Transverse Vibrations of an Elastic Beam With a Moving Dynamic Absorber

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Abstract. In this work, mathematical model of the joint transverse vibrations of a linear elastic characteristic beam and hysteresis type elastic dissipative characteristic dynamic absorber are determined analytically depending on the parameters and variables of the system. In this case the dynamic absorber is moving on the beam with constant velocity. The amplitude-frequency characteristics and transfer functions were determined analytically and conclusions were drawn based on numerical analysis. In particular, the change in the ratio which shows natural frequency of the dynamic absorber to natural frequency of the beam with the change in the ratio which shows natural frequency of the system to natural frequency of the beam is shown based on numerical calculations. In addition, the ratio of the dynamic absorber amplitude to the beam amplitudes is analyzed analytically and numerically. It is found that this ratio is a decreasing function.

INTRODUCTION

Nowadays special attention is paid to damping harmful vibrations caused by external influences in the most advanced devices widely used in mechanics, automation, aerospace and other fields, increasing their efficiency and ensuring long-term reliable operation. In this regard, one of the urgent tasks is to conduct scientific research on the existing problems associated with vibrations of advanced devices, develop mathematical models, evaluate their dynamics, select optimal parameters and provide necessary recommendations.

Nowadays, a lot of research has been conducted and continues on the issues of mathematical modeling, damping of harmful vibrations, the use of various types of dynamic absorbers and determining their optimal parameters, and vibration control.

The work [1] is related to the design of control forces at nodal points to prevent displacement and deflection of selected points in vibrating structural elements under harmonic excitations. It is shown that the dominant vibrations at the desired points can be eliminated using reflection forces. A simple and accurate closed-form expression for the control force is obtained using the dynamic Green's function. It is shown that under certain conditions this control force can be created using passive elements such as springs and shock absorbers. The effectiveness of this method is demonstrated using numerical examples in various cases.

The article [2] considers the issue of damping the vibrations of a distributed parameter system using a dynamic absorber with elastic dissipative properties of the hysteresis type. First, an equation is formed that represents the amplitude-frequency characteristic of the system. Then, two frequency domains with multiple solutions are identified in the system and the amplitude-frequency characteristic is analyzed at different system parameters. The behavior of the system under various external influences is studied.

This article [3] studies the motion of two identical non-parallel plates under the influence of a continuously moving dynamic absorber. Fourth-order differential equations with partial derivatives are formulated and solved using an analytical method. First, the resulting second-order ordinary differential equations are solved asymptotically. The differential transformation method is a semi-analytical method, and its application to second-order ordinary differential equations to obtain a non-oscillating series solution is shown. The vibration equations obtained using the Laplace transform and approximation methods are solved in the MAPLE program. The effects of the load speed and the elasticity of the elastic layer on the vibrations of double-plate systems are graphically shown. At the same time, it was observed that the transverse deflection of each plate increased with increasing values of different velocities over time for the moving load.

In this article [4], the vibration of a beam with a moving mass is studied. The mass moves from one end to the other with a velocity having constant and slowing time intervals. The vibration of the midpoint of the beam after the mass stops moving is analyzed. In this case, a mathematical model of the system is obtained using the finite element method. The relationship between the natural frequency of the system and the velocity of the moving mass is determined and its effect on vibration is evaluated. It is found that by choosing the right velocity, the vibration reduction is close to 70% during movement and 80% after stopping.

The work [5] studies the issue of using dynamic absorbers to control vibrations of systems under earthquakes and other external dynamic forces. Analytical expressions of the natural frequency are determined. The use of passive, active and hybrid dynamic absorbers in dynamic vibration damping is analyzed. Recommendations are given for improving dynamic absorbers.

This work [6] is concerned with the vibrations of beams equipped with dynamic absorbers subjected to random excitations. Generalized function theory is used to determine the changes in parameters in different states of a dynamic absorber. In this case, the characteristic equation is obtained from the determinant of a 4x4 matrix and the eigenfunctions are determined from it. The corresponding orthogonality conditions for the eigenfunctions are obtained, and the stochastic vibrations under the influence of white noise are evaluated. Several boundary value problems are solved under the influence of random moving loads.

The article [7] studies the use of dynamic absorbers to optimize the vibrations of a clamped rod. The effect of changing the parameters of the dynamic absorber on the rod vibrations is evaluated. Initially, the equations of motion are obtained and the Den Hartog optimization method is used to optimize the parameters. Experiments are conducted to verify the correctness of the theoretical foundations obtained and deviations from the theoretical results are discussed.

In the work [8], dynamic absorbers are evaluated as promising technologies among the devices and methods for damping vibrations of structural elements, including struts, due to their mechanical simplicity and reliable operation. Analytical reviews of the damping of structural vibrations using dynamic absorbers are presented.

The work [9] is aimed at optimizing the parameters of a dynamic absorber using the method of determining invariant points and maximizing $H\infty$. The optimal values of the parameters of a single-mass and tuned-mass dynamic absorber are determined to minimize the resonance amplitude of the system. The invariant points are determined and the tuning frequency is minimized. As a result, the optimal parameters of the dynamic absorber are determined and the effect of each parameter on the system is evaluated. The decrease in the vibration amplitude around the resonant frequency of the system is analyzed. It is shown that the change in the single-mass leads to better efficiency in the resonant vibration range and also expands the effective frequency range of the dynamic absorber vibration.

The work [10] notes that dynamic absorbers have long been used as passive control devices to reduce harmful vibrations, and the most widely used of this type of devices are tuned-mass dynamic absorbers. In addition, the optimal parameters of three-element tuned mass dynamic absorbers for the main structures due to ground acceleration were studied. Unlike conventional tuned mass dynamic absorbers, three-element tuned mass dynamic absorbers include two spring elements, one of which is connected in series with a damping element. The optimal parameters were obtained using a simulated algorithm. Numerical results showed that three-element tuned mass dynamic absorbers are effective in reducing the vibrations of structures.

The work [11] presented that tuned mass dynamic absorbers are mechanisms that serve to reduce the response of vibrations under seismic loads and improve the performance of the system. Using the developed algorithm, the parameters of the dynamic absorber were optimized. The mathematical model of the dynamic absorber was obtained based on the model of the spring and the mass load attached to it. Using these differential equations, it was possible to estimate the vibrations under the influence of seismic loads. An algorithm was used to find the optimal parameters corresponding to the minimum vibrations. The simulation results showed that the obtained algorithm can find the optimal values of parameters such as mass, stiffness and damping coefficients that reduce the vibrations in the system. According to the analysis, the best combination of algorithm parameters and the probability of intersection were determined to determine the optimal matching values of the parameters. The maximum amplitude value of the system without dynamic absorbers was reduced from 0.24259 m to 0.034385 m with the use of dynamic absorbers.

The work [12] shows that in practical engineering, nonlinear characteristics of a dynamic absorber arise due to the installation of devices that limit the movement in the system, and it is found that ignoring such nonlinearity negatively affects the efficiency of the dynamic absorber. In order to increase the efficiency of the dynamic absorber, the parameters of a nonlinear tuned mass dynamic absorber are analyzed, taking into account linear and cubic nonlinearities. A model of a nonlinear tuned mass dynamic absorber, that is, a system with one degree of freedom, is developed, and the complex variable averaging method is used to obtain the vibration expression of the system under resonance conditions.

In the works [13-16], an analytical approach to obtain a frequency-dependent expression of vibrations of a system equipped with dynamic absorbers with one degree of freedom is developed, and several modes are introduced for precise tuning of the dynamic absorber. The damping element of the dynamic absorber installed on the body increases the stiffness of its elastic element. These elements achieved an 87.1% reduction in the amplitude of the experimentally tuned frequency-dependent vibration function of the cutting tool with dynamic absorbers. Recommendations for determining the optimal values of the parameters are developed.

In the works [17-20], the combined nonlinear vibrations of hysteresis-type dissipative rods, beams and fluid-coupled dynamic absorbers under the influence of kinematic, random, and random parametric excitations were structurally modeled and their dynamics evaluated.

One of the important tasks is scientific research on mathematical modeling and dynamics assessment of the complex joint motion of distributed parameter and concentrated mass systems, that is, determining the optimal parameters depending on the structural parameters using the transfer function, and improving the methods of mathematical modeling and dynamics assessment.

MATERIALS AND METHODS

The differential equations of joint transverse vibrations of a linear elastic characteristic beam with hysteresis type elastic dissipative characteristic dynamic absorber, which is moving on the beam are determined as following:

$$\ddot{q}_i+p_i^2 q_i-\mu\mu_0 n_0^2 u_{i0}\left(1+(-\theta_1+j\theta_2)\left(D_0+f(\zeta_{ot})\right)\right)\zeta\delta(x-x_0)\times$$
$$\times\ H\left(\frac{1}{v}-t\right)=-\ d_i\frac{\partial^2 w_0}{\partial t^2};$$
$$u_{i0}\ddot{q}_i+\ddot{\zeta}+n_0^2(1+(-\theta_1+j\theta_2)\left(D_0+f(\zeta_{ot})\right))\zeta=-\frac{\partial^2 w_0}{\partial t^2}, \qquad (1)$$

where

$\mu=\frac{m}{\rho Al}$; $\mu_0=\frac{l}{d_{2i}}$; $d_i=\frac{d_{1i}}{d_{2i}}$; $d_{1i}=\int_0^l u_i\,dx$; $d_{2i}=\int_0^l u_i^2\,dx$; $\theta_1,\theta_2=\theta_{22}sign(\omega)$ are constant coefficients that depend on the elastic dissipative properties of the dynamic absorber material and are determined from the hysteresis loop; j^2=-1; D_0 is a parameter determined from the hysteresis node; $f(\zeta_{ot})$ is being the decrement of vibrations, ζ_{ot} is a function of the absolute value of the relative deformation; $q_i(t)$ is a function of time and represents the displacement of the beam; $u_i(x)$ are considered orthogonal functions; p_i and n_0 are the natural frequency of the beam and the dynamic absorber; $u_{i0}=u_i(x_0)$; $x_0=vt$ is the point where the dynamic absorber is located; v is the velocity of the dynamic absorber; t is time; $\delta(x)$ is Dirac's delta function; $H(\frac{l}{v})$ is Heaviside function; l is The length of the beam; c, m are the stiffness and mass of the dynamic absorber, respectively; ζ is relative deformation of the dynamic absorber; w_0 is displacement of the base.

The system of equations (1) represents the differential equations of vibrations for the combined transverse vibrations of the beam with linear elastic characteristics and the moving dynamic absorber with a hysteresis-type elastic dissipative characteristic.

Using the system of differential equations (1), it is possible to determine the transfer function to analyze the dynamics of the beam that is protected from the considered vibrations. First, for this purpose, let's transform the system of differential equations into an algebraic form using the differential operator $S=\frac{d}{dt}$.

$$[S^2+p_i^2]q_i-\mu\mu_0 n_0^2 u_{i0}(1+(-\theta_1+j\theta_2)\left(D_0+f(\zeta_{ot})\right))\zeta H\left(\frac{1}{v}-t\right)=-\ d_i W_0;$$
$$u_{i0}S^2 q_i+[S^2+n_0^2\left(1+(-\theta_1+j\theta_2)\left(D_0+f(\zeta_{ot})\right)\right)]\zeta=-W_0, \qquad (2)$$

where $W_0=\frac{\partial^2 w_0}{\partial t^2}$.

The system of equations (2) has $S^2=-\omega^2$ and also $H(\frac{1}{v}-t)$ the Heaviside functions $\frac{1}{v}-t>0$ when it is $H\left(\frac{1}{v}-t\right)=1$, taking into account its property, we solve it with respect to the variables q_i and ζ.

$$q_i(t)=-\frac{A_{1*}+jA_{2*}}{B_{1*}+jB_{2*}}W_0;$$

$$\zeta(t)=-\frac{A_{3*}+jA_{4*}}{B_{1*}+jB_{2*}}W_0, \qquad (3)$$

where

$$A_{1*}=-\omega^2+(1+d_i\mu\mu_0u_{i0})n_0^2(1-\theta_1(D_0+f(\zeta_{ot}));$$
$$A_{2*}=(1+d_i\mu\mu_0u_{i0})n_0^2\theta_2(D_0+f(\zeta_{ot}));$$
$$A_{3*}=(-\omega^2+p_i^2)d_i+u_{i0}\omega^2;$$
$$A_{4*}=0;$$
$$B_{1*}=[-\omega^2+p_i^2]\left[-\omega^2+n_0^2\left(1-\theta_1(D_0+f(\zeta_{ot}))\right)\right]-\mu\mu_0n_0^2u_{i0}^2\omega^2\times$$
$$\times(1-\theta_1(D_0+f(\zeta_{ot})));$$
$$B_{2*}=[-\omega^2+p_i^2]n_0^2\left(1+\theta_2(D_0+f(\zeta_{ot}))\right)-\mu\mu_0n_0^2u_{i0}^2\theta_2(D_0+f(\zeta_{ot})))\omega^2.$$

The obtained system of equations (3) allows us to determine and numerically analyze the amplitude-frequency characteristics and transfer function of the vibrations of the beam with elastic dissipative characteristics in combination with a dynamic absorber with hysteresis-type elastic dissipative characteristics.

RESULTS AND DISCUSSION

The expression for the absolute acceleration of the beam protected from the considered vibrations, $W_i=\frac{\partial^2 w_i}{\partial t^2}$ and the ratio of the desired acceleration expression to the base acceleration expression can be written as follows:

$$W_i(j\omega,x)=1+u_i\omega^2\frac{A_{1*}+jA_{2*}}{B_{1*}+jB_{2*}}. \qquad (4)$$

The expression (4) represents the transfer function of the transverse vibrations of the beam with linear elastic characteristics combined with the dynamic absorber of the hysteresis-type elastic dissipative characteristic. This transfer function allows for evaluating the effectiveness of the hysteresis-type elastic dissipative dynamic absorber in damping the vibrations of the elastic beam and determining the optimal parameters of the dynamic absorber.

Let's assume that the moving dynamic absorber moves along the length of the beam $l=0.5\ m$ and moves at a constant velocity $v=0.25\ m/s$. In that case, the position of the point over time $t=1\ s$. The point will be at $x_0=0.25\ m$. This point is located at the midpoint of the beam's length.

Let's consider the following case, namely $\theta_1=0$; $\theta_2=\theta_{22}=const$. In that case:

$$\left[(1+(M_0-A_0-2)\theta_{22})\frac{n_0^2}{p_i^2}\right]\frac{\omega^4}{p_i^4}+[-1+(A_0+2)\theta_{22}+(A_0+1)\times$$
$$\times(2\theta_{22}-1)\frac{n_0^2}{p_i^2}]\frac{n_0^2}{p_i^2}\frac{\omega^2}{p_i^2}-(A_0+1)(2\theta_{22}-1)\frac{n_0^4}{p_i^4}=0;$$
$$(5)$$
$$\left[(1+(M_0-A_0)\theta_{22})\frac{n_0^2}{p_i^2}\right]\frac{\omega^4}{p_i^4}+[-1-A_0\theta_{22}-(A_0+1)\times$$
$$\times(2\theta_{22}+1)\frac{n_0^2}{p_i^2}]\frac{n_0^2}{p_i^2}\frac{\omega^2}{p_i^2}+(-A_0+1)\frac{n_0^4}{p_i^4}=0;$$
$$A_0=d_i\mu\mu_0u_{i0};\ M_0=\mu\mu_0u_{i0}^2.$$

In order to determine the relationship between the ratios $\frac{n_0}{p_i}$ and $\frac{\omega}{p_i}$, we take the system parameters as follows when plotting the graphs of these equations:

$$\mu=0.1;\ \mu_0=\frac{l}{d_{2i}}=\frac{0.5}{0.1248092022}=4.006114863;$$
$$d_i=\frac{d_{1i}}{d_{2i}}=\frac{0.09134867751}{0.1248092022}=0.7319065894;$$
$$A_0=d_i\mu\mu_0u_{i0}=-0.2055342254;\ M_0=\mu\mu_0u_{i0}^2=0.01968491707;$$
$$\theta_{20}=\frac{1}{\pi};$$

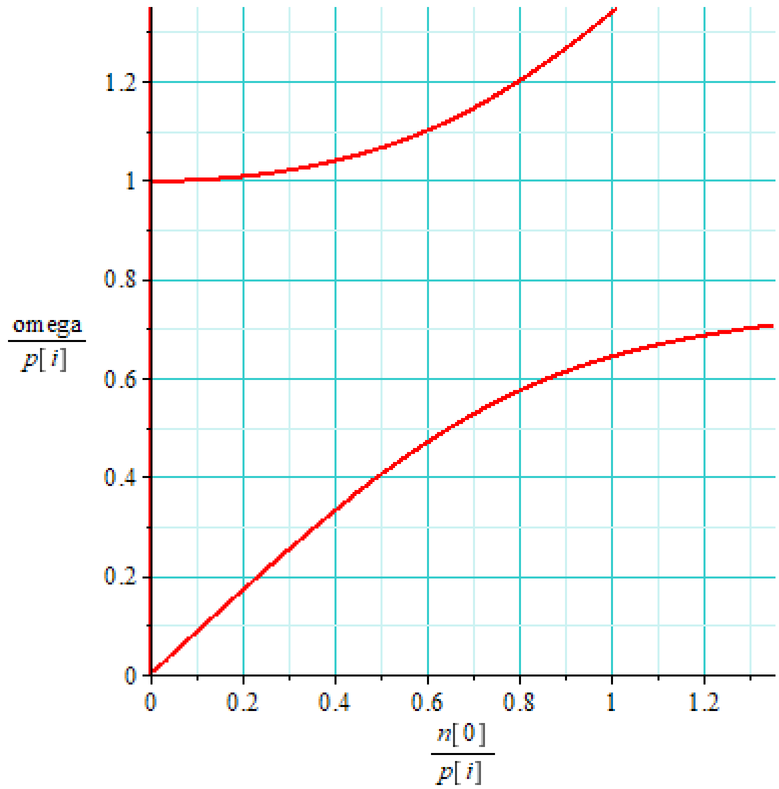

FIGURE 1. *Graph of the change in the function* $\frac{\omega}{p_i}$*, defined by equation (5), depending on* $\frac{n_0}{p_i}$*.*

In Fig. 1, the equation (5) defines in functions variation graph of $\frac{\omega}{p_i}$ is given depending on $\frac{n_0}{p_i}$. From these graphs, it can be concluded that, $0<\frac{n_0}{p_i}<0.18$ when it is $\frac{\omega}{p_i}$ the ratio takes values equal to one. In this range of the ratio $\frac{n_0}{p_i}$ and at the previously given parameter values p_i and ω the frequencies converge. As a result, an increase in amplitudes is observed. $0.18 \leq \frac{n_0}{p_i}$ when it is $\frac{\omega}{p_i}$ the ratio does not take values equal to one. In this range of the ratio $\frac{n_0}{p_i}$ and at the previously given parameter values, an increase in amplitudes is not observed. Thus, for this case the ratio $\frac{n_0}{p_i}$ should be [0.18;1.0] it is advisable to take the ratio within the given interval.

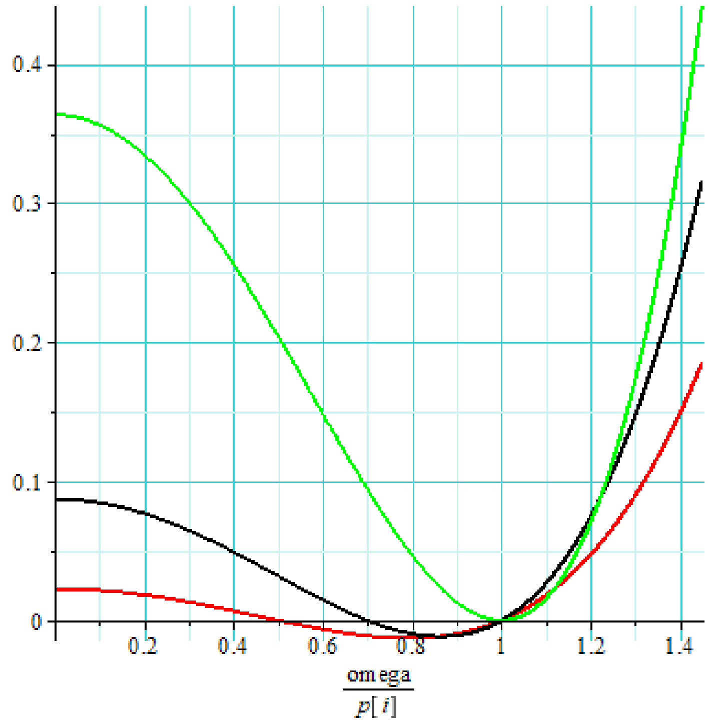

FIGURE 2. The graph of the function defined by the equation (4) as a function of $\frac{\omega}{p_i}$ ($\frac{n_0}{p_i}$=0.5; 0.7; 1.0 ($red; black; green$)).

In Fig.2, variations of the function defined by the equation (4) depending on frequency ratio $\frac{n_0}{p_i}$=0.5; 0.7; 1.0 ($red; black; green$) are shown. From these graphs it can be concluded that, $\frac{n_0}{p_i}$ with a decrease in the ratio from one value $\frac{\omega}{p_i}$ the values of the ratio are also reduced by one value, and the zeros of the function are two multiplied by one. So given that this function represents a denominator in the amplitude-frequency characteristic, the approximation of this function to the zero value induces resonance. Therefore, for the selected value of the parameters $\frac{n_0}{p_i}$=1.0 the value gives rise to one large value of the amplitude. $\frac{n_0}{p_i}$=0.5; 0.7 in those cases, however, two large values of the amplitude arise.

Let's analyze the ratio of the absolute displacement of the moving dynamic absorber to the absolute displacement of the beam. For this purpose:

$$\left|\frac{\zeta_a}{q_{ia}}\right| = \left|\frac{EI}{m\omega^2}\frac{\partial^3 u_i}{\partial x^3}_{|x=x_0}\right|. \quad (5)$$

Let's plot the graph of the ratio (5).

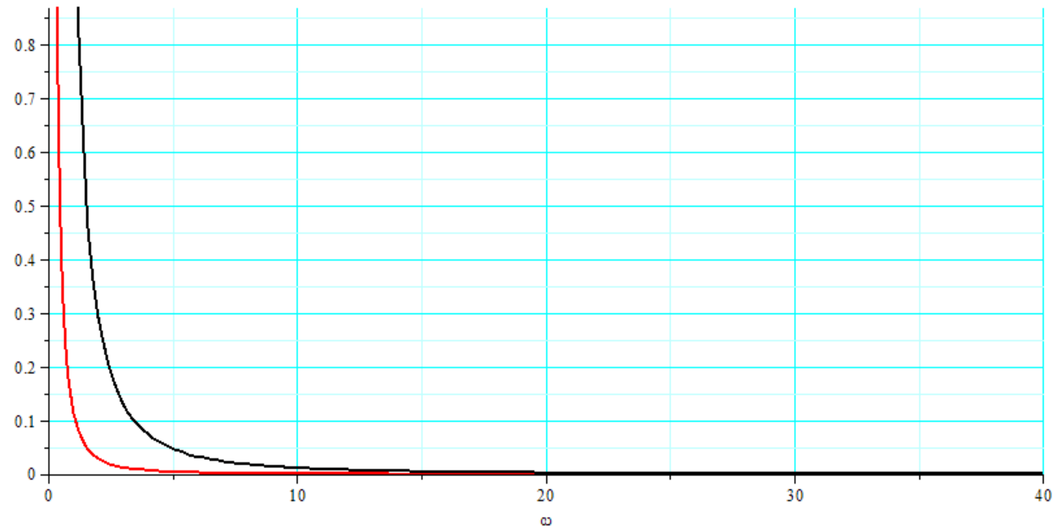

FIGURE 3. Graph of the function $\left|\frac{\zeta_a}{q_{ia}}\right|$ depends on the frequency.
The change in this case is as follows: the first mode of vibration (red) and the second mode of vibration (black).

The graphs in Fig 3 show the frequency-dependent variation of the amplitude ratio $\left|\frac{\zeta_a}{q_{ia}}\right|$ for the above parameter values and μ=0.1 for the first (red) and second (black) modes of vibration. From this it can be seen that with increasing frequency, the ratio of the absolute displacement of the moving dynamic absorber to the absolute displacement of the beam decreases faster in the first modes of vibration than in the second modes of vibration.

CONCLUSION

1. The vibrations of the beam with elastic dissipative characteristics together with a dynamic absorber with hysteresis-type elastic dissipative characteristics were mathematically modeled in an analytical form depending on the system parameters.
2. The amplitude-frequency characteristics of the vibrations of the beam with elastic dissipative characteristics in combination with a dynamic absorber with hysteresis-type elastic dissipative characteristics were determined.
3. The analytical expression of the transfer function for the vibrations of a beam with linear elastic characteristics, combined with a moving hysteresis-type elastic dissipative dynamic absorber, was determined and analyzed depending on the system parameters and variables.
4. It was showed that with increasing frequency, the ratio of the absolute displacement of the moving dynamic absorber to the absolute displacement of the beam decreases faster in the first modes of vibration than in the second modes of vibration.

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