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Longitudinal Vibrations of Cylindrical Bodies of Complex Composite Shape in an Elastic Medium

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Abstract. The study of oscillations and motion of bodies in elastic media is currently one of the most pressing scientific problems in various fields of geotechnics, construction, military equipment and mechanical engineering. In particular, the complex shape of the tip of cylindrical bodies significantly complicates their interaction with an elastic or elastic-plastic medium. Such bodies are found in models of piles, submarines, probes (medical injections), various geotechnical devices and even ballistic bodies hitting the ground.
Oscillations occurring in the longitudinal direction in an elastic medium reflect the dynamic connection of the body with the medium. The motion of bodies of this shape is significantly affected by the resistance force of the medium, deformation properties, stress distribution in the contact zone and geometric parameters. If the tip of a cylindrical body has a complex shape, that is, conical, parabolic or other complex surfaces, then the oscillations, resistance force, absorption and energy release will have significantly more complex properties.
This article theoretically analyzes the longitudinal vibration of a cylindrical body with a complex shape at the tip in an elastic medium.

Keywords: longitudinal motion, cylindrical body, elastic medium, elastic modulus, resistance force, impulsive load, impact motion, mathematical modeling, viscoelasticity, pile driving.

INTRODUCTION

Let us consider the problem of vertical vibrations of a cylindrical shell of radius a, length L, with a flat front end and embedded in a soil layer of radius R and thickness H, modeled by an elastic medium. In contrast to the Winkler model proposed in [1, 2], to solve the problem we use the variational methods proposed in the works in the case of axisymmetric deformation of the layer [3]

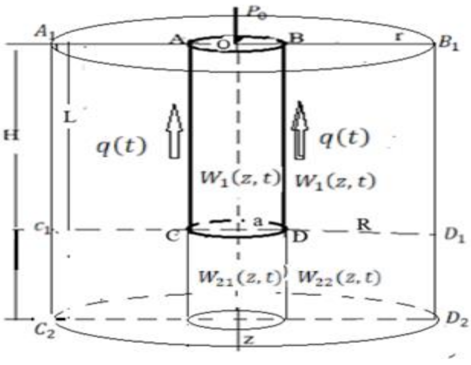


FIGURE 1. Diagram of the motion of a cylindrical shell (elastic rod) in a layer of elastic medium

We'll direct the 0z axis vertically downwards along the cylinder axis [13], $w(r,z,t)$ and $u(r,z,t)$ the 0r axis perpendicular to it. (Fig. 1) We'll denote the axial and radial displacements of the layer particles in axisymmetric coordinates coordinates (r,z), t - time [4].

We'll represent the stress tensor components as

$$\sigma_{zz} = \frac{E_0}{1-\nu_0^2}\left[\frac{\partial w}{\partial z}, +\nu_0\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\right], \tag{1}$$

$$\sigma_{rr} = \frac{E_0}{1-\nu_0^2}\left(\nu_0\frac{\partial w}{\partial z}+\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\right], \tag{2}$$

$$\tau_{zr} = \frac{E_0}{2(1+\nu_0)}\left(\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\right) \tag{3}$$

E_0 and ν_0 are Young's modulus and Poisson's ratio of the soil environment [5].

We assume that the cylinder performs only vertical movement under the action of the distributed force and is [6] henceforth taken as $u(r,z,t) \approx 0$.

METHOD OF RESEARCH

The displacement of particles in the layer is represented as [7]

$$w(r,z,t) = W_1(z,t)\varphi(r) \quad at \quad 0<z<L,\ a \le r < R \tag{4}$$

$$w(r,z,t) = W_{21}(r,t)\psi(z) \quad \text{at} \quad L<z<H,\ 0 \le r < a \tag{5}$$

$$w(r,z,t) = W_{22}(r,t)\psi(z) \quad \text{at} \quad L<z<H,\ a \le r < R \tag{6}$$

Following the work [3], we compile the work of external normal forces for zone $0<z<L$, $0 \le r < R$ in relation to the selected annular strip [12], where it is accepted $\varphi(a)=1$.

$$2\pi\int_a^R \frac{\partial \sigma_{zz}}{\partial z}\varphi(r)rdr - 2\pi\int_a^R \tau_{rz}\varphi'(r)rdr - p(z,t) = 0 \tag{7}$$

where $p(z,t)$ - intensity of tangential forces on the lateral surface of the cylinder

$q(z,t)$ - normal pressure distributed at the lower end (disk) of the cylinder

The equation of motion of a cylinder (rod) is written as [8]

$$EF\frac{\partial^2 W_1}{\partial z^2} - \rho F\frac{\partial^2 W_1}{\partial t^2} - p(z,t) = 0 \tag{8}$$

Considering $\varphi(a) = 1$,, we put the expression $p(z,t)$ from (7) into equation (8)

$$\left(EF+2s_1\right)\frac{\partial^2 W_1}{\partial z^2}-\left(\rho F+m_1\right)\frac{\partial^2 W_1}{\partial t^2}-k_1 W_1=0 \tag{9}$$

Equation (9) is integrated when the boundary conditions are satisfied [9]

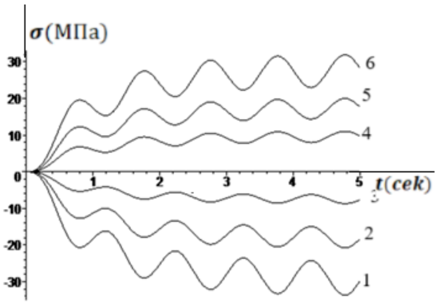
$$\frac{\partial W_1}{\partial z}=\frac{P_0(t)}{EF} \quad \text{at } z=0\ ; \quad \frac{\partial W_1(L.t)}{\partial z}=-\beta W_1 \quad \text{at } z=L\ ; \quad \beta=\pi a^2 k_2+2s_2\varphi'(a)/EF$$

RESULTS AND DISCUSSION

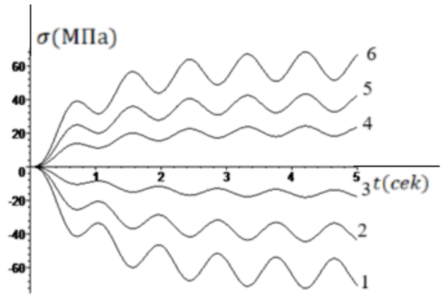
Figures 1 and 2 show the curves of the dependence of displacement on time in different sections of the rod for two given Young's modulus E_0 [10].

In calculations it is accepted [11] $\psi(z)=\frac{sh\gamma(H-z)}{sh\gamma(H-L)}$, $P_0=P_{00}\exp(-\alpha_0 t)$]

$E_0 = 30\cdot 10^6\ Pa$, $v_0 = 0.3$, $E = 4\cdot 10^{10}$Pa, $p_0 = 1800\ kg/m^3$, $= 6000 kg/m^3$, $L = 5\ m$, $H = 10\ m$,$a = 0.2m$, $R = 5m$, $\gamma = 1$,$P_{00} = 2\cdot 10^4\ N$, $\alpha_0 = 1$ (1/c).



(a)



(b)

FIGURE 2. Change in stress σ(MPa) over time t(cek) for two given Young's modulus of the medium in different cross-sections of the rod: 1) z=1 m, 2) z=3 m, 3) z=4 m, 4) z=4.5 m, 5) z=4.6 m, 6) z=4.9 m; $(a) E_0 = 30\cdot 10^6 Pa$, $(b)\ E_0 = 80\cdot 10^6 Pa$

CONCLUSION

From the analysis of the graphs presented in Fig. 2 it follows that the graphs of stress in the middle of the rod are symmetrical and their dependence on time is oscillatory. In this case, for sections z<4, the rod is in a compressed state and at z>4 in a tensile state. An increase in the Young's modulus of the base leads to an increase in stress in the sections of the rod.

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