Stability of Random Vibrations of an Imperfectly Elastic Circular Plate

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Abstract: This work is devoted to the problem of checking the stability of transverse vibrations of a circular plate with elastic dissipative characteristics of the hysteresis type under the influence of random excitations. The expression of the mean square values of the plate was determined and the stability was checked using the vertical tangents method. The stability conditions were determined analytically depending on the structural parameters of the system and analyzed as a result of numerical calculations.

Key words: circular plate, hysteresis, dissipative, vibration, mean square value, stability, random excitation.

INTRODUCTION

Circular plate-type devices are widely used in all areas of technology. The issues of checking the stability of the dynamics of vibrations of circular plates in various processes, determining the dynamic characteristics of the plate materials under various boundary conditions, taking into account the nonlinear dissipative properties, are of great relevance. A lot of scientific research is being conducted on the dynamics of plates and stability issues.

The work [1] presented the equivalent linearization of nonlinearity in mechanical systems under random excitations, a mathematical model of nonlinear dissipative forces and hysteresis-type elastic dissipative characteristics, and the application of the averaging method for systems with hysteresis-type elastic dissipative characteristics. The concept of the hysteresis operator is introduced into the mathematical model of the hysteresis-type elastic dissipative characteristic, and its definition, properties, and classes are analyzed.

The work [2] investigated the transverse vibrations of a plate using finite difference methods and the transverse vibrations of a plate resting on an elastic base under the influence of kinematic excitations in combination with several dynamic dampers. The natural frequencies and natural vibration modes of plates of various shapes were investigated using the finite difference method for different boundary conditions. The experimental results were compared with analytical solutions and the reliability of the results was demonstrated.

In the work [3], the nonlinear vibration of a circular plate with clamped edges, which is widely used in marine structures, under the influence of an external hydrostatic force, was studied. The Helmholtz-Duffing system of equations was obtained by reducing the static and dynamic deflections and applying the Galerkin method. The effect of the static force on the dynamics, i.e., hardening, asymmetry and softening, was studied by numerically solving the system of differential equations of motion. An analytical solution of the vibration near resonance was obtained for the first form of vibration. The analytical solution allowed a theoretical assessment of the effect of the static force on the dynamics. The numerical and analytical results were compared at small values of the static deflection and the correctness of the results was shown. The proposed analysis method for the vibration of a circular plate is based on the fact that it can be applied to forms formed by rods, membranes and their combinations.

In the work [4], analyzed the free vibrations of composite annular circular plates. The equations of equilibrium, differential equations related to displacements and properties were obtained. The generalized eigenvalue problem for the frequency parameter and the associated eigenvector of the coefficient properties in the equations was numerically solved. The effect of the parameters on the frequency itself was investigated and suggestions were made to check the frequencies or to recommend them. By checking the reliability of the obtained results, recommendations were developed.

In the works [5-7], free vibrations of circular plates made of porous material were studied. The circular plates were assumed to be thin and their longitudinal deformations were ignored. The properties of the porous material were assumed to vary depending on the thickness of the plates according to the given functions. Differential equations of motion were obtained using Hamilton's variational principle and classical plate theory. Free and fixed supports were considered as boundary conditions for the boundary problem. The effects of some parameters, such as the distribution of pores and the compression of pores, on the natural frequency and stress were depicted in graphs, and conclusions were given.

In the works [8-9], solutions for the free vibration characteristics of thin circular plates supported on a Winkler-type elastic foundation were considered based on classical plate theory. Parameter-dependent analyses were performed to evaluate the effect of the displacement and stiffness of the elastic foundation on the natural frequencies of circular plates. It was found that the boundary conditions and the presence of an elastic foundation affect the characteristics of free transverse vibrations of a circular plate. The natural frequencies of vibrations for varying values of the stiffness parameter of the Winkler-type foundation were determined based on numerical calculations. Twelve vibration modes are presented in tabular and graphical form for ease of use of the obtained results in the design process.

In the works [10-11], nonlinear vibrations of rigid bodies and composite circular plates were studied in the articles. The nonlinear differential equations of motion were obtained using the generalized Hamiltonian principle and the von Karman plate theory. The solutions were determined by the finite element method and their correctness was checked. The inclusion of weight forces introduces additional linear and quadratic nonlinear terms into the dynamic model. As a result, the inclusion of weights leads to an increase in the natural frequency. It is shown that the vibrations of circular plates are asymmetric when the effect of weights is taken into account.

In the works [12-13], frequencies of circular plates reinforced with rigid concentric rings are determined in the articles. The natural frequencies of such circular plates are calculated for various boundary conditions and different values of the radius of the inner ring support. Various forms of plate vibrations are determined and presented in tabular form for use in the design process. The influence of boundary conditions and the radius of the concentric ring support on the natural frequencies of the plate is studied. It is shown that the determined frequency values serve to assess the accuracy of other numerical methods used, and recommendations are given.

In the work [14], analyzes the vibrations of a circular plate under various mixed boundary conditions. The methods for studying the vibrations of a circular plate are discussed and possible applications are highlighted. The methods for studying the problems of bending of a circular plate are analyzed.

In the works [15-16], the oscillations of the plate layer and the stability issues under the influence of various forces were considered. In the considered problems, cases were considered where several plate layers are connected by a linear elastic characteristic. The eigenfrequencies and critical forces in longitudinal bending were expressed analytically. The expression of the eigenfrequencies depending on the number of plate layers was obtained. The eigenfrequencies were found using the Galerkin method. The minimum of the unstable areas was determined depending on the system parameters. Variational methods were used to analyze the equations of motion, and conclusions were drawn.

Mathematical modeling of nonlinear mechanical systems, study of dynamics and exploring of stability of vibrations and instructions for selection of parameters corresponding to stable vibrations, in particular materials of mechanical systems, are given in the works [17-21]. Expressions of modal mass and modal stiffness are expressed analytically. By means of these expressions, the issues of choosing and modeling system materials were also solved.

In this work, we consider the issues of determining the root mean square values of nonlinear vibrations of an imperfectly elastic circular plate under the influence of random excitations and verifying their stability.

MATERIALS AND METHODS

The expression for the root mean square values of the nonlinear vibrations of a circular plate with imperfect elastic characteristics under the influence of random excitations is as follows:

$$\sigma_T^2 = \int_{-\infty}^{+\infty} |A(\omega)|^2 S_{W_0}(\omega)\, d\omega, \tag{1}$$

where $A(\omega)$ is the amplitude-frequency characteristic; $S_{W_0}(\omega)$ is the spectral density of the base acceleration; ω is the vibration frequency.

The expression for the amplitude-frequency characteristic for nonlinear transverse vibrations of a circular plate with hysteresis-type elastic dissipative characteristics is obtained as follows [22]:

$$A(\omega) = \frac{d_*}{(1-\eta_{1t}R_{1t}-\nu_{1t}R_{2t})\omega_{01}^2 - \omega^2 + (\eta_{2t}R_{1t}+\nu_{2t}R_{2t})\omega_{01}^2 j}, \tag{2}$$

where ω_{01} is the natural frequency of the plate; η_{1t}, η_{2t}, ν_{1t}, ν_{2t} are constant coefficients determined from the hysteresis surface of the plate material [23];

$$R_{1t} = \frac{3D}{\omega_{01}^2 \rho h d_1}\sum_{i_1=0}^{k_1} C_{i_1}\frac{h^{i_1}}{2^{i_1}(i_1+3)}|\sigma_T|^i G_{i_1};\, G_{i_1} = \iint_S PQ\left[\frac{\partial^2}{\partial r^2}\left(\beta_1|\beta_1|_a^{i_1}\right) + \left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\beta_2|\beta_2|_a^{i_1}\right]ds;$$

$$R_{2t} = \frac{6D(1-\mu)}{\omega_{01}^2 \rho h d_1}\sum_{i_2=0}^{k_2} K_{i_2}\frac{h^{i_2}}{2^{i_2}(i_2+3)}|\sigma_T|^i H_{i_2};\, H_{i_2} = \iint_S PQ(\frac{1}{r}\frac{\partial^2}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial}{\partial \theta})(\beta_3|\beta_3|_a^{i_2})ds;$$

$$\beta_1 = Q\frac{\partial^2 P}{\partial r^2} + \mu\left(\frac{1}{r}Q\frac{\partial P}{\partial r} + \frac{1}{r^2}P\frac{\partial^2 Q}{\partial \theta^2}\right);\quad \beta_2 = \frac{1}{r}Q\frac{\partial P}{\partial r} + \frac{1}{r^2}P\frac{\partial^2 Q}{\partial \theta^2} + \mu Q\frac{\partial^2 P}{\partial r^2};$$

$$\beta_3 = \frac{1}{r}\frac{\partial P}{\partial r}\frac{\partial Q}{\partial \theta} - \frac{1}{r^2}P\frac{\partial Q}{\partial \theta};\quad d_* = \frac{d_2}{d_1};\, d_1 = \int_0^{2\pi} Q^2\, d\theta \int_{r_0}^{R_0} P^2\, dr;\, d_2 = \int_0^{2\pi} Q d\theta \int_{r_0}^{R_0} P\, dr;$$

$C_{i_1}(i_1 = 0, \dots, k_1)$, $K_{i_2}(i_2 = 0, \dots, k_2)$ - α_{1i}, α_{2i}, α_{3i} and z_i are hysteresis parameters determined from experimentally selected lines $\alpha_1 = f_r(z)$, $\alpha_2 = f_\theta(z)$, $\alpha_3 = g(z)$ at points corresponding to the coordinates of the cyclic deformations of the material [24]; P and Q are functions of radius r and angle θ, respectively; R_0 is the radius of the circular plate; r_0 is the radius of the inner sphere given the boundary conditions; $D = \frac{Eh^3}{12(1-\mu^2)}$ is cylindrical rigidity of the plate; E is Young's modulus; h is plate thickness; μ is Poisson's ratio; ρ is density of the plate material; $j = \sqrt{-1}$.

Considering the amplitude-frequency characteristic expression (2), the expression for the mean square values (1) for nonlinear transverse random vibrations of a circular plate with hysteresis-type elastic dissipative characteristic is as follows:

$$\sigma_T^2 = \int_{-\infty}^{+\infty} \frac{d_*^2 S_{W_0}(\omega)}{[(1-\eta_{1t}R_{1t}-\nu_{1t}R_{2t})\omega_{01}^2 - \omega^2]^2 + [(\eta_{2t}R_{1t}+\nu_{2t}R_{2t})\omega_{01}^2]^2} d\omega. \tag{3}$$

We can express the spectral density of the fundamental accelerations in the following form [23]:

$$S_{W_0}(\omega) = \frac{D_{W_0}\alpha\omega_c^3}{\pi(\omega_c^2 - \omega^2 + j\alpha\omega_c\omega)(\omega_c^2 - \omega^2 - j\alpha\omega_c\omega)}, \tag{4}$$

where D_{W_0} is the dispersion of the base acceleration; α is a parameter characterizing the width of the vibration spectrum; ω_c is the frequency in the vibration spectrum at which the probability of vibration is high.

We substitute the expression for the spectral density of the base acceleration (4) into the expression for the mean square value (3)

$$\sigma_T^2 = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{d_*^2 D_{W_0}\alpha\omega_c^3}{([(1-\eta_{1t}R_{1t}-\nu_{1t}R_{2t})\omega_{01}^2 - \omega^2]^2 + [(\eta_{2t}R_{1t}+\nu_{2t}R_{2t})\omega_{01}^2]^2)([\omega_c^2 - \omega^2]^2 + [\alpha\omega_c\omega]^2)} d\omega \tag{5}$$

We calculate the resulting integral expression using the method presented in [1]. To do this, we write it as follows:

$$\sigma_T^2 = \frac{d_*^2 D_{W_0}\alpha\omega_c^3}{\pi}\int_{-\infty}^{+\infty} \frac{Z_4(\omega)}{X_4(i\omega)X_4(-i\omega)} d\omega, \tag{6}$$

where

$$Z_4(\omega) = b_3\omega^6 + b_2\omega^4 + b_1\omega^2 + b_0;$$

$$X_4(i\omega) = a_4(i\omega)^4 + a_3(i\omega)^3 + a_2(i\omega)^2 + a_1(i\omega)^1 + a_0;$$

$$a_4 = 1;\, a_3 = -\alpha\omega_c - p_1\omega_{01};\; p_1 = \left(2(p_2 - (1-\eta_{1t}R_{1t}-\nu_{1t}R_{2t}))\right)^{\frac{1}{2}};$$

$$p_2 = ((1 - \eta_{1t} R_{1t} - \nu_{1t} R_{2t})^2 + (\eta_{2t} R_{1t} + \nu_{2t} R_{2t})^2)^{\frac{1}{2}};$$
$$a_2 = \alpha p_1 \omega_c \omega_{01} + p_2 \omega_{01}^2 + \omega_c^2;\ a_1 = -\alpha p_2 \omega_c \omega_{01}^2 - p_1 \omega_{01} \omega_c^2;\ a_0 = p_2 \omega_c^2 \omega_{01}^2;$$
$$b_5 = b_4 = b_3 = b_2 = b_1 = 0;\ b_0 = 1.$$

We express the expression for the mean square values (5) using the integral (6) in the following form:

$$\sigma_T^2 = \frac{d_*^2 D_{W_0}(-p_1 \alpha^2 \omega_{01} \omega_c^2 - \alpha(p_1^2 \omega_c \omega_{01}^2 + \omega_c^3) - p_1 p_2 \omega_{01}^3)}{\alpha p_1 p_2 \omega_{01}^3 (p_2^2 \omega_{01}^4 + \alpha p_1 p_2 \omega_c \omega_{01}^3 + (\alpha^2 p_2 - 2p_2 + p_1^2)\omega_{01}^2 \omega_c^2 + \alpha p_1 \omega_{01} \omega_c^3 + \omega_c^4)}, \tag{7}$$

We will check the stability of random vibrations of a circular plate with elastic dissipative characteristics of the hysteresis type. For this, we will use the method of vertical stresses. (7) The condition for the existence of a vertical stress transferred to the graph of the mean square value is as follows:

$$\frac{d\sigma_T}{d\omega_c} = \frac{\frac{\partial f}{\partial \omega_c}}{1 - \frac{\partial f}{\partial \sigma_T}} = \infty, \tag{8}$$

where

$$f = [\frac{d_*^2 D_{W_0}(-p_1 \alpha^2 \omega_{01} \omega_c^2 - \alpha(p_1^2 \omega_c \omega_{01}^2 + \omega_c^3) - p_1 p_2 \omega_{01}^3)}{\alpha p_1 p_2 \omega_{01}^3 (p_2^2 \omega_{01}^4 + \alpha p_1 p_2 \omega_c \omega_{01}^3 + (\alpha^2 p_2 - 2p_2 + p_1^2)\omega_{01}^2 \omega_c^2 + \alpha p_1 \omega_{01} \omega_c^3 + \omega_c^4)}]^{\frac{1}{2}}.$$

Assuming that $\frac{\partial f}{\partial \omega_c} \neq 0$, we derive the following equation from the condition for the existence of vertical tangents (8):

$$1 - \frac{1}{2f_1}\frac{\partial f_1}{\partial \sigma_T} + \frac{1}{2f_2}\frac{\partial f_2}{\partial \sigma_T} = 0, \tag{9}$$

where

$$f_1 = d_*^2 D_{W_0}(-p_1 \alpha^2 \omega_{01} \omega_c^2 - \alpha(p_1^2 \omega_c \omega_{01}^2 + \omega_c^3) - p_1 p_2 \omega_{01}^3);$$
$$f_2 = (p_2^2 \omega_{01}^4 + \alpha p_1 p_2 \omega_c \omega_{01}^3 + (\alpha^2 p_2 - 2p_2 + p_1^2)\omega_{01}^2 \omega_c^2 + \alpha p_1 \omega_{01} \omega_c^3 + \omega_c^4)\alpha p_1 p_2 \omega_{01}^3.$$

If equality (9) is valid for any of the values of the parameters, the nonlinear stationary vibrations of the considered circular plate with elastic dissipative characteristics of the hysteresis type under the influence of random excitations will be unstable. Otherwise, stationary motion will stability.

RESULTS AND DISCUSSION

We numerically analyze the random nonlinear stationary vibrations of the circular plate under consideration.
For the circular plate material, we obtain 40X steel and its dimensions as follows [25]:

$$E = 2.08 \cdot 10^{11} \frac{N}{m^2};\ \rho = 7810\ \frac{kg}{m^3};\ \mu = 0.3;\ r = 0.12\ m;\ R = 0.135\ m;\ \ h = 0.12 \cdot 10^{-2} m;$$
$$D = \frac{Eh^3}{12(1-\mu^2)} = 32.91428571 Nm;$$
$$C_0 = 0;\ \ C_1 = 10.18332;\ \ C_2 = 43264;\ \ C_3 = -88943591;\ K_0 = 0;\ K_1 = 11.963;$$
$$K_2 = -8959.997;\ \ K_3 = 4426666;\ \eta_1 = \nu_1 = \frac{3}{4};\ \ \eta_2 = \nu_2 = \frac{1}{\pi};\ \ \omega_{01} = 1329.312616\ s^{-1};$$
$$P(r) = J_0(\lambda_* r) - \frac{J_0(\lambda_* a)}{I_0(\lambda_* a)} I_0(\lambda_* r) = J_0(26.633r) + 0.05570816212 I_0(26.633r);\ \ Q(\theta) = 1;$$
$$d_1 = \int_0^{2\pi} Q^2\, d\theta \int_{r_0}^{R} P^2\, dr = 0.000017239031\pi;$$
$$d_2 = \int_0^{2\pi} Q d\theta \int_{r_0}^{R} P\, dr = 0.0005327511318\pi;\ \ d_* = \frac{d_2}{d_1} = 30.9037748.$$

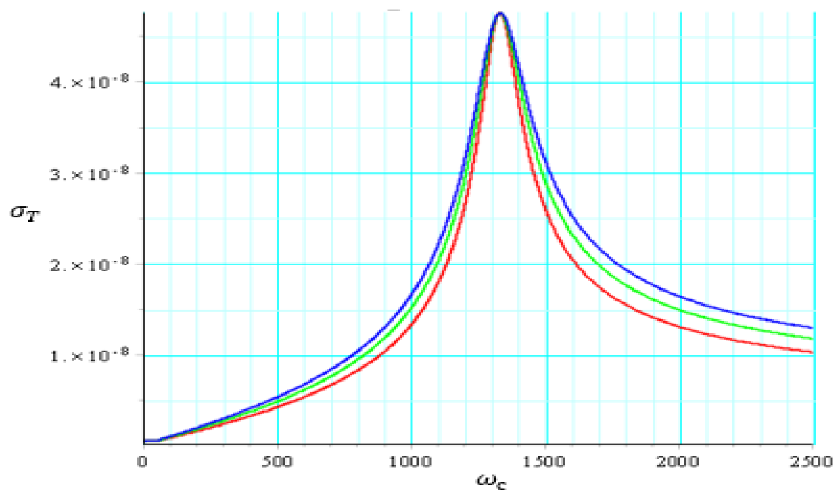

FIGURE 1. Graph of the root mean square expression (7)

In Fig. 1 the graphs of the expression of the mean square values (7) are presented for the values of the parameter characterizing the width of the vibration spectrum $\alpha = 0.01; 0.015; 0.02$ (red, green, blue). From these graphs, it can be concluded that a change in the parameter characterizing the width of the vibration spectrum does not lead to a change in the mean square value around the resonant frequency. However, increasing this parameter leads to an increase in the mean square values due to the broadening of the vibration spectrum at frequencies not around the resonant frequency.

The graphs of the expression of the stability condition (9) for different values of the parameter α are depicted in Fig. 2.

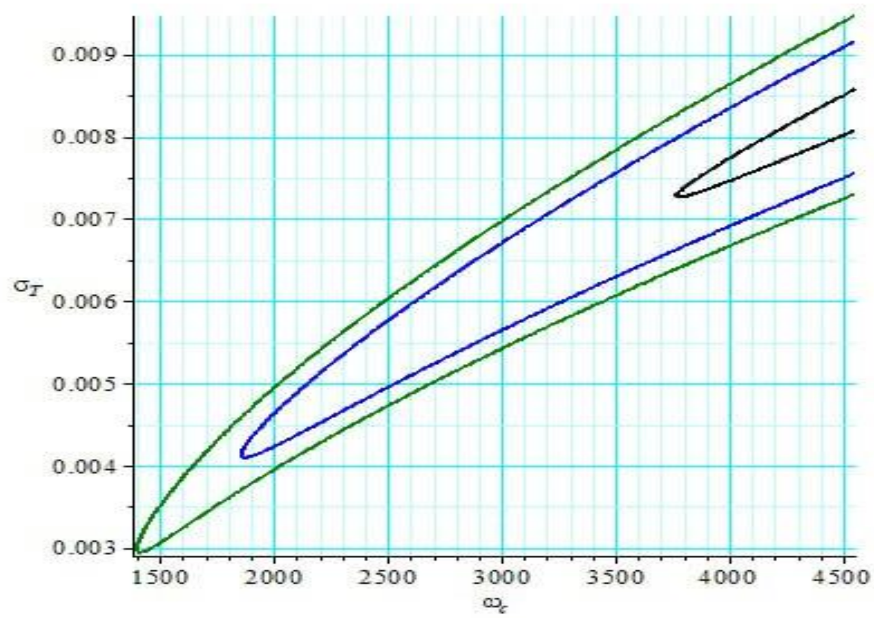

FIGURE 2. Graph of the expression for the stability condition (9)

In Fig. 2, the change in the stabiliity boundaries and areas determined by the stability condition (9) is analyzed by changing the parameter characterizing the spectrum width $\alpha = 0.1\,; 0.2; 0.25$ $(green,\ blue,\ black)$. From these graphs, we can see that when $\alpha = 0.1$, the unstable areas in the given intervals $\omega_c \in [800; 4500]$ and $\sigma_T \in [0.001; 0.01]$ are larger than in the remaining cases $\alpha = 0.2$ and $\alpha = 0.25$. The largest stability area in the given intervals is formed when $\alpha = 0.25$.

CONCLUSION

Analytical expressions for the root mean square values of the displacements in transverse vibrations of a circular plate under the influence of random excitations were obtained. Expressions for the mean square values were determined for the complex representation of the spectral density of the fundamental acceleration. The mean square values and analytical expressions for the stability spheres determined depending on the structural parameters of the system allow us to evaluate the dynamics of transverse vibrations of the imperfectly elastic circular plate under random excitations and to verify the stability. A change in the parameter characterizing the width of the vibration spectrum does not lead to a change in the mean square value around the resonant frequency. However, an increase in this parameter leads to an increase in the mean square value due to the expansion of the vibration spectrum at frequencies not around the resonant frequency. As a result, the disappearance of stability movements can be observed.

The presented methodology allows us to check the stability of the base accelerations in random processes in terms of their parameters, taking into account the nonlinear dissipative characteristics of the plate in different forms of spectral density.

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