Analysis of Dynamic Absorber Efficiency in Random Vibrations of a Beam

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Abstract. In this work, the mean square values of displacements and base accelerations in the vibrations of a hysteresis-type elastic dissipative characteristic beam with variable cross-section in combination with a hysteresis-type elastic dissipative characteristic dynamic absorber under the influence of random excitations were determined analytically depending on the system parameters. The effectiveness of the dynamic absorber was evaluated based on the expressions of the mean square values of displacements and base accelerations. The change in the damping efficiency at different values of the parameters was investigated. In particular, the change in the mean square values of displacements and base accelerations around the resonance frequency with a change in the parameter characterizing the spectral width was analyzed and conclusions were drawn.

Keywords: beam, dynamic absorber, hysteresis, random vibration, mean square value.

INTRODUCTION

In modern technology and equipment, the study of vibrations, taking into account the nonlinear dissipative characteristics of materials in machines, mechanisms, devices and their elements, and the determination of their dynamic characteristics as a result of stability and verification of their stability condition are considered urgent problems in terms of ensuring their long-term effective operation.

There are many scientific research works on mathematical modeling of vibrations of distributed parameter systems, taking into account their dissipative characteristics, and on checking the stability of their damping and harmful vibrations. In particular:

The work [1] considered transverse vibrations of a beam with a uniformly decreasing thickness starting from one end. The vibration velocity was analyzed. It was shown that the obtained differential equations have an exact solution. Based on these solutions, a new method for solving the beam equation when the thickness change is not parabolic is proposed, and conclusions are given.

The work [2] presented numerical methods for analyzing vibrations of beams with variable cross-section. The Ostrogradsky-Hamilton principle was used to obtain the equations of longitudinal, torsional and transverse vibrations of beams. The boundary value problems given by differential calculation methods under various boundary conditions were solved and analyzed.

The work [3] studied nonlinear free and forced vibrations of discrete and continuous systems. The systems are mathematically modeled using Lagrange equations, and solution methods are developed.

The work [4] theoretically studied nonlinear systems depending on the change in the stability parameters of motion. Theorems are presented for the separation of stable and unstable areas of system motion. The stability of motion of systems represented by non-homogeneous differential equations with periodic coefficients is analyzed.

The work [5] provides theoretical information on the harmonic balance method used to investigate the vibration of nonlinear systems. The nature of the nonlinear coefficients in the differential equation of motion of the system is analyzed. In addition, a method for determining the characteristic equation for systems with hysteresis-type elastic dissipative characteristics is shown and justified.

In the work [6], vibration forms of beams protected from vibrations were investigated by conducting experiments. The changes of the mode shapes with the change of frequencies are shown in the graphs, the conclusions about the effect of the frequencies on the mode shapes are drawn and the necessary recommendations are given.

In the work [7], the problems of determining the mode shapes, frequencies and optimal parameters of the joint vibrations of the second beam mounted on its free end as a dynamic absorber for a beam with one end free and the other end clamped were considered. In vibration damping, in addition to the ratio of the mass of the dynamic absorber to the mass of the beam, the damping efficiency is shown depending on the elastic properties of the beam installed as a dynamic absorber and its length. It was found that the flexibility properties and its length compared to the traditional dynamic absorber provided additional possibilities in the necessary design of this dynamic absorber.

In works [8-15], the theoretical foundations and applications of studying the dynamics and motion stability of nonlinear systems under the influence of various excitations are presented, and the issues of linearizing nonlinear characteristics are considered. The effectiveness of dynamic absorbers in various processes is evaluated.

Mathematical modeling of nonlinear mechanical systems, studying their dynamics, exploring the stability of their vibrations and reducing harmful vibrations at low and high frequencies were solved in the works [16-22]. The Stability behavior of the system was checked at different values of the parameters, conclusions were obtained as a result of numerical calculations.

The problem of determining the mean square values of nonlinear vibrations of a hysteresis-type elastic beam with a variable cross-section and a dynamic absorber under the influence of random excitations is one of the urgent problem.

MATERIALS AND METHODS

In this work, the problem of determining the mean square values of nonlinear vibrations of a hysteresis-type elastic beam with a variable cross-section and a dynamic absorber under the influence of random excitations was considered and numerically analyzed.

The system of differential equations of motion in transverse random vibrations of a hysteresis-type elastic beam with a variable cross-section and a dynamic absorber is expressed as follows as a result of the method of expanding the system into a series by eigenforms [22]:

$$
\begin{aligned}
&\ddot{q}_i + \{(1 + K_0(-\gamma_1 + j\gamma_2))p_i^2 + \frac{3E}{\rho F d_{2i}}(-\gamma_1 + j\gamma_2) \times \\
&\times \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l I(x) u_i \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)dx + \frac{E}{\rho F d_{2i}} \int_0^l \frac{\partial^2 I}{\partial x^2} u_i \frac{\partial^2 u_i}{\partial x^2} \times \\
&\times \left[1 + K_0(-\gamma_1 + j\gamma_2) + 3(-\gamma_1 + j\gamma_2) \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k(k+3)} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right] dx + \\
&+ \frac{2E}{\rho F d_{2i}} \int_0^l \frac{\partial I}{\partial x} u_i \frac{\partial}{\partial x}\Big[\frac{\partial^2 u_i}{\partial x^2}[1 + K_0(-\gamma_1 + j\gamma_2) + 3(-\gamma_1 + j\gamma_2) \times \\
&\times \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k(k+3)} \left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\Big]\Big] dx\} q_i - \mu\mu_0 u_{i0} n_0^2 (1 + (-\varphi_1 + j\varphi_2) \times \\
&\times (H_0 + f(\sigma_{\zeta_{ot}})))\zeta = - d_i \frac{\partial^2 w_0}{\partial t^2}; \qquad (1) \\
&u_{i0}\ddot{q}_i + \ddot{\zeta} + n_0^2\left(1 + (-\varphi_1 + j\varphi_2)\left(H_0 + f(\sigma_{\zeta_{ot}})\right)\right)\zeta = -\frac{\partial^2 w_0}{\partial t^2},
\end{aligned}
$$

where $u = u(x)$; $u_{i0} = u(x_0)$; $n_0^2 = \frac{c}{m}$; $\mu = \frac{m}{\rho F l}$; $\mu_0 = \frac{l}{d_{2i}}$; $d_i = \frac{d_{1i}}{d_{2i}}$; $d_{1i} = \int_0^l u_i\, dx$; $d_{2i} = \int_0^l u_i^2\, dx$; $\gamma_1, \gamma_{2*} = \gamma_2 sign\omega$ coefficients determined from the nonlinear functional expressing the dissipative properties of the beam material; E is the modulus of elasticity; $j = \sqrt{-1}$; ω is frequency of vibrations; c, m are the stiffness and mass of the dynamic absorber, respectively; ρ, F are the density and cross-sectional area of the beam material, respectively; $\delta(x)$ is the Dirac delta function; w_a is absolute displacement of the beam; σ_{ia} are absolute values of the mean square deviations of the beam; $f(\sigma_{\zeta_{ot}})$ is decrement of vibrations, $\sigma_{\zeta_{ot}}$ is a function of the mean square values of the relative deformation, $f(\zeta_{ot}) = H_1\sigma_{\zeta_{ot}} + H_2\sigma_{\zeta_{ot}}^2 + \cdots + H_s\sigma_{\zeta_{ot}}^s$, $H_0, H_1, \dots, H_s$ are parameters of the hysteresis node determined experimentally, depending on the damping properties of the dynamic absorber material; φ_1, φ_2 are coefficients depending on the dissipative properties of the dynamic absorber material, determined from the hysteresis surface; $K_0, K_1, \dots, K_r$ are experimentally determined parameters of the hysteresis node, which depend on the damping properties of the beam material; p_i are natural frequencies of the beam; $I(x) = \frac{bh^3}{12}$ is moment of inertia; $b = b(x)$, h and l are width, height and length of the beam, respectively; w is displacement of the base.

First, let's reduce the system of differential equations (1) to a system of algebraic equations using the differential operator $S = \frac{d}{dt}$. Introducing the substitution $S = j\omega$ into the resulting system of algebraic equations, it is possible to determine the variables q_i $(i = \overline{1,n})$ and ζ:

$$q_i = \frac{A_{10} + jB_{10}}{A_2 + jB_2} W_0;$$

$$\zeta = \frac{A_{20} + jB_{20}}{A_2 + jB_2} W_0, \quad (2)$$

where

$$A_{10} = d_i\omega^2 - (d_i + \mu\mu_0 u_{i0}) n_0^2 N_{01}; \quad B_{10} = -(d_i + \mu\mu_0 u_{i0}) n_0^2 N_{02};$$
$$N_{01} = 1 - \varphi_1\big(H_0 + f(\zeta_{ot})\big); \quad N_{02} = \varphi_2\big(H_0 + f(\zeta_{ot})\big);$$
$$A_{20} = -p_i^2 N_{11} - p_i^2 - (u_{i0}\, d_i - 1)\omega^2; \quad B_{20} = -p_i^2 N_{12};$$

$$N_{11} = -K_0\gamma_1 - \frac{3E}{\rho F d_{2i} p_i^2}\gamma_1 \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l I(x) u_i \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right) dx +$$
$$+ \frac{E}{\rho F d_{2i} p_i^2} \int_0^l \frac{\partial^2 I(x)}{\partial x^2} u_i \frac{\partial^2 u_i}{\partial x^2}\left[1 - K_0\gamma_{1_1} - 3\gamma_1 \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k(k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right] dx + \frac{2E}{\rho F d_{2i} p_i^2} \times$$
$$\times \int_0^l \frac{\partial I(x)}{\partial x} u_i \frac{\partial}{\partial x}[\frac{\partial^2 u_i}{\partial x^2}[1 - K_0\gamma_1 - 3\gamma_1 \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k(k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k]]dx;$$

$$N_{12} = K_0\gamma_2 + \frac{3E}{\rho F d_{2i} p_i^2}\gamma_2 \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k(k+3)} \int_0^l I(x) u_i \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right) dx +$$
$$+ \frac{E}{\rho F d_{2i} p_i^2} \int_0^l \frac{\partial^2 I(x)}{\partial x^2} u_i \frac{\partial^2 u_i}{\partial x^2}\left[1 + K_0\gamma_2 + 3\gamma_2 \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k(k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right] dx + \frac{2E}{\rho F d_{2i} p_i^2} \times$$
$$\times \int_0^l \frac{\partial I(x)}{\partial x} u_i \frac{\partial}{\partial x}\left[\frac{\partial^2 u_i}{\partial x^2}\left[1 + K_0\gamma_2 + 3\gamma_2 \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k(k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right]\right] dx;$$

$$A_2 = (p_i^2 - \omega^2 + p_i^2 N_{11})(n_0^2 N_{01} - \omega^2) - p_i^2 n_0^2 N_{12} N_{02} - u_{i0}^2 \mu\mu_0 \omega^2 n_0^2 N_{01};$$
$$B_2 = (p_i^2 - \omega^2 + p_i^2 N_{11}) n_0^2 N_{02} + p_i^2 N_{12}(n_0^2 N_{01} - \omega^2) - u_{i0}^2 \mu\mu_0 \omega^2 n_0^2 N_{02}.$$

Taking into account that the absolute acceleration of a hysteresis-type dissipative characteristic of a vibration-protected beam with variable cross-section consists of the sum of the relative acceleration and the base acceleration, and using expressions (2), we write the expression for the root mean square values of the absolute accelerations of the vibration-protected beam as follows:

$$\sigma_{W_0}^2 = \int_{-\infty}^{\infty} \left|1 - u_i\omega^2 \frac{A_{10} + jB_{10}}{A_2 + jB_2}\right|^2 S_{W_0}(\omega) d\omega. \quad (3)$$

In order to calculate this integral expression, we obtain the following expression for the spectral density of the fundamental accelerations:

$$S_{W_0}(\omega) = \frac{D_{W_0}\alpha\omega_э^3}{\pi(\omega_э^2 - \omega^2 + j\alpha\omega_э\omega)(\omega_э^2 - \omega^2 - j\alpha\omega_э\omega)}, \quad (4)$$

where D_{W_0} is the dispersion of the fundamental acceleration; $\omega_э$ is the frequency with the highest probability in the vibration spectrum; α is a parameter characterizing the width of the vibration spectrum.

As a result, the integral (3) is expressed in the following form:

$$\sigma_{W_0}^2 = R_* \int_{-\infty}^{+\infty} \frac{Y_6(\omega)}{Z_6(i\omega) Z_6(-i\omega)} d\omega, \quad (5)$$

where

$$Y_6(\omega) = a_{35}\omega^{10} + a_{34}\omega^8 + a_{33}\omega^6 + a_{32}\omega^4 + a_{31}\omega^2 + a_{30};$$

$$Z_6(i\omega) = t_6(i\omega)^6 + t_5(i\omega)^5 + t_4(i\omega)^4 + t_3(i\omega)^3 + t_2(i\omega)^2 + t_1(i\omega)^1 + t_0;$$
$$a_{35} = 0; a_{34} = (1 - d_i u_i)^2;$$
$$a_{33} = 2(\big((\mu\mu_0 u_0 + d_i)u_i - u_0^2\mu\mu_0 + 1\big)N_{01}n_0^2 - p_i^2(N_{11} + 1))(1 - d_i u_i);$$
$$a_{32} = (N_{01}^2 + N_{02}^2)(\mu\mu_0 u_0(u_0 - u_i) - d_i u_i + 1)^2 n_0^4 +$$
$$+2\left(\mu\mu_0 u_0(u_0 - u_i)\big((N_{11} + 1)N_{01} + N_{12}N_{02}\big) - 2(N_{11} + 1)N_{01}(d_i u_i - 1)\right)n_0^2 p_i^2 + ((N_{11} + 1)^2 + N_{12}^2)p_i^4;$$
$$a_{31} = -2[(N_{11} + 1)(N_{01}^2 + N_{02}^2)(\mu\mu_0 u_0(u_0 - u_i) - d_i u_i + 1)n_0^2 + N_{01}((N_{11} + 1)^2 + N_{12}^2)p_i^2]n_0^2 p_i^2;$$
$$a_{30} = ((N_{11} + 1)^2 + N_{12}^2)(N_{01}^2 + N_{02}^2)n_0^4 p_i^4.$$

The values of the integral (5) are as follows:

$$\sigma_{W_0}^2 = \frac{\pi R_*}{t_5}\frac{\begin{vmatrix} a_{35} & a_{34} & a_{33} & a_{32} & a_{31} & a_{30} \\ -t_6 & t_4 & -t_2 & t_0 & 0 & 0 \\ 0 & -t_5 & t_3 & -t_1 & 0 & 0 \\ 0 & t_6 & -t_4 & t_2 & -t_0 & 0 \\ 0 & 0 & t_5 & -t_3 & t_1 & 0 \\ 0 & 0 & -t_6 & t_4 & -t_2 & t_0 \end{vmatrix}}{\begin{vmatrix} t_5 & -t_3 & t_1 & 0 & 0 & 0 \\ -t_6 & t_4 & -t_2 & t_0 & 0 & 0 \\ 0 & -t_5 & t_3 & -t_1 & 0 & 0 \\ 0 & t_6 & -t_4 & t_2 & -t_0 & 0 \\ 0 & 0 & t_5 & -t_3 & t_1 & 0 \\ 0 & 0 & -t_6 & t_4 & -t_2 & t_0 \end{vmatrix}}, \quad (6)$$

where $R_* = \frac{D_{W_0}\alpha\omega_{\text{э}}^3}{\pi}$; $t_6 = 1; t_5 = \alpha\omega_{\text{э}} + \delta_0; t_4 = \alpha\omega_{\text{э}}\delta_0 + \omega_{\text{э}}^2 + \delta_1; t_3 = \omega_{\text{э}}^2\delta_0 + \alpha\omega_{\text{э}}\delta_1 + \delta_2;$

$$t_2 = \omega_{\text{э}}^2\delta_1 + \alpha\omega_{\text{э}}\delta_2 + \delta_3; t_1 = \omega_{\text{э}}^2\delta_2 + \alpha\omega_{\text{э}}\delta_3; t_0 = \omega_{\text{э}}^2\delta_3;$$
$$\delta_3 = p_i^2 n_0^2[\big((N_{11} + 1)N_{01} - N_{02}N_{12}\big)^2 + ((N_{11} + 1)N_{02} + N_{01}N_{12})^2]^{\frac{1}{2}};$$
$$\delta_0^2 - 2\delta_1 = \Delta_0;$$
$$\delta_1^2 - 2\delta_0\delta_2 + 2\delta_3 = \Delta_1; \quad (7)$$
$$\delta_2^2 - 2\delta_1\delta_3 = \Delta_2,$$
$$\Delta_0 = -2(p_i^2(1 + N_{11}) + (1 + \mu\mu_0 u_{i0}^2)n_0^2 N_{01});$$
$$\Delta_1 = 2p_i^2 n_0^2\big((1 + N_{11})N_{01} - N_{12}N_{02}\big) + \Big((1 + \mu\mu_0 u_{i0}^2)n_0^2 N_{01} + p_i^2(1 + N_{11})\Big)^2 + \Big((1 + \mu\mu_0 u_{i0}^2)n_0^2 N_{02} + p_i^2 N_{12}\Big)^2;$$
$$\Delta_2 = -2n_0^2 p_i^2[(N_{01}^2 + N_{02}^2)(1 + N_{11})(1 + \mu\mu_0 u_{i0}^2)n_0^2 + N_{01}p_i^2((1 + N_{11})^2 + N_{12}^2)].$$

We calculate the integral representing the root mean square displacements of a beam with a variable cross-section and elastic dissipative characteristics of the hysteresis type, and after some simplifications we write

$$\sigma_{q_i}^2 = \frac{1}{b_1 b_2 p_i^3}\frac{d_i^2 D_{W_0}\left(-b_1\alpha^2 p_i\,\omega_{\text{э}}^2 - \alpha\left(b_1^2\omega_{\text{э}} p_i^2 + \omega_{\text{э}}^3\right) - b_1 b_2 p_i^3\right)}{b_2^2 p_i^4 + \alpha b_1 b_2\omega_{\text{э}} p_i^3 + (\alpha^2 b_2 - 2b_2 + b_1^2)p_i^2\omega_{\text{э}}^2 + \alpha b_1 p_i\,\omega_{\text{э}}^3 + \omega_{\text{э}}^4}, \quad (8)$$

where

$$b_1 = [2(b_2 - 1 + N_{10i})]^{\frac{1}{2}};$$
$$b_2 = [(1 - N_{10i})^2 + (N_{20i})^2]^{\frac{1}{2}};$$
$$N_{10i} = \gamma_1 K_0 + \frac{1}{p_i^2}[\gamma_1\frac{3E}{\rho F d_{2i}}\sum_{k=1}^{n} K_k\sigma_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l I(x)u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)dx -$$
$$-\frac{E}{\rho F d_{2i}}\int_0^l\frac{\partial^2 I(x)}{\partial x^2}u_i\frac{\partial^2 u_i}{\partial x^2}\left[1 - \gamma_1 K_0 - 3\gamma_1\sum_{k=1}^{n} K_k\sigma_{ia}^k\frac{h^k}{2^k(k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right]dx - \frac{2E}{\rho F d_{2i}}\times$$
$$\times\int_0^l\frac{\partial I(x)}{\partial x}u_i\frac{\partial}{\partial x}\left[\frac{\partial^2 u_i}{\partial x^2}\left(1 - \gamma_1 K_0 - 3\gamma_1\sum_{k=1}^{n} K_k\sigma_{ia}^k\frac{h^k}{2^k(k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)\right]dx;$$
$$N_{20i} = \gamma_2 K_0 + \frac{1}{p_i^2}[\frac{3E}{\rho F d_{2i}}\gamma_2\sum_{k=1}^{n} K_k\sigma_{ia}^k\frac{h^k}{2^k(k+3)}\int_0^l I(x)u_i\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_i}{\partial x^2}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)dx +$$
$$+\frac{E}{\rho F d_{2i}}\int_0^l\frac{\partial^2 I(x)}{\partial x^2}u_i\frac{\partial^2 u_i}{\partial x^2}\left[\gamma_2 K_0 + 3\gamma_2\sum_{k=1}^{n} K_k\sigma_{ia}^k\frac{h^k}{2^k(k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right]dx + \frac{2E}{\rho F d_{2i}}\times$$

$$\times \int_0^l \frac{\partial I(x)}{\partial x} u_i \frac{\partial}{\partial x}\left[\frac{\partial^2 u_i}{\partial x^2}\left(\gamma_2 K_0 + 3\gamma_2 \sum_{k=1}^{n} K_k \sigma_{ia}^k \frac{h^k}{2^k (k+3)}\left|\frac{\partial^2 u_i}{\partial x^2}\right|^k\right)\right] dx;$$

The expression of the mean square values (8) makes it possible to determine and analyze the dynamics and stability of the beam's vibrations under the influence of random excitations at different values of the parameters.

RESULTS AND DISCUSSION

The beam material was 40X grade steel ($E = 2.08 \cdot 10^{11} \frac{N}{m^2}$; $\rho = 7810\ \frac{kg}{m^3}$), and for the boundary value problem, a beam with one end clamped and the other end free was considered.

We numerically analyze the vibrations of a beam with one end fixed and one end free. The parameters are taken as follows: $l = 0.25;\ \gamma_1 = \frac{3}{4};\ \gamma_2 = \frac{1}{\pi}; h = 2 \cdot 10^{-3}; K_0 = H_0 = 0;\ K_1 = 6.760624; K_2 = -8278.5937; K_3 = 5894761;$

For the case where the cross section is variable:

$$d_{11} = 195.6830992;\ d_{21} = 1.984833412 \cdot 10^{10}; \mu = 0.5; \mu_0 = 1.259551550 \cdot 10^{-11};$$

$$d_1 = 9.858918034 \cdot 10^{-9};\ I_{1l} = 1.941963501 \cdot 10^{-11} m^4; m_1 = \mu \rho S_l h = 0.2335032198\ kg;$$

$$N_{11} = 0.2160785846 \cdot 10^{-5} - 0.3059812236 \cdot 10^{-2} \sigma_{ia} + 269.8256554 \sigma_{ia}^2 - 1.471908755 \cdot 10^7 \sigma_{ia}^3;$$

$$N_{12} = 0.32892743 \cdot 10^{-5} + 0.1311697993 \cdot 10^{-2} \sigma_{ia} - 117.8134994 \sigma_{ia}^2 + 6.67251608 \cdot 10^6 \sigma_{ia}^3.$$

For the case where the cross section is unchanged:

$$N_{11} = -0.1988432491 \cdot 10^{-2} \sigma_{ia} + 174.097791 \sigma_{ia}^2 - 9.648472811 \cdot 10^6 \sigma_{ia}^3;$$

$$N_{12} = 0.84391696 \cdot 10^{-3} \sigma_{ia} - 73.88939738 \sigma_{ia}^2 + 4.094939045 \cdot 10^6 \sigma_{ia}^3.$$

In order to analyze the deviation of the resonance curves from the vertical, we draw graphs of the N_{11} function in cases where the cross-section of the beam is variable and constant.

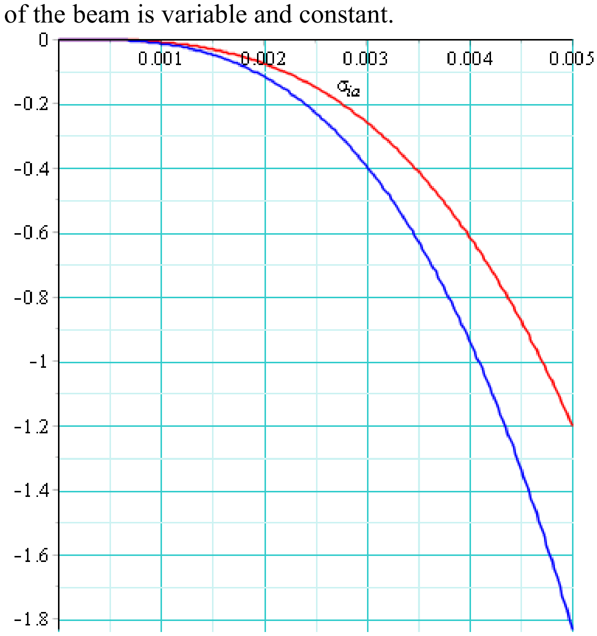


Figure 1. Graphs of the deviation of the resonance curve from the vertical in cases where the cross-section of the beam is variable (blue) and constant (red)

Figure 1 shows the graphs of the deviation of the resonance curve from the vertical in the cases where the cross-sectional width of the beam is variable (blue) according to the law $b = 0.02 + 0.01 \sin(80x)$and $b = 0.02$ is constant (red). From this it can be said that the deviation of the resonance curve from the vertical is less when the cross-sectional width of the beam is variable than when it is constant.

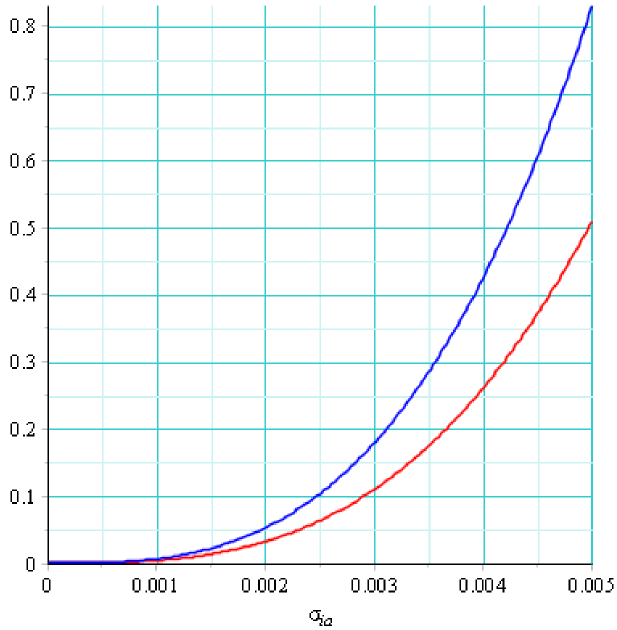


Figure 2. Energy distribution graphs for variable (blue) and constant (red) cross-sections of the beam

Figure2 shows the energy dissipation graphs for the cases where the cross-sectional width of the beam varies according to the law b=0.02+0.01 sin(80x) (blue) and b=0.02 is constant (red). From this it can be said that the energy dissipation is greater when the cross-sectional width of the beam varies than when it is constant.

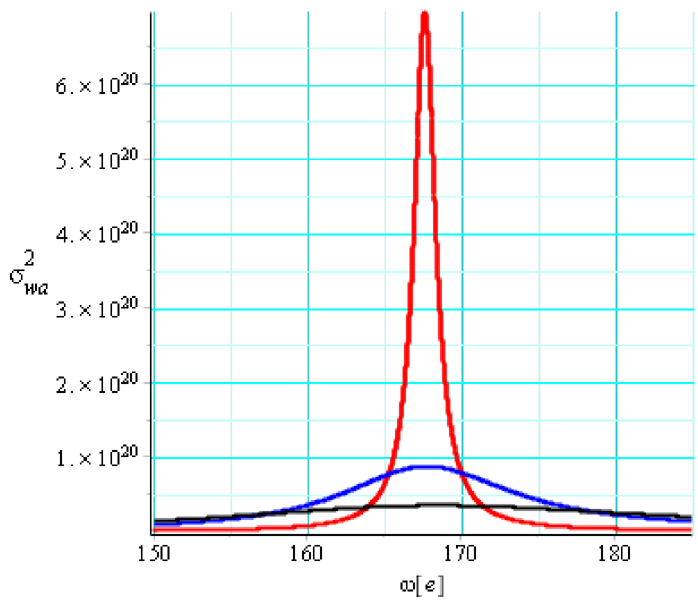
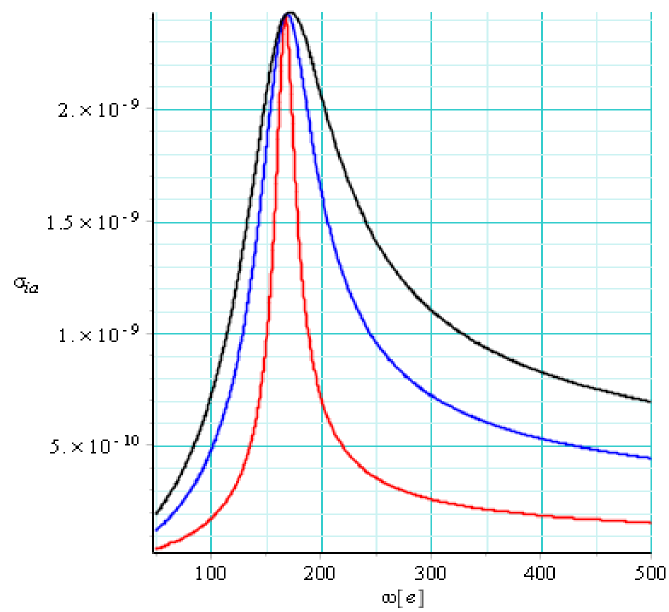


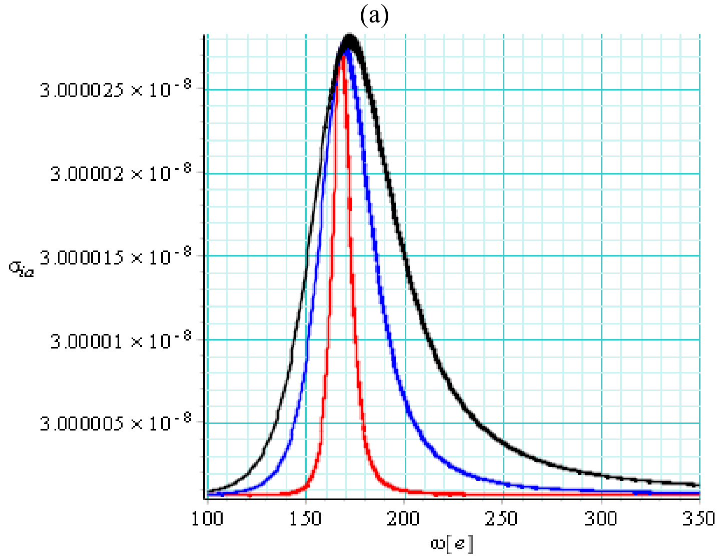
Figure 3. (6) Graphs of root mean square absolute accelerations
($\alpha = 0.01\ (red); 0.08(blue); 0.1(black)$.*)*

Figure 3 shows graphs of the change in the mean square values of the absolute accelerations of the beam (6) depending on the frequency at which the vibration probability is high at the values of the parameter α *(*$\alpha = 0.01\ (red); 0.08(blue); 0.1(black)$. From these graphs, it can be seen that with an increase in the parameter characterizing the spectrum width, the root mean square values of the absolute accelerations of the beam, which vary according to the law $b = 0.02 + 0.01\sin(80x)$ decrease.

In order to analyze the dynamics of vibrations of a cross-section with elastic dissipative characteristics of the hysteresis type with a width of $b = 0.02 + 0.01\sin(80x)$ and a constant width of $b = 0.02$ under the influence of random excitations, we analyze the change in the graphs of the expression of the mean square values (8) depending on the frequency at which the vibration probability is high.



(a)



(b)

Figure 4. Changing the mean square values (8)
$\alpha = 0.01\ (red)); 0.08(blue); 0.1(black)$.

In the graphs in Fig. 4-a), the graphs of the expression of the mean square value (8) are presented for the cases where the cross-sectional width of the beam varies according to the law $b = 0.02 + 0.01\sin(80x)$ and in Fig. 4-b) $b = 0.02$ is constant. From the graphs presented for both cases, it can be said that the mean square values of the beam with a variable cross-section are almost ten times smaller than in the case where the cross-section is constant. In addition, in both cases, an increase in the parameter α, which characterizes the width of the vibration spectrum, does not change the mean square value around the resonant frequency, but at a sufficient distance from the resonant frequency leads to an increase in the mean square value.

CONCLUSION

The expressions of the mean square values of the displacements and base accelerations of a beam with a hysteresis-type elastic dissipative characteristic with a variable cross-section were numerically analyzed. The changes in the mean square values of the displacements and base accelerations at different values of the parameters were shown on the basis of graphs, and the corresponding conclusions were drawn. It was shown that the deviation of the resonance curve from the vertical was less in the case of a beam with a hysteresis-type elastic dissipative characteristic with a variable cross-section than in the case of a fixed one. It was shown that the energy dissipation was greater in the case of a beam with a hysteresis-type elastic dissipative characteristic with a variable cross-section than in the case of a fixed one. The nonlinear vibrations of a hysteresis-type dissipative beam with a variable cross-section under the influence of random excitations were mathematically modeled and their mean square values were determined analytically for the general case depending on the system parameters. The expressions of the root mean square values of the displacements of a hysteresis-type dissipative beam with a variable cross-section were numerically analyzed. The changes in the root mean square values of the displacements at different values of the parameters were shown on the basis of graphs and the corresponding conclusions were drawn.

REFERENCES

1. A. N. Zotov, A. P. Tokarev, A Dynamic Vibration Absorber with Adjustable Stiffness mechanics of machines Volume **52**, pp. 525–531, (2023).
2. Y.Cheng, D. Li., Ch. Li, Dynamic vibration absorbers for vibration control within a frequency band Contents lists available at Science Direct journal homepage: www.elsevier.com/locate/jsvi 0022-460X/$ - see front matter & (2010) Elsevier Ltd. All rights reserved. doi:10.1016/j.jsv.2010.10.018.
3. O. Drachev, I. Turbin, I. Amirdzhanova, T. Varentsova, V. Petrova, Togliatti State University, Togliatti, Russian Federation Dynamic vibration absorber for shaft machining MATEC Web of Conferences 329, 03045 (2020). https://doi.org/10.1051/matecconf/202032903045.
4. A. T. Zhakash, E. A. Dzhakashova, O. M. Tursynbay, Numerical methods for calculating vibrations of straight beams of variable cross-section International Scientific Journal Theoretical & Applied Science, (2019). http://T-Science.org.
5. N. Zh. Kinash, V. B. Kashuba, D. H. Nguyen, Dynamic Damping Modes in Systems with Several Degrees of Freedom Systems. Methods. Technologies. N.Zh. Kinash et al. Dynamic Damping Modes, No. **1**(33) pp. 19-28 19 UDC 62.752, 621:534.833; 888.6, (2017) DOI: 10.18324/2077-5415-2017-1-19-28.
6. T. M. Mahesha, K. Ranjith, Experimental Inverstigation of Transverse Vibration Characteristics of Beams. International Journal for Research in Applied Science & Engineering Technology (IJRASET). Volume **7**, pp.265-273, (2019).
7. H. Yingyu, W. Waion, Ch. Li, Optimal design of a beam-based dynamic vibration absorber using fixed-points theory. Journal of Sound and Vibration. Volume **421**, pp. 111-131, (2018).
8. Y. V. Nemirovsky, A. V. Mishchenko, R. F. Terletskii, Dynamic analysis of composite beams with variable cross-section Computational Continuum Mechanics@journal-icmm. 2 v.8, 2015. Journal of Mathematical Sciences, Vol. **223**, No. 1, (2017).
9. J. Kisilowski, R. Kowalik, Numerical Testing of Switch Point Dynamics-A Curved Beam with a Variable Cross-Section Materials (Basel), **13**(3), 701, (2020). doi: 10.3390/ma13030701.
10. M. A. Pavlovsky, L. M. Ryzhkov, V. B. Yakovenko, O. M. Dusmatov, Nonlinear problems of the dynamics of vibration protection systems. - K.: Tekhnika, - p. 204. (1997).
11. A. Szekre´nyes, Natural vibration-induced parametric excitation in delaminated Kirchhoff plates. Journal of Composite Materials, Volume **50**(17), p.2337–2364, (2016).
12. L. A. Baragunova, M. M. Shogenova, O. M. Shogenov, E. A. Yafaunov, Free vibrations of variablesection beams taking rotational and frictional forces into account Bulletin of Science and Research Center of Construction, (40)-7-20. https: //doi.org/10.37538/2224-9494-2024-1
13. V. P. Pavlov, L. R. Nusratullina, E.M. Nusratullin, Refining the results of numerical calculations of the frequencies of transverse vibrations of a beam of variable cross-section with elastic fixation by extrapolation method UGATU Bulletin, Vol. **27**, No. 3 (101), pp. 29–37 (2023). http://journal.ugatu.su
14. G. A. Bogdanova, Z. V. Ivanova, Application of dynamic vibration absorbers to increase the stability of communication tower structures to wind effects. Bulletin of the St. Petersburg State Transport University, Vol. **20**, No. 3, (2023).

15. L. Hans, B. Viktor, J. Mattias, S.L. Grétarsson, Nonlinear dynamic absorber to reduce vibration in hand-held impact machines International Conference on Engineering Vibration Ljubljana, Slovenia, 7-10 September (2015).
16. O. Drachev, I. Turbin, I. Amirdzhanova, T. Varentsova, V. Petrova, Togliatti State University, Togliatti, Russian Federation Dynamic vibration absorber for shaft machining MATEC Web of Conferences 329, 03045 (2020) https://doi.org/10.1051/matecconf/202032903045.
17. Mirsaidov M. M., Dusmatov O .M., Khodjabekov M. U. The problem of mathematical modeling of a vibration protected beam under kinematic excitations. IOP Conf. Series: Materials Science and Engineering, 1030 (2021) 012069, https://doi:10.1088/1757-899X/1030/1/012069
18. M. M. Mirsaidov, O. M. Dusmatov, M. U. Khodjabekov, Mode Shapes of Transverse Vibrations of Beams Protected from Vibrations in Kinematic Excitations. Lecture Notes in Civil Engineering 170, pp. 217-227, (2021). https://doi:org/10.1007/978-3-030-79983-0_20
19. M. M. Mirsaidov, O. M. Dusmatov, M. U. Khodjabekov, Stability of nonlinear vibrations of plate protected from vibrations. Journal of Physics: Conference Series, 1921 (2021) 012097, https://doi:10.1088/1742-6596/1921/1/012097
20. M. M. Mirsaidov, O. M. Dusmatov, M. U. Khodjabekov, Mode Shapes of Hysteresis Type Elastic Dissipative Characteristic Plate Protected from Vibrations. Lecture Notes in Civil Engineering 282, pp. 127-140, (2022). https://doi.org/10.1007/978-3-031-10853-2_12.
21. O. M. Dusmatov, M. U. Khodjabekov, F. U. Kasimova, Dynamics of a beam with variable cross-section protected from vibration. Web of Conferences 549, 00001 (2024), https://doi.org/10.1051/e3sconf/202454900001.
22. O. M. Dusmatov, F.U. Kasimova, The problem of damping nonlinear transverse vibrations of an elastic beam under random excitations. "SamDU Scientific Information" scientific journal No. **3**, (2025).