case, the black line is installed at points  $l/6$  and  $5l/6$ . From the graphs, we can see that the priority areas approach each other as the points where the dynamic dampers are installed are moved symmetrically away from the center of the rod.

Figure 5 shows the changes in the instability region with varying locations of the dynamic vibration absorbers. As the distance between the installation points of the dynamic vibration absorbers increases, the two regions of unstable amplitudes diverge.



**FIGURE 5.** AFCs when changing the mass ratio of 0.04 and the installation location of DVDs  $1/3$ ,  $2/3$  (red),  $1/4$ ,  $3/4$  (blue),  $1/5$ ,  $4/5$  (yellow),  $1/6$ ,  $5/6$  (black).

## CONCLUSION

The problem of optimizing the transverse vibrations of an elastic beam with two parallel installed dynamic vibration dampers with elastic elements during harmonic vibrations of the base is considered.

A principle has been developed for calculating the installation of dynamic vibration dampers for an elastic beam, taking into account the amplitude-frequency characteristics corresponding to transverse vibrations.

Analyzes of system vibrations were carried out with changes in the locations of the DHA installations and changes in the ratio of the masses of the DHA to the mass of the beam.

## REFERENCES

1. Ch.-M. Lee and V. N. Goverdovskiy, Vibration Protection Systems with Negative and Quasi-Zero Stiffness. (Cambridge University Press, 2021) pp. 25 – 51. DOI: <https://doi.org/10.1017/9781108874540.004>
2. E. E. Perepelkin, B. I. Sadovnikov, N. G. Inozemtseva, M. V.K limenko, Investigation of the dynamics of transverse oscillations of a vertical rod under gravity, friction, and thermal expansion. <https://doi.org/10.1016/j.pnucene.2024.105419>
3. D. Cubero and F. Renzoni, Vibrational mechanics in higher dimension: Tuning potential landscapes. <https://doi.org/10.1103/PhysRevE.103.032203>
4. E. E. Perepelkin, B. I. Sadovnikov, N. G. Inozemtseva, M. V. Klimenko, Investigation of the dynamics of transverse oscillations of a vertical rod under gravity, friction, and thermal expansion. <https://doi.org/10.1016/j.pnucene.2024.105419>
5. Yu. Hua, W. Xie, J. Xie, Novel Rod-Sprung-Mass Model to Investigate Characteristics of Building Structural Vibration Induced by Railways. Journal of Building Engineering, (2024).
6. M. H Zainulabidin, N. Jaini, “Vibration Analysis of a Rod Structure Attached with a Dynamic Vibration Absorber”, Engineering, Applied Mechanics and Materials, **3** (2013). DOI: 10.4028/www.scientific.net/AMM.315.315, Corpus ID: 108440341.
7. V. Shpachuk, A. Rubanenko, Y. Vashchenko O. Beketov, “Influence of mechanical and structural parameters of the rod with mass damper on the natural frequencies of transverse vibrations”. № 134 (2017), ISSN 0869-1231.
8. O. M. Dusmatov, Modeling the dynamics of vibroprotection systems (Fan Publishing House, Tashkent, 1997), p.167.
9. G. S. Pisarenko, O. E. Boginich, Oscillations of kinematically excited mechanical systems with allowance for energy dissipation, (Naukova dumka, Kyiv, 1981), p.219.
10. G. S. Pisarenko, A. P. Yakovlev, V. V. Matveev. Vibration-absorbing properties of structural materials. (Naukova dumka, Kyiv, 1971). –p.327.

11. M. Mirsaidov, K. Mamasoliev. "Contact Interactions of Multi-Layer Plates with a Combined Base". AIP Conference Proceedings, (2022), 2637, 050001. <https://doi.org/10.1063/12.0013538>
12. R. Khalmuradov, Kh. Ismoilov, Z. Khudoyberdiyev, B. Babajanov, AIP Conf. Proc. 3177, 050011 (2025) <https://doi.org/10.1063/5.0295160>
13. M. Mirsaidov and etc. "Stability of nonlinear vibrations of vibroprotected plate". In Journal of Physics: Conference Series, 1921, (2021), <https://doi.org/10.1088/1742-6596/1921/1/012097>
14. M. Mirsaidov, O. Dusmatov, M. Khodjabekov, "Stability of Nonlinear Vibrations of Elastic Plate and Dynamic Absorber in Random Excitations". E3S Web of Conferences 410, 03014 (2023). [doi.org/10.1051/e3sconf/202341003014](https://doi.org/10.1051/e3sconf/202341003014).
15. O. Dusmatov, Kh. Buranov, T. Absalomov, "On nonlinear vibrations of an elastic rod with a dynamic damper/ Int. Conf. on modern problems math. physics and information technology". Tashkent, (2004), p. 156-158.
16. O. Dusmatov, Kh. Buranov, A. Kudratov, "Stability of an elastic rod with dynamic absorbers under harmonic transverse vibrations" AIP Conf. Proc. 3177, 080002 (2025). <https://doi.org/10.1063/5.0294900>
17. Kh. Buranov, Q. Li, V. L. Popov, "Area and boundary contributions to friction in an adhesive contact of power-law indenters" AIP Conf. Proc. 3177, 040002 (2025). <https://doi.org/10.1063/5.0294715>
18. M. U. Khodjabekov, Kh. M. Buranov, A. E. Qudratov, "Modal Mass and Stiffness of Hysteresis Type Elastic Dissipative Characteristic Plate". International Conference on Actual Problems of Applied Mechanics AIP Conference Proceedings 2637, 050004 (2022). <https://doi.org/10.1063/5.0118292>