Dynamics of a System of Two Rigid Beams on an Elastic Single-Layer Foundation With Mechanical Characteristics Varying With Depth

Kamola Khaydarova[1, a)], Batirjan Mardonov[2, b)], Dilshod Kholikov[1, 3, c)], Rustambek Usmonov[1, d)], Ibrokhim Madatov[1, e)]

[1] Samarkand State University
[2] Samarkand, Uzbekistan Institute of Mechanics and Seismic Stability of Structures named after M.T. Urazbayev, Tashkent, Uzbekistan
[3] Samarkand campus of Zarmed University

[a)] Corresponding author: xaydarovakamolaxakimovna@gmail.com
[b)] batsam@list.ru
[c)] dilshodxoliqov2586@mail.ru
[d)] rustambekusmonov@gmail.com
[e)]ibrokhimmadatov@gmail.com

Abstract: To assess the change in the resistance of soil foundations during their deformation, calculation models are used that schematize the relationship between the load on the soil mass and its settlement. Soil foundation models are theoretical generalizations of experimental data on the patterns of deformation of foundations under load.

Keywords: Approximation, reactive resistance of the soil, two-dimensional elastic layer, seismogram, seismic load, variational principle.

INTRODUCTION

The soil massifs of construction sites are composed of different types of soil and differ significantly in their distribution properties. Based on the consideration of these properties, a distinction is made between the general deformation model (an example is the linearly deformed half-space model) and various local deformation models proposed in the works [1], among which the most widespread is the Winkler model for the front and rear faces of structures and the base.[2-4]. In contrast to this model, in work [5], based on the general variational principle, a technical theory for calculating a structure on an elastic foundation is proposed, which is more accurate and at the same time simpler than the theory of an elastic half-space.[8].

This theory is very flexible and allows solving not only the basic problems of calculating beams and slabs on an elastic foundation, but also a number of more complex issues related to the calculation of shells taking into account lateral loading and the underlying layer, the dynamics and stability of structures on an elastic foundation. In this case, the heterogeneous base is considered as a single-layer or multi-layer model, the properties of which are described by two or more characteristics.[9].

Let us consider a flat deformed state of a single-layer elastic foundation with a variable propagation velocity of a longitudinal wave along the depth of the layer at a constant Poisson's ratio.[10].

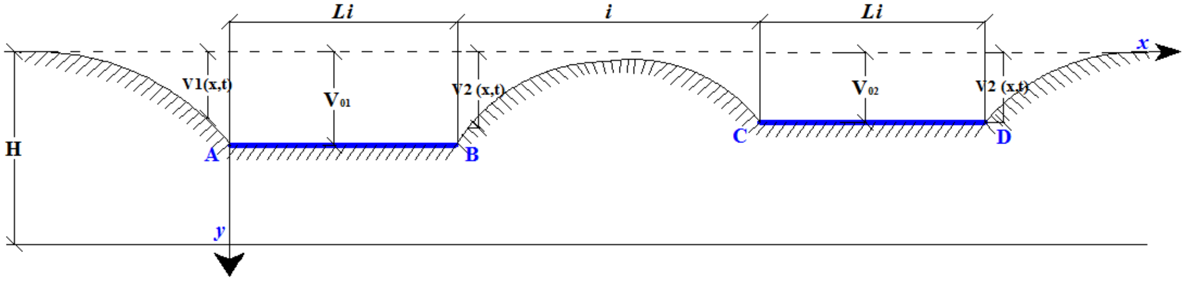

FIGURE 1. Scheme of deformation of the layer in contact with rigid beams

We assume that the upper boundary of the layer contacts two rigid beams AB and CD, and the lower boundary of the layer is fixed. We establish the origin of coordinates in the initial section of the beam AB and direct the axis 0x along the horizontal direction, and the axis 0y perpendicular to it from top to bottom (Fig. 1).

FORMULATION OF THE PROBLEM

The lengths of each beam are denoted by $L_1$and L_2 the distance between the beams by l is denoted by $u(x,y,t), v(x,y,t)$ the displacements of particles along the axes $0x$ and 0y, respectively. In what follows, we assume $u(x,y,t)=0$. Using the variational principle of V.Z. Vlasov [5], we reduce the motion of the medium to one-dimensional, according to which we represent the displacement $v(x,y,t)$through the generalized displacement $V(x,t)$ and the transverse distribution function $\psi(y)$using the formula;

$$v(x,y,t)=V(x,t)\psi(y)$$

where the function $\psi(y)$ is determined by the physical content of the problem and approximates the deformed state of the layer in the transverse direction. Let us consider the case when beams AB and CD perform movements according to the laws $V_{01}(t)$ and $V_{02}(t)$ under the action of vertical forces[11].

$$P_{01}(t) \; and \; P_{02}(t).$$

In this case, we choose the function $v(x,t)$in the following form

$$v(x,y,t)==V_1(x,t)\psi(y) \quad \text{at} -\infty<x<0 \tag{1}$$

$$v(x,y,t)==V_{01}(t)\psi(y) \quad \text{at } 0<x<L_1 \tag{2}$$

$$v(x,y,t)==V_2(x,t)\psi(y) \quad \text{at } L_1 \; <x<L_1+l \tag{3}$$

$$v(x,y,t)==V_{02}(t)\psi(y) \quad \text{at } L_1+l<x<L_1+l+L_2 \tag{4}$$

$$v(x,y,t)==V_3(x,t)\psi(y) \quad \text{at } L_1+l+L_2 \; <x<\infty \tag{5}$$

Stresses σ_x, σ_y and $\tau_{yx}=\tau_{xy}$are calculated using the formulas

$$\sigma_x=\frac{E_0(y)\nu_0}{1-\nu_0^2}\frac{\partial v}{\partial y}$$
$$\sigma_y=\frac{E_0(y)}{1-\nu_0^2}\frac{\partial v}{\partial y} \tag{6}$$
$$\tau_{yx}=\frac{E_0(y)}{2(1+\nu_0)}\frac{\partial v}{\partial x}$$

In the case of plane deformation, the quantities $E_0(y))$ and $\nu_0$are determined through the modulus of elasticity $\mathrm{E_{rp}}(y))$ and the Poisson's ratio ν_{rp} of the soil according to the formulas[12].

$$E_0=\frac{\mathrm{E_{rp}}(y)}{1-\nu_{\mathrm{rp}}^2}, \nu_0=\frac{\nu_{\mathrm{rp}}}{1-\nu_{\mathrm{rp}}}$$

Assuming $\psi(0)=1$ and following the work [1], we compose an expression for the work of all forces of the selected element on the possible displacement $v(x,y,t)$

$$\delta\int_0^H\frac{\partial\tau_{yx}}{\partial x}\psi(y)dy-\delta\int_0^H\sigma_y\psi'(y)dy-\delta\int_0^H\rho(y)\frac{\partial^2 v}{\partial t^2}\psi(y)dy+q(x,t)=0 \tag{7}$$

Where δ is the thickness of the selected element, , $q(x,t)$is the contact force between the beam and the foundation. Taking into account the dependencies $\mathrm{E_{rp}}=\rho_{\mathrm{rp}}(y)c_{\mathrm{rp}}^2(y)$, where $\rho_{\mathrm{rp}}(y)$ and $\mathrm{c_{rp}}(y)$ are, respectively, the density and propagation velocity of the longitudinal wave in the soil layer, taking into account formulas (1)-(5), from equality (7) we obtain [13].

$$2s\frac{\partial^2 V_1}{\partial x^2}-kV_1-m_0\frac{\partial^2 V_1}{\partial t^2}=0 \qquad -\infty<x<0 \tag{8}$$

$$-kV_{01} - m_0 \frac{\partial^2 V_{01}}{\partial t^2} + q_1 = 0 \qquad 0< x < L_1 \tag{9}$$

$$2s \frac{\partial^2 V_2}{\partial x^2} - kV_2 - m_0 \frac{\partial^2 V_2}{\partial t^2} = 0 \qquad L_1 \; < x < L_1 + l \tag{10}$$

$$-kV_{02} - m_0 \frac{\partial^2 V_{02}}{\partial t^2} + q_2 = 0 \qquad L_1 + l < x < L_1 + l + L_2 \tag{11}$$

$$2s \frac{\partial^2 V_1}{\partial x^2} - kV_3 - m_0 \frac{\partial^2 V_3}{\partial t^2} = 0 \qquad L_1 + l + L_2 \; < x < \infty \tag{12}$$

where

$$s = \frac{\delta \rho_{cp} c_{cp}^2}{4(1+\nu_0)} \int_0^H \bar{E}_0(y) \psi^2(y)\, dy,$$

$$k = \frac{\delta \rho_{cp} c_{cp}^2}{1-\nu_0^2} \int_0^H \bar{E}_0(y) \psi'^2(y)\, dy,$$

$$m_0 = \delta\rho_{cp} \int_0^H \bar{\rho} \psi^2(y) dy, \, m_{01} = \delta\rho_{cp} \int_0^H \bar{\rho} (\psi - \psi^2) dy$$

$$\bar{E}_0(y) = E_0(y)/\rho_{cp} c_{cp}^2, \bar{\rho} = \rho/\rho_{cp}$$

ρ_{cp} and c_{cp}-average density and propagation speed of longitudinal wave in soil environment

SOLUTION METHOD

Further we assume that $\psi(0) = 1$.

The displacements $V_{01}(t)$ and $V_{02}(t)$ satisfy the equations of motion

$$m_1 \ddot{V}_{01} = -q_1 L_1 + 2s[V_1{}'(0,t) - V_2{}'(L_1,t)] + P_{01}(t) \tag{13}$$

$$m_2 \ddot{V}_{02} = -q_2 L_2 + 2s[V_2{}'(L_1 + l, t) - V_3{}'(L_1 + l + L_2, t)] + P_{02}(t) \tag{14}$$

Having determined the expressions q_1 and q_2 from (9) and (11), we reduce equations (3) and (14) to the form

$$(m_1 + m_0 L_1)\ddot{V}_{01} + kV_{01} - 2sV_1{}'(0,t) + 2sV_2{}'(L_1,t) = 0 \tag{15}$$

$$(m_1 + m_0 L_2)\ddot{V}_{02} + kV_{02} - 2sV_2{}'(L_1 + l, t) + 2sV_3{}'(L_1 + l + L_2, t) = 0 \tag{16}$$

Equations (8), (10) and (12) are wave equations satisfying the zero initial and following boundary conditions

$$V_1(x,t) \to 0 \; at \; x \to -\infty. \; V_1(0,t) = V_{01}, \tag{17}$$

$$V_2(L_1, t) = V_{01}, V_2(L_1 + l, t) = V_{02} \tag{18}$$

$$V_3(L_1 + l + L_2, t) = V_{02}, V_2(x,t) \to 0 \; at \; x \to \infty \tag{19}$$

Solutions of equations (8), (10) and (12), satisfying conditions (17)-(19) are presented in the form [6]

$$V_1 = V_{01}\left(t + \frac{x}{a}\right) + cx \int_0^{t+x/a} V_{01}(\tau) \frac{J_1(c\sqrt{a^2(t-\tau)^2 - x^2})}{\sqrt{a^2(t-\tau)^2 - x^2}} d\tau \quad t > 0 \quad -\infty < x < 0$$

$$V_2 = \varphi\left(t - \frac{x - L_1}{a}\right) - c(x - L_1)a \int_0^{t + \frac{x - L_1}{a}} \varphi(\tau) \frac{J_1(c\sqrt{a^2(t-\tau)^2 - (x-L_1)^2})}{\sqrt{a^2(t-\tau)^2 - (x-L_1)^2}} d\tau +$$

$$+\psi\left(t - \frac{L_1 + l - x}{a}\right) + c(x - L_1 - l)a \int_0^{t + (x - L_1 - l)/a} \psi(\tau) \frac{J_1(c\sqrt{a^2(t-\tau)^2 - (x-L_1-l)^2})}{\sqrt{a^2(t-\tau)^2 - (x-L_1-l)^2}} d\tau \;\; L_1 < x < L_1 + l,$$

$$V_3 = V_{02}\left(t - \frac{x - L_1 - l - L_2}{a}\right) + c(x - L_1 - l - -L_2)a \int_0^{t + (x - L_1 - l - L_2)/a} V_{02}(\tau) \frac{J_1(c\sqrt{a^2(t-\tau)^2 - x^2})}{\sqrt{a^2(t-\tau)^2 - x^2}} d\tau \quad t > 0 \quad L_1 + l < x < \infty$$

Where

$$\varphi = (\psi_1 - \varphi_1)/2 \, , \varphi = \frac{(\varphi_1 + \psi_1)}{2}, \; a = \sqrt{2s/m_0} \; c = \; \sqrt{k/2s}$$

The functions φ_1 and ψ_1 are solutions of integral equations [14]

$$\varphi_1(t) = V_{01}(t) + V_{02}(t) - \varphi_1\left(t - \frac{l}{a}\right) - cl \int_0^{t - \frac{l}{a}} \varphi_1(\tau) \frac{J_1\left(c\sqrt{a^2(t-\tau)^2 - l^2}\right)}{\sqrt{a^2(t-\tau)^2 - l^2}} d\tau$$

$$\psi_1(t) = V_{01}(t) - V_{02}(t) + \psi_1\left(t - \frac{l}{a}\right) + cl \int_0^{t - \frac{l}{a}} \varphi_1(\tau) \frac{J_1\left(c\sqrt{a^2(t-\tau)^2 - l^2}\right)}{\sqrt{a^2(t-\tau)^2 - l^2}} d\tau$$

Substituting the expressions V_1, V_2 and V_3 into equations (15) and (16), we obtain

$$(m_1 + m_0 L_1)\ddot{V}_{01} + kV_{01} + \frac{2s}{a}\dot{V}_{01} + 2sc \int_0^t V_{01}(\tau) \frac{J_1(c\, a(t-\tau))}{(t-\tau).} d\tau - \frac{2s}{a}\dot{V}_{02} - 2sc \int_0^t V_{02}(\tau) \frac{J_1(c\, a(t-\tau))}{(t-\tau)} d\tau$$

$$(m_2 + m_0 L_1)\ddot{V}_{02} + kV_{02} + \frac{2s}{a}\dot{V}_{02} + 2sc \int_0^t V_{02}(\tau) \frac{J_1(c\, a(t-\tau))}{(t-\tau)} d\tau - \frac{2s}{a}\dot{V}_{01} - 2sc \int_0^t V_{01}(\tau) \frac{J_1(c\, a(t-\tau))}{(t-\tau)} d\tau$$

REFERENCES

1. К. Джонсон Механика контактного взаимодействия. М., Мир 189.510 с.
2. Л. А. Галин, О гипотезе Циммермана _Винклера для балок. Прикладная математика и механика. Т.7. вып.4.(1943).
3. Б. Н. Жемочкин, А. П. Синицын, Практические методы расчета фундаментов балок и плит на упругом основании (без гипотез Винклера). Стройиздат, М.,(1947).
4. Г. И. Глушков, Расчет сооружений заглубленных в грунт. М.Стройиздат. (1977). 295.с.
5. Т.Р.Б. Рашидов, С. В. Кузнецов, Б. М. Мардонов, И. Мирзаев, Прикладные задачи сейсомдинамики сооружений.Ташкент. (2019). 268 с.
6. В. З. Власов, Н. Н. Леонтьев, Балки, плиты и оболочки на упругом основании.М. Изд. Физмат.лит. (1960). 490 с.
7. Б. М. Будак, А. А. Самарский, А.Н. Тихонов, Сборник задач по математической физике. М.Физмат. лит (1956). 682 с.
8. D. Kholiqov, J. Abdurazzoqov, R. Usmonov, K. Xaydarova, Free torsional vibration of an elastic thin-walled cylindrical shell with variable cross sectio// AIP 2024/11/27 060029-1. https://doi.org/10.1063/5.0241748
9. B. Mardonov, K. X. Xaydarova, D.M. Ismatova, Горизонтально- вращательные колебания сооружений с фундаментом, взаимодействующим с основанием по билинейному закону при сейсмических воздействиях, Problems of architecture and construction **2** (3), 824-827
10. X.K. Xakimovna, I.D. Maxmudovna, Prospects for the application of brick walls in modern housing construction under construction in uzbekistan. Indonesian journal of law and economics review **19** (1), 10.21070/ijler. v19i1
11. K. Xaydarova, Bir qatlamli elastik asosda joylashgan to‘sinning seysmik kuchlar ta’siridagi tebranishining V.Z. Vlasov usuli asosida analitik tadqiqi, Yashil iqtisodiyot va taraqqiyot, № **5**-son, 148-152 b. (2025)
12. B. Mardonov K. Xaydarova, I. Turayev, E. Yadigarov, Dynamics of a rigid beam on an elastic single-layer foundation with mechanical characteristics varying with depth, Innovative: International Multi-disciplinary Journal ofApplied Technology (ISSN 2995-486X) VOLUME ISSUE 4 (2025).
13. R. Khalmurodov, Kh. Ismoilov, Frequency Analysis of Axisymmetric Vibrations of a Conical Shell in a Deformable Medium, AIP Conference Proceedings 3244, 060032 (2024). https://doi.org/10.1063/5.0241498. (Q4 Scopus).
14. Kh. Khudoynazarov, K. Mamasoliyev, E. Ismoilov, Non-stationary influence of a transverse-isotropic cylindrical shell with a viscous compressed fluid, AIP Conf. Proc. 3177, 050005 (2025) https://doi.org/10.1063/5.0294882