Random Parametric Transverse Vibrations of a Beam With Elastic Dissipative Characteristics of the Hysteresis Type

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Abstract: This work deals with the problem of investigating the dynamics of nonlinear random parametric transverse vibrations of a vibration-protected beam. The nonlinear single-valued function representing the dissipative property of the elastic damping element material of the beam was taken into account in the form of a linear function using the statistical linearization method. In numerical calculations, the linearization coefficients were determined based on the Pisarenko-Boginich's hypothesis. Using the Ito method, an analytical expression of the mean square values of the vibration-protected beam was determined, and conclusions were drawn based on numerical calculations.

Key words: imperfectly elastic, strain, hysteresis, dissipative, random parametric vibration, mean square value.

INTRODUCTION

In engineering and technology, the problems of mathematical modeling of vibrations of various types of devices, taking into account the nonlinear elastic dissipative characteristics of their materials, determining their dynamic characteristics in various processes, and ultimately selecting their structural parameters to ensure long-term effective operation and durability are relevant. A large number of scientific studies are being conducted to study the vibrations of various mechanical systems with lumped and distributed masses with nonlinear characteristics, evaluate their dynamics, and verify their stability.

In the article [1], nonlinear parametric vibrations of a beam with a dynamic absorber under the influence of external excitations are studied taking into account the elasticity and damping properties of materials. The linearization method is used to solve nonlinear differential equations of motion of the system. The non-stationary and stationary values of the amplitude and phase of vibrations are determined analytically. The stability conditions of stationary motion are obtained based on the Rous-Hurwitz criterion. The effect of changing the parameter values on the amplitude-frequency characteristic constructed based on the calculation results is shown.

The article [2] deals with the study of nonlinear parametric vibrations of a beam combined with an element with friction properties under the influence of random excitations. The nonlinearity is taken in the form of a cubic degree polynomial of the Winkler type. The differential equation describing the vibrations of the beam is determined, it is shown that it does not have an exact analytical solution, and an approximate solution is proposed based on the linearization method . The proposed solution is compared with the proposed solution using a numerical approach using Monte Carlo, and the results are shown to be in good agreement.

The work [3] shows that the nonlinearity of the hysteresis type with elastic dissipative characteristic is widespread in engineering fields and many mathematical models have been developed to describe it. Theoretically, it is shown that the restoring forces of the hysteresis type are usually divided into equivalent stiffness and equivalent damping. It is proved that due to the complexity of the hysteresis type nonlinearity, it is difficult to obtain analytical expressions for these equivalent components. A method for studying the oscillations of systems with hysteresis type elastic dissipative characteristic with a high number of parametrically excited degrees of freedom is proposed. Expressions for the equivalent stiffness and equivalent damping coefficients are obtained using the Bouc-Wen model. The mean square values are determined and numerically analyzed using the stochastic averaging method.

In the article [4], bridges reinforcement for usable geometric linear not been cable bending angle and bridge vibration the secret of the effect into account received without random movement sunder the influence parametric vibrations studied. Random in motion cable soft characteristic maybe as maybe and bridge plate joint movement differential equations system. This system of differential equations is transformed into Ito differential equations and the Milstein-Platen method is used for numerical analysis. In order to avoid the influence of the parametric diffusion coefficient in Ito differential equations, an iterative method for solving random differential vibrations of the beam is proposed. The amplitude, spectral density and density function changes are analyzed and the results obtained by this method are compared with those obtained by the Gaussian method.

This paper [5] discusses the different transitions from periodicity to quasi-periodicity, intermittency and chaos in systems with hysteresis. Three types of hysteresis are considered: the Bouc-Wen law of hysteresis, the Masing law and the pseudoelastic constitutive law typical of shape memory alloys. The first two rate-independent models do not account for heat transfers, while in the third case the thermodynamic transformations are taken into account. It is shown that these systems share similar trends for the loss of stability of the fundamental response to highly nonlinear responses of various kinds.

In this work [6], contribution to propose a metaheuristic-based parametric identification process for the design of the Bouc–Wen–Baber–Noori hysteresis model and evaluate the results by using some established experimental investigation methods. To fulfill this aim, the Fuzzy Adaptive Charged System Search is proposed for optimization in which a fuzzy-logic-based parameter tuning process is utilized to achieve better performance in comparison with the standard Charged System Search algorithm. For nonlinear dynamic analysis, an Iterative Hysteretic Analysis process is also intbeamuced for conducting the precise analysis of the structure with exact solutions. Comparing the metaheuristic-based results to the experimental findings demonstrates that the proposed algorithm is capable of providing very competitive results. Besides, the proposed adaptive method is capable of pbeamucing very competitive results in comparison with different optimization algorithms.

The steady-state dynamic response of a structure isolated by a nonlinear wire rope spring operating in the direction of gravity is experimentally studied in [7]. The isolated structure consists of two cantilever beams with a lumped mass at the tip. The force-displacement cycles provided by the isolator show a hysteretic behavior due to inter-wire friction and geometric nonlinearities. The restoring force is nonsymmetric exhibiting softening under compression and hardening under tension. The device rheological response is identified using experimental data and a suitable mechanical model. The frequency response curves for increasing levels of the vertical base excitation are obtained for the standalone device, the isolated and non-isolated structure. The expected softening trend of the isolation system and the increase of the displacement amplitude at low frequencies are ascertained both theoretically and experimentally.

This paper [8] deals with the investigation of the random vibration of a Bouc-Wen hysteretic system under Poisson white noise excitations. The solution of the generalized Fokker-Planck-Kolmogorov equation is expressed in the form of a radial basis and a neural network with Gaussian activation functions. As an example to illustrate the process, a steel fiber reinforced concrete column loaded with Poisson white noise is studied. The effects of several important parameters of the system and excitation on the stochastic response are evaluated and the obtained results are compared with those obtained by Monte Carlo simulation. Numerical results show that the radial basis and neural network method can accurately analyze the stationary response with significantly higher computational efficiency.

The work [9] presents methods for using stochastic methods in solving problems of protecting various mechanical systems from harmful vibrations in random processes, and develops recommendations.

The work [10] studies the stability of parametric vibrations of a plate under the influence of external pulsed wind. In engineering practice, it is shown that the air flow outside a car or aircraft always exhibits a pulsating characteristic, which turns elastic structural components and the external air flow into a parametric excitation system. The parametric vibration equation of a plate under the influence of a pulsating external air flow is obtained using Hamilton's principle. The linear potential flow theory is used to calculate the aerodynamic force. The stability of solutions is analyzed using Floquet theory, and the correctness of the results is shown using numerical simulations. The influence of plate parameters on the stability of vibrations is studied, and some practical conclusions are proposed from simulations and analyses for the optimal design of a plate in an aerodynamic environment.

The work [11] considered the vibrations of a distributed linear mechanical system under the influence of positional forces. Using the decomposition method, the conditions under which the problem of analyzing the stability of solutions of second-order differential equations can be solved were obtained. The direct Lyapunov method was used for the decomposition. In this case, special forms of the Lyapunov-Krasovsky functionals were proposed. The stability conditions obtained in the analytical form were numerically analyzed and the correctness of the results was shown by comparisons.

The work [12] checks the stability of solutions of Ito differential equations obtained by several numerical methods in dimensional Wiener processes. Positive operators on positive cones from the Krein-Perron-Frobenius theory are used for the boundary of the obtained solutions of Ito differential equations. In addition, the problem of determining the exact intervals of the asymptotic mean square for systems affected by state-dependent noisy excitations and the asymptotic probability quantities for systems affected by state-independent noisy excitations is also solved. The advantages and disadvantages of the methods for checking asymptotic stability are analyzed. Recommendations for their application are given.

In the works [13], the Euler-Bernoulli beam with viscoelastic characteristics and solves problems aimed at improving the performance of beam during their movement. In addition, the influence of the inertia factor on the stability limits of the beam is determined. In this case, the beam is taken as a system with linear or exponentially distributed parameters along the length. The results show that a decrease in the density gradient parameter and an increase in the elastic modulus gradient parameter increase the eigenfrequencies and expand the stability limit of the beam. It is found that the density and elastic modulus gradients are inversely proportional in the oscillatory motion of the beam.

In the works [14], the vibrations of a rectangular plate mounted on elastic springs at its edges were investigated using approximating series. The expression for the eigen frequency was determined depending on the system parameters and numerically analyzed. The analytical expression for the eigen modes of vibration satisfying the boundary conditions was proposed in a new form, that is, in the form of adding additional polynomials to the Fourier series.

The work [15] investigated the transverse vibrations of a plate passing between rollers rotating around two fixed axes using asymptotic methods. The dynamic model of nonlinear forces was obtained using the Duffing equation. The damping and stiffness coefficients of the rollers were analyzed, and their optimal values for damping plate vibrations were determined.

The work [16] systematically describes the basic principles of the theory of random functions used in various practical areas. Much attention is paid to the correlation theory of random processes and the determination of probabilistic properties of dynamical systems. Along with systems described by ordinary differential equations, systems described by partial differential equations (systems with distributed parameters) are also studied. The problem of determining the transfer function of a linear system that minimizes the error variance for given characteristics of the useful signal and noise is highlighted.

In the works [17-23], the influence of material characteristics on the vibrations of mechanical systems was studied. Motions of mechanical systems are modeled taking into account dissipative characteristics. Based on the obtained model, the behavior of the mechanical system under various external influences was studied and conclusions were drawn.

One of the current problems is the investigation of the dynamics and stability of nonlinear vibrations of a beam with hysteresis-type elastic dissipative characteristics under the influence of random parametric excitations.

MATERIAL AND METHODS

Let's consider the problem of investigating the dynamics of nonlinear transverse random parametric vibrations of a beam with elastic dissipative characteristics of the hysteresis type.

The differential equation of motion for random parametric transverse vibrations of a beam with elastic dissipative characteristics of the hysteresis type is as follows:

$$EI\frac{\partial^4 w}{\partial x^4}+\frac{24}{h^3}EI\nu\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}\int_0^{\frac{h}{2}}\varphi(a)z^2dz\right)+m\frac{\partial^2 w}{\partial t^2}=-m\omega_{01}^2\xi_0(t)q_i, \quad (1)$$

where m is the mass per unit length of the beam; w is the deflection of the beam; E is Young's module; I is the moment of inertia; $\xi_0(t)$ is a variable representing a stationary normal random process; ω_{01} is the natural frequency of the baem; h is beam height; ω is frequency of vibrations; ν_1, ν_2 are linearization coefficients representing the dissipative properties of the beam material [17]; x is the coordinate; $\varphi(a)$ is function representing energy dissipation; z is axis perpendicular to the beam; $q_i = q_i(t)$ is time-dependent function; $i^2=-1$;

Let's look for the solution to equation (1) as follows:

$$w_i(x,t)=\sum_{i=1}^{\infty}u_i(x)q_i(t). \quad (2)$$

where, the mode shapes of the beam $u_i(x)$ satisfies the following equation:

$$EI\frac{\partial^4 u_i}{\partial x^4}-m\omega_{01}^2u_i=0\,. \quad (3)$$

If the function $q_i(t)$ is taken in the form $q_i = q_{ia} cos\theta$, it is possible to write the expression for the relative energy distribution (q_{ia} is amplitude of q_i and θ is phase).

$$\varphi(a) = \sum_{j_1=0}^{S_1} C_{j_1} q_a^{j_1} \left| z \frac{\partial^2 u_i}{\partial x^2} \right|^{j_1}, \tag{4}$$

where C_{j_1} are parameters determined from the hysteresis loop.

Substituting expression (4) into differential equation (1) and applying the Bubnov-Galerkin method, it is possible to obtain the following equation for determining the variable $q_i(t)$:

$$\ddot{q}_i + \omega_{01}^2 (1 + \nu R) q_i = -\omega_{01}^2 \xi_0(t) q_i(t), \tag{5}$$

where

$$R = \mathrm{C}_0 + \frac{3EI}{\omega_0^2 m d_1} \sum_{j_1=1}^{s_1} C_{j_1} q_a^{j_1} \frac{h^{j_1}}{2^{j_1}(j_1+3)} G_{j_1}; \tag{6}$$

$$G_{j_1} = \int_0^l u_i(x) \frac{\partial^2}{\partial x^2} \left(u_i''(x) |u_i''(x)|^{j_1}\right) dx; \;\; u_i''(x) = \frac{\partial^2 u_i(x)}{\partial x^2}; \;\; d_1 = \int_0^l u_i^2(x) dx.$$

Let's look for the solution of differential equation (5) as following:

$$q_i(t) = A(t) e^{i\omega t} + B(t) e^{-i\omega t}, \tag{7}$$

where $A(t), B(t)$ are slowly variable functions and they their amplitude value satisfies the condition $\langle q_{ia} \rangle = 2\sqrt{\langle A(t) \rangle \langle B(t) \rangle}$.

According to differential equation (5) and it's solution (7), it is possible to get following first orderly differential equations:

$$\dot{A} = \frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2 (1 + (-\nu_1 + i\nu_2)R)\right)(A + Be^{-2i\omega t}) - \frac{1}{2i\omega}\left(\omega_{01}^2 \xi_0(t)\right)(A + Be^{-2i\omega t});$$

$$\dot{B} = -\frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2 (1 + (-\nu_1 - i\nu_2)R)\right)(Ae^{2i\omega t} + B) + \frac{1}{2i\omega}\left(\omega_{01}^2 \xi_0(t)\right)(Ae^{2i\omega t} + B). \tag{8}$$

In order to reduce the system of differential equations (8) to the system of Ito equations by using stochastic averaging method. In this case, the system of differential equations (8) is expressed as follows:

$$\dot{X}_s(t) = f_s(X,t) + \sum_{r=1}^2 G_{sr}(X,t) \xi_{0r}(t), \;\; (s = 1,2) \tag{9}$$

where

$$\dot{X}_1 = \dot{A}; \;\; \dot{X}_2 = \dot{B};$$

$$\dot{X}_1 = f_1 + G_{11}\xi_0(t); \;\; \dot{X}_2 = f_2 + G_{22}\xi_0(t);$$

$$f_1(X,t) = \frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2 (1 + (-\nu_1 + i\nu_2)R)\right)A + \frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2 (1 + (-\nu_1 - i\nu_2)R)\right)Be^{-2i\omega t};$$

$$f_2(X,t) = -\frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2 (1 + (-\nu_1 - i\nu_2)R)\right)B - \frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2 (1 + (-\nu_1 + i\nu_2)R)\right)Ae^{2i\omega t};$$

$$G_{11} = -\frac{\omega_{01}^2}{2i\omega}(A + Be^{-2i\omega t}); \;\; G_{22} = \frac{\omega_{01}^2}{2i\omega}(Ae^{2i\omega t} + B).$$

As a result, the system of differential equations.

$$dX_s(t) = Y_s(X)dt + \sum_{r=1}^2 H_{sr}(X) d\xi_{0r}(t), \;\; (s = 1,2) \tag{10}$$

where

$$Y_s = M_t \left\{ f_s(X,t) + \sum_{l=1}^2 \sum_{m,n=1}^2 \int_{-\infty}^0 G_{lm}(X, t+\tau) \frac{\partial G_{sn}(X,t)}{\partial X_l} E[\xi_{0n}(t)\xi_{0m}(t + +\tau)] d\tau \right\}; \tag{11}$$

$$[HH^T]_{sr} = M_t \left\{ \sum_{m,n=1}^2 \int_{-\infty}^{\infty} G_{sn}(X,t) G_{rm}(X, t+\tau) E[\xi_{0n}(t)\xi_{0m}(t+\tau)] d\tau \right\}; \tag{12}$$

$M_t\{\cdot\} = \lim_{n\to\infty} \frac{1}{T}\int_0^T \{\cdot\} dt$ is time averaging operator; $E[\cdot]$ is mathematical expectation; τ is correlation time.

If $\xi_{0r}(t) = \xi_0(t)$ is stationary normal random process, $\langle d\xi_{0r}(t) \rangle = d\langle \xi_{0r}(t) \rangle = d\langle \xi_0(t) \rangle = 0$ then the differential equations (10) will be expressed by following:

$$\frac{d\langle X_s(t) \rangle}{dt} = Y_s(\langle X \rangle). \quad (s = 1,2) \tag{13}$$

or

$$\frac{d\langle A \rangle}{dt} = \left(p_1 + \frac{\pi}{2} p_3^2 \left(S(0) - \psi(2\omega)\right)\right) \langle A \rangle;$$

$$\frac{d\langle B \rangle}{dt} = \left(p_2 - \frac{\pi}{2} p_3^2 \left(S(2\omega) - S(0)\right)\right) \langle B \rangle, \tag{14}$$

where

$$p_1 = \frac{1}{2i\omega}(\omega^2 - \omega_{01}^2 (1 + (-\nu_1 + i\nu_2)R));$$

$$p_2 = \frac{1}{2i\omega}\left(\omega^2 - \omega_{01}^2 (1 + (-\nu_1 - i\nu_2)R)\right);$$

$$p_3 = \frac{\omega_{01}^2}{2i\omega}.$$

The spectral densities $S(0)$, $S(2\omega)$, $\psi(2\omega)$ of a stationary normal random process $\xi_0(t)$ are defined as follows [16]:

$$S(2\omega) = \frac{1}{\pi}\int_{-\infty}^{0} R(\tau) cos\omega\tau d\tau\,; \;\; \psi(2\omega) = \frac{1}{\pi}\int_{-\infty}^{0} R(\tau) sin\omega\tau d\tau,$$

where $R(\tau) = E[\xi_{0n}(t)\xi_{0m}(t+\tau)] = \langle \xi_{0n}(t)\xi_{0m}(t+\tau)\rangle$ is correlation function.

Let's look for the solution to the system of differential equations (14) as follows:

$$A = A_0(t)e^{-\lambda t};$$
$$B = B_0(t)e^{-\lambda t}, \tag{15}$$

where A_0, B_0 are amplitude values of random parametric excitations of the beam; λ is characteristic number.
Based on the given formulas, we analyze the random parametric vibrations of the beam.

RESULT AND DISCUSSION

According to solutions of the differential equations (15) and the differential equations it is possible to write characteristic equation of the system. The roots of the characteristic equation are as follows:

$$\lambda_1 = -\frac{\omega_{01}^2}{2\omega}\left(\nu_2 R + \frac{\pi\omega_{01}^2}{4\omega}\big(S(0) - \psi(2\omega)\big)\right) + i\left(\frac{\omega_{01}^2}{2\omega}(1-\nu_1 R) - \frac{\omega}{2}\right);$$
$$\lambda_2 = \frac{\omega_{01}^2}{2\omega}\left(\nu_2 R + \frac{\pi\omega_{01}^2}{4\omega}\big(S(2\omega) - S(0)\big)\right) + i\left(\frac{\omega_{01}^2}{2\omega}(1-\nu_1 R) - \frac{\omega}{2}\right). \tag{16}$$

Determined roots of the characteristic give the opportunity to investigate mean square value and to check stability of the solution of the considered system.

According to the stability theory, motion is asymptotic stable when the real parts of roots of characteristic equation are negative. As a result, the borders between stable and unstable vibrations are followings:

$$\nu_2 R + \frac{\pi\omega_{01}^2}{4\omega}\big(S(0) - \psi(2\omega)\big) = 0;$$
$$\nu_2 R + \frac{\pi\omega_{01}^2}{4\omega}\big(S(2\omega) - S(0)\big) = 0. \tag{17}$$

According to expression (6), it is possible to write following when $s_1 = 2$ [17]:

$$\nu_2 R = \nu_2\left(C_0 + \frac{3EIh}{8\omega_{01}^2 m d_1}C_1\sigma_{ia} + \frac{3EIh^2}{20\omega_{01}^2 m d_1}C_2\sigma_{ia}^2\right). \tag{18}$$

It is possible to get the expression which gives chance to investigate mean square values of the considered system from equalities (17) and expression (18).

$$C_0 + \frac{3EIh}{8\omega_{01}^2 m d_1}C_1\sigma_{ia} + \frac{3EIh^2}{20\omega_{01}^2 m d_1}C_2\sigma_{ia}^2 + \frac{\pi\omega_{01}^2}{8\omega\nu_2}\big(S(2\omega) - \psi(2\omega)\big) = 0. \tag{19}$$

Let's take the expression for the spectral density $S(\omega)$ in the following form [16]:

$$S(\omega) = \frac{\sigma_\xi^2}{\pi}\cdot\frac{v}{\omega^2 + v^2}. \tag{20}$$

where σ_ξ is the mean square value of the base excitation; v is a dominant frequency.

Let's define the correlation function based on the spectral density expression (20). For this, we use the following relationship [16]:

$$R(\tau) = 2\int_0^\infty S(\omega) cos\omega\tau d\omega. \tag{21}$$

When calculating the correlation function (21), we take into account that it is a trigonometric function $cos\omega\tau = (e^{i\omega\tau} + e^{-i\omega\tau})/2$. Then, if substitute the spectral density expression (20) into the correlation function.

$$R(\tau) = 2\int_0^\infty \frac{\sigma_\xi^2}{\pi}\cdot\frac{v}{\omega^2 + v^2}\frac{e^{i\omega\tau} + e^{-i\omega\tau}}{2}d\omega. \tag{22}$$

The expression $\frac{v}{\omega^2 + v^2}$ can be replaced by the expression with the following approximate:

$$\frac{q}{\omega^2 + v^2} \approx \frac{1}{v}e^{-\frac{\omega^2}{v^2}}. \tag{23}$$

The approximate replacement has sufficient accuracy. It can be seen in Fig.1.

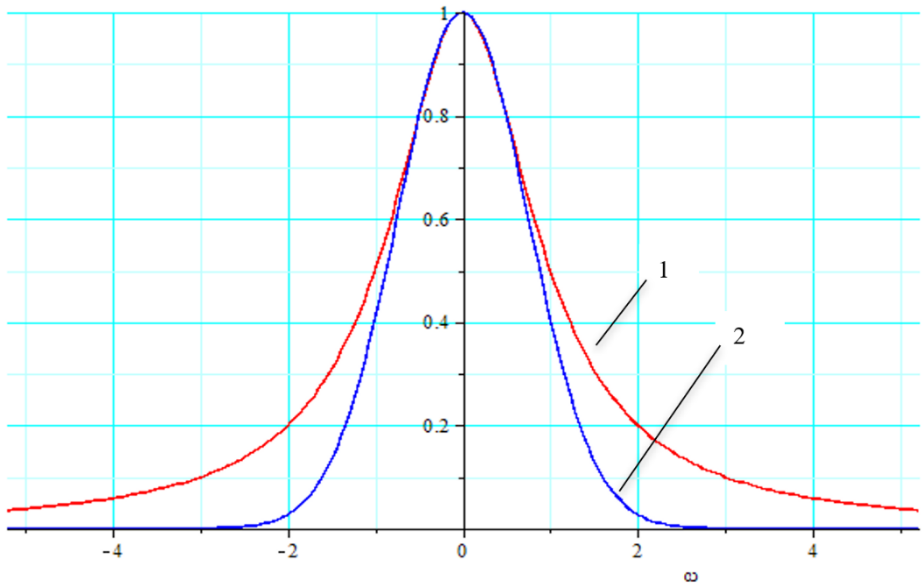

FIGURE 1. The graph of the functions $\frac{q}{\omega^2+v^2}$ and $\frac{1}{v}e^{-\frac{\omega^2}{v^2}}$, 1, 2 respectively, (v=1).

Considering the substitution (23),

$$R(\tau)=\frac{\sigma_\xi^2}{\pi v}\int_0^\infty\left(e^{i\omega\tau-\frac{\omega^2}{v^2}}+e^{-i\omega\tau-\frac{\omega^2}{v^2}}\right)d\omega=\sigma_\xi^2 e^{-v|\tau|}. \tag{24}$$

Determined (24) correlation function according to $\psi(2\omega)$ spectral density expression as follows will be [16]:

$$\psi(2\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}R(\tau)\sin\omega\tau d\tau=\frac{\sigma_\xi^2}{2\pi}\int_{-\infty}^{\infty}e^{-q|\tau|}\sin\omega\tau d\tau. \tag{25}$$

This spectral density expression in calculation according to trigonometric function $\sin\omega\tau=(e^{i\omega\tau}-e^{-i\omega\tau})/2i$ is as following:

$$\psi(2\omega)=\frac{\sigma_\xi^2}{2\pi}\int_{-\infty}^{\infty}e^{-v|\tau|}\cdot\frac{e^{i\omega\tau}-e^{-i\omega\tau}}{2i}d\tau=0. \tag{26}$$

It is possible to write the expression (19) as follows according to the determined spectral density expressions:

$$\frac{3EI}{md_1}\left(\frac{h}{2}C_1\sigma_{ia}+\frac{h^2}{5}C_2\sigma_{ia}^2\right)+\frac{\Omega_1^2\sigma_\xi^2}{v_2\Omega_2(4\Omega_2^2+1)}=0, \tag{27}$$

where $\Omega_1=\frac{\omega_{01}^2}{v};\Omega_2=\frac{\omega}{v}$.

The resulting expression (27) is an expression for determining the mean square values of the displacements. Using this expression, it is possible to determine, verify the stability, and numerically analyze the mean square values of the displacements of a beam with hysteresis-type elastic dissipative characteristics under the influence of random parametric excitations.

CONCLUSIONS

The mean square values of the displacements in random parametric transverse vibrations of a beam with elastic dissipative characteristics of the hysteresis type were determined analytically depending on the system parameters. The expression of the mean square values of these displacements allows us to evaluate the dynamics of the beam during its random parametric transverse vibrations. It can be seen that the root mean square values of the displacements depend on the dissipative properties of the beam material, its modulus of elasticity, its dimensions, mass, specific vibration modes and frequencies. In addition, the expression of the mean square values of the determined displacements allows us to verify and numerically analyze the stability of random parametric transverse vibrations of a beam with elastic dissipative characteristics of the hysteresis type.

REFERENCES

1. A. A Alifov, Oscillations under a nonlinear parametric action and combinations of delays. St. Petersburg State Polytechnical University Journal. Physics and Mathematics, **17**(1) (2024) 47–55.DOI:https://doi.org/10.18721/JPM.17105
2. G Malara, B Pomaro, P Spanos, Nonlinear stochastic vibration of a variable cross-section beam with a fractional derivative element. International Journal of Non-Linear Mechanics, **135**, 103770, (2021). 10.1016/j.ijnonlinmec.2021.103770.
3. M. L Deng, W. Q Zhu, Q F Lü, Stationary response of MDOF hysteretic system under random excitation. International Journal of Non–Linear Mechanics, **170** (2025) 104994. https://doi.org/10.1016/j.ijnonlinmec.2024.104994
4. F Wang, X Chen, H Xiang, Parametric Vibration Model and Response Analysis of Cable–Beam Coupling Under Random Excitation. Journal of Vibration Engineering Technologies, **11**, 2373–2386 (2023). https://doi.org/10.1007/s42417-022-00708-4
5. W Lacarbonara, F Vestroni, Nonlinear phenomena in hysteretic systems. IUTAM Symposium on 50 Years of Chaos: Applied and Theoretical. Procedia IUTAM **5** (2012) 69 – 77. doi:10.1016/j.piutam.2012.06.010
6. N M Rahbari, H Veladi, M Azizi, P Sareh, S Talatahari, Design and evaluation of hysteresis models for structural systems using a fuzzy adaptive charged system search, Decision Analytics Journal, **6** (2023) 100147. https://doi.org/10.1016/j.dajour.2022.100147
7. A Salvatore, B Carboni, L. Q Chen, W Lacarbonara, Experimental Dynamic Response of a Nonlinear Wire Rope Insulator. Nonlinear Dynamics and Control. Springer, Cham. (2020) https://doi.org/10.1007/978-3-030-34747-5_9
8. L. C Chen, Z Yuan, J. M. Qian, J. Q. Sun, Random vibration of hysteretic systems under Poisson white noise excitations. Applied Mathematics and Mechanics (English Edition), **44**(2), 207–220 (2023) https://doi.org/10.1007/s10483-023-2941-6
9. M. A Pavlovskii, L. M Ryzhkov, Random parametric oscillations of elastic systems with hysteresis energy dissipation. Soviet Applied Mechanics, **26**, 890–895 (1990). https://doi.org/10.1007/BF00888776
10. C. Zhiwei , Y Guo, Parametric Vibration Stability Analysis of a Pyramid Lattice Sandwich Plate Subjected to Pulsatile External Air fl ow. Acta Mechanica Solida Sinica, (2023) 36:95–104. https://doi.org/10.1007/s10338-022-00367-8 .
11. A Yu Aleksandrov, A. A Tikhonov, Stability analysis of mechanical systems with distributed delay via decomposition. Vestnik of Saint Petersburg University. Applied Mathematics. Computer Science. Control Processes, (2021), vol. **17**, iss.1, pp. 13–26. https://doi.org/10.21638/11701/spbu10.2021.102
12. H Schurz, A brief review on stability investigations of numerical methods for systems of stochastic differential equations. Networks and Heterogeneous Media. (2024). 19. 355-383. 10.3934/nhm.2024016.
13. A. Shariati, D. W Jung, H M Sedighi, K. Z Krzysztof, H Mostafa, S Maryam, Stability and Dynamics of Viscoelastic Moving Rayleigh Beams with an Asymmetrical Distribution of Material Parameters. Symmetry. (2020) doi:10.3390/sym12040586
14. J Changjian, Y Linquan, Sh Jiping, H Tonghao, L Cheng, In-plane free vibration of functionally graded rectangular plates with elastic restraint. Global Journal of Engineering Sciences, Volume **2**(3), (2019). pp. 1-8. GJES.MS.ID.000537. https://doi.org/10.33552/GJES.2019.02.000537
15. L.Wang, Y. Zhao, Q. Zhu, Y. Liu, Q. Han, Nonlinear vibration characteristic analysis of roller-plate system based on asymptotic methods. ISIJ International, Volume **60**. (2020). pp. 1237–1244.
16. A. A. Sveshnikov, Applied Methods of the Theory of Random Functions. Moscow, (1968).
17. M. A. Pavlovsky, L. M Ryzhkov, V. B Yakovenko, O. M. Dusmatov, Nonlinear problems of the dynamics of vibration protection systems. - K.: Tekhnika, (1997). - p. 204.
18. M. M Mirsaidov, O. M Dusmatov, M U Khodjabekov, Stability of nonlinear vibrations of plate protected from vibrations. Journal of Physics: Conference Series, **1921** (2021) 012097, https://doi:10.1088/1742-6596/1921/1/012097
19. M. M Mirsaidov, O. M Dusmatov, M U Khodjabekov, Mathematical modeling of hysteresis type elastic dissipative characteristic plate protected from vibration. International Conference on Actual Problems of Applied Mechanics - APAM-2021, AIP Conf. Proc. 2637, 060009-1–060009-7;(2021) https://doi.org/10.1063/5.0118289
20. M. M Mirsaidov, O. M Dusmatov, M U Khodjabekov, Mode Shapes of Transverse Vibrations of Beam Protected from Vibrations in Kinematic Excitations. Lecture Notes in Civil Engineering 170,(2021),pp. 217-227, https://doi:org/10.1007/978-3-030-79983-0_20

21. O. M Dusmatov, M U Khodjabekov, B Tashov, Determination of Modal Mass and Stiffness in Longitudinal Vibrations of the Beam.Aip Conference Proceedings, 3244(1), 060023.(2024) https://doi.org/10.1063/5.0241687
22. M. M Mirsaidov, O. M Dusmatov, M U Khodjabekov, Stability of Nonlinear Vibrations of Elastic Plate and Dynamic Absorber in Random Excitations E3s Web of Conferences, 410,03014.(2023) https://doi.org/10.1051/e3sconf/202341003014
23. M. M Mirsaidov, O. M Dusmatov, M U Khodjabekov,Mode Shapes of Hysteresis Type Elastic Dissipative Characteristic Plate Protected from Vibrations. Lecture Notes in Civil Engineering **282** , (2023), pp . 127-140 , https://doi.org/10.1007/978-3-031-10853-2_12