3rd International Conference
Advanced Mechanics: Structure,
Materials, Tribology

Study of deflections of bending layered structures taking into account transverse shear deformations

AIPCP25-CF-AMSMT2025-00063 | Article

Study of deflections of bending layered structures taking into account transverse shear deformations

I. Khamzaev[1], S. Khasanov[2], N. Bibutov[3], E. Umarov[1,b)], O. Khaydarova[1], B. Ergashov[1], U. A. Nishonov[4,a)]

[1]Fergana State Technical University, Republic of Uzbekistan, Fergana
[2]Tashkent State Technical University, Republic of Uzbekistan, Tashkent
[3]Bukhara State Technical Universitya, Republic of Uzbekistan, Bukhara
[4]Samarkand State University named after Sharof Rashidov, 140104, University Boulevard 15, Samarkand, Uzbekistan

[a)] Corresponding author: utkirn1978@mail.ru
[b)] elmurod.umarov@fstu.uz

Abstract: The article considers the influence of shear stresses on the deformation of structural elements subject to bending. It is known that the structural element subject to bending is a beam, and the main problem is to check its rigidity. To do this, it is necessary to determine the displacements that occur under the action of external forces. This problem is solved based on the integration of the main differential equation of the curved axis of the beam. However, in general, the bending of the beam is also significantly affected by shear stresses that occur on the surfaces of its cross section. The shear deformation caused by these stresses, in some cases, can have a significant effect on the deformation of the beam, that is, on its rigidity. The paper considers the study of a three-layer rod consisting of a lightweight porous material, such as foam plastic, called a filler, connected on the outer surfaces with thin metal sheets (bearing layers).

Key words: deformation, beam, shear, layers, bearing layer, rigidity, filler, deflection, shear stresses, bending moments, rod, element, displacement function, stress function, diagrams, coefficient.

INTRODUCTION

Let us consider, in contrast to cases, when determining deflections, in addition to the deformation of curvature created by bending moments, we take into account the shear of the beam elements.

The shear deformation of the rod element is created by transverse forces -Q, which are caused by the average shear of the element [4,5,6].

$$\gamma_{av} = \frac{kQ}{GA} \tag{1}$$

where k is a coefficient that takes into account the uneven distribution of tangential stresses in the cross-section; its value depends on the cross-section of the beam.

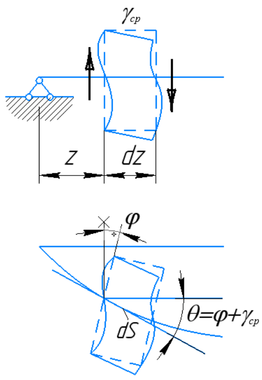

FIGURE 1. General view of a rotated cross-section of a beam after deformation, considering shear deformation.

LITERATURE REVIEW AND METHODS

If we still denote the angle of rotation of the cross section by φ, and the angle of inclination of the tangent to the axis of the bent beam by θ, then it is easy to see that the segment of the axis dS as an element of the tangent curve $v = v(z)$ in addition to the angle φ will rotate due to the shift by the angle γ_{av} so that the total angle of inclination of the tangent θ will be.

$$\theta = \varphi + \gamma_{av} \tag{2}$$

Since the cross-section will not be normal to the axis of the beam when taking into account the shifts [5,6,7].

Based on the equality θ=φ′, instead of $\varphi = \frac{dv}{dz} = v'(a)$ we get.

$$\varphi + \gamma_{av} = v' \tag{3}$$

and a system of differential equations

$$\frac{d\varphi}{dz} = -\frac{M}{EJ}$$

$$\frac{d\varphi}{dz} = \varphi$$

taking into account the shifts will have the form

$$\left.\begin{aligned} \frac{d\varphi}{dz} &= -\frac{M}{EJ} \\ \frac{dv}{dz} &= \varphi + \frac{kQ}{GA} \end{aligned}\right\} \tag{4}$$

Substituting φ from the second line into the first, we obtain instead the second-order equation $\frac{d^2v}{dz^2} = -\frac{M}{EJ}$ taking into account the shifts.

$$\frac{d^2v}{dz^2} = -\frac{M}{EJ} + \frac{d}{dz}\left(\frac{kQ}{GA}\right) \tag{5}$$

When integrating equations (4) or (5), it should be borne in mind that two separate displacement functions are subject to determination: the deflection function v(z) and the section rotation angle function φ(z) the latter are no longer related by deflections (a), as was the case without taking into account the shifts. Their dependence is determined by equality. (2) This creates its own peculiarities in the formulation of boundary conditions and conditions of

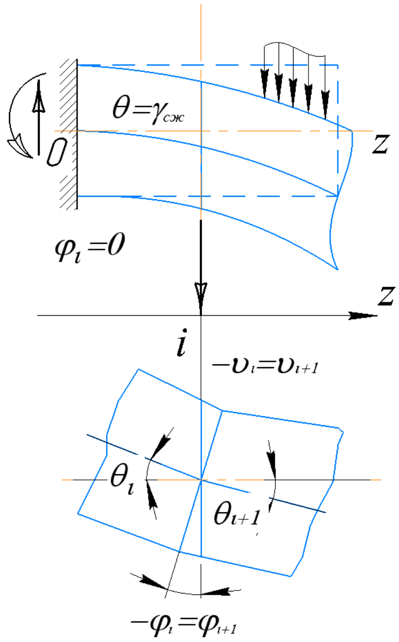

FIGURE 2. Example for determining the deflection of a homogeneous cantilever beam with a rectangular cross-section, taking shear deformation into account.

continuity of displacements at the boundaries of the integration sections. For example, in the embedment, the boundary conditions at z=0 will be: 1) v(0)=0 and 2) φ(0)=0. In this case, from the second condition and equality (2) it follows that the tangent in the embedment will turn by an angle of $\theta = \gamma_{cp}$.

At the boundary of the sections at the *i-th* point the conditions of continuity of displacements will be $v_i = v_{i+1}$ and $\varphi_i = \varphi_{i+1}$ In this case the angles of inclination of the tangents can be different $\theta_i \neq \theta_{i+1}$, if $\theta_i = \theta_{i+1}$.

Example. Find the deflections of a homogeneous cantilever beam of rectangular cross-section taking into account the shear deformation from a uniformly distributed load. (Fig. 3)

Solution. We use the system of equations (4). From the first equation we find $\varphi = -\int_0^z \frac{Mdz}{EJ} + \varphi_0$ from the second equation $v = \int_0^z \varphi dz + \int_0^z \frac{kQ}{GA} dz + v_0$

In the case under consideration, $M = \frac{ql^2}{2} - \frac{qz^2}{2} + qlz$ and $Q = ql - qz$, in addition, in the seal $v_0=0$ $\varphi_0=0$, therefore, for the expressions φ and v we obtain the following expression

$$\varphi = \frac{ql^2 z}{2EJ} - \frac{qlz^2}{2EJ} + \frac{q^2}{6EJ}; \quad v = \frac{ql^2 z^2}{4EJ} - \frac{qlz^3}{6EJ} + \frac{qz^4}{24EJ} + \frac{k}{GA}\left(qlz - \frac{q^2}{2}\right)$$

The obtained expressions for the deflections v can be represented as the sum of two functions $v = v_M + v_Q$ where $v = \frac{ql^2 z^2}{4EJ} - \frac{qlz^3}{6EJ} + \frac{qz^4}{24EJ}$ - deflections caused by bending moments $v_Q = \frac{k}{GA}\left(qlz + \frac{qz^2}{2}\right)$ - deflections from shear deformations caused by transverse forces.

The diagram of v_Q and v_M is shown in Fig. 3. Their sum will give the general line of deflections.

Let's calculate the deflection at the end of the beam $v_Q(l) = f_Q$ and $v_M(l) = f_M$ compare them quantitatively. At $z=l$ we find f_Q, f_M and their ratios:

$$f_Q = \frac{kql^2}{2GA};\ f_M = \frac{ql^4}{8GA};\ \frac{f_Q}{f_M} = 4\frac{E}{G}\frac{J}{Al^2}k \qquad (6)$$

In the case of a homogeneous beam we have $E = 2(1+\mu)G$; $J = bh^3/12$; $A = bh$, $k = 6/5$. Sloping $l/h = 5$ and $\mu = 0,25$ we get

$$\frac{fQ}{fM} = \frac{12}{15}(1-\mu)\frac{h^2}{l^2} = \frac{12}{15}\cdot 1,25\frac{1}{5^2} = \frac{1}{25}$$

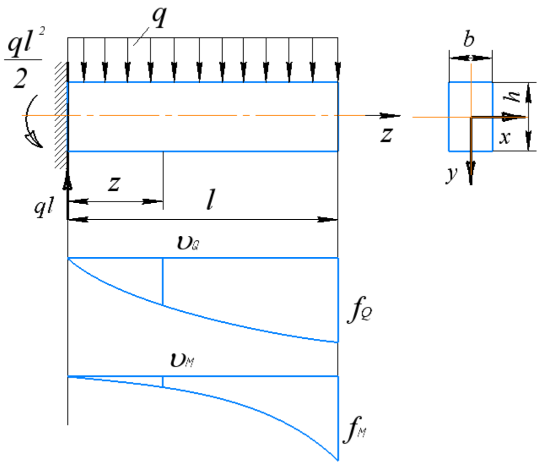

FIGURE 3. Deflection of the beam from shear deformations v_Q and from bending v_M

i.e. the deflection caused by shear deformations is 25 times less than the deflection caused by bending moments. The shear deformation can be justifiably neglected, as it was accepted according to the hypotheses of Kirchhoff, they can be neglected except for layered structures. But in engineering and construction, multilayer elements of structures are widely used [1,2,3,4].

RESULTS

Let us consider a three-layer rod consisting of a lightweight porous material, such as foam plastic, called a filler, connected on the outer surfaces with thin metal sheets (bearing layers). Three-layer panels are usually used, which have high heat-insulating and bearing properties and, in addition, very low weight. Let us estimate the effect of shear deformations in such a three-layer beam, using the calculation scheme and formulas obtained in the example in Fig. 4. The modulus of elasticity of the filler is many times less than the modulus of elasticity of the bearing layers. Therefore, it is considered that the entire bending moment in the section is taken by the bearing layers, and the filler perceives only the shear stresses τ, which in this case will be constant across its thickness. Then, in formulas (6), the rigidity EJ will relate only to the bearing layers and will be

$$EJ = E2в\delta(h/2)^2 = Ев\delta h^2/2$$

where E – modulus of elasticity of the material of the supporting (layers) sheets
δ – sheet thickness

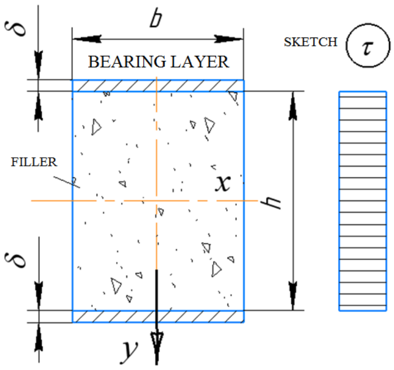

FIGURE 4. Calculation diagram of a three-layer rod and the law of shear stress distribution in the core material.

At the same time, the shear rigidity GA will relate only to the filler and will be GA=Gвh, where G is the filler shear modulus. Due to the uniformity of the distribution of τ across the filler thickness, the coefficient k in formula (1).

CONCLUSION

Substituting the given values of EJ and GA into formula (6) for f_Q/f_M, we obtain the desired ratio of the deflection from transverse forces to the deflection from bending moments of a three-layer beam:

$$\frac{f_Q}{f_M} = 2\frac{E\delta h^2}{Ghl^2}$$

Let h/*l*=1/5; δ/h=10^{-2} и E/G=10^3, $f_Q / f_M = 0{,}8$ i.e. shear deformations increase the deflection f_M from bending moments by 80% and in this case shear deformations should in no case be neglected.

Usually multilayer elements of structures, road and airfield pavements are calculated taking into account shear deformation. These issues are considered in detail in the works.

REFERENCES

1. I. H. Hamzayev, & E. S. Umarov, Resistance of Materials. Textbook., (Fergana., Classic, 2022)).
2. G. S. Pisarenko et al. *Strength of Materials*. (Kyiv, 1986), 775 pages.
3. *Finite Element Method: A Textbook for Universities*, edited by P.M. Varvak. (Kyiv: Vysha Shkola, 1981), 176 pages.
4. *Calculation of Nonhomogeneous Shallow Shells and Plates Using the Finite Element Method: Monograph*, supervised by V.G. Piskunov. (Kyiv, 1987), 200 pages.
5. V. V. Bolotin, Yu.N. Novichkov, *Mechanics of Multilayered Structures*. (Moscow: Mashinostroenie, 1980), 375 pages.
6. I. Khamzaev, et.al., E3S Web of Conferences **538**, 01015 (2024)
7. I. Khamzaev, et.al., EPJ Web of Conferences **318**, 04011 (2025)

AIP Publishing

LICENSE TO PUBLISH AGREEMENT FOR CONFERENCE PROCEEDINGS

This License to Publish must be signed and returned to the Proceedings Editor before the manuscript can be published. If you have questions about how to submit the form, please contact the AIP Publishing Conference Proceedings office (confproc@aip.org). For questions regarding the copyright terms and conditions of this License, please contact AIP Publishing's Office of Rights and Permissions, 1305 Walt Whitman Road, Suite 300, Melville, NY 11747-4300 USA; Phone 516-576-2268; Email: rights@aip.org.

Article Title ("Work"):

Study of deflections of bending layered structures taking into account transverse shear deformations

All Author(s):

I. Khamzaev, S. Khasanov, N. Bibutov, E. Umarov, O. Khaydarova, B. Ergashov, U. A. Nishonov

Title of Conference: AMSMT2025

Name(s) of Editor(s) Valentin L. Popov

All Copyright Owner(s), if not Author(s):

(Please list **all** copyright owner(s) by name. In the case of a Work Made for Hire, the employer(s) or commissioning party(ies) are the copyright owner(s). For large groups of copyright owners, attach a separate list to this form.)

Copyright Ownership and Grant of Rights

For the purposes of this License, the "Work" consists of all content within the article itself and made available as part of the article, including but not limited to the abstract, tables, figures, graphs, images, and multimedia files, as well as any subsequent errata. "Supplementary Material" consists of material that is associated with the article but linked to or accessed separately (available electronically only), including but not limited to data sets and any additional files.

This Agreement is an Exclusive License to Publish not a Transfer of Copyright. Copyright to the Work remains with the Author(s) or, in the case of a Work Made for Hire, with the Author(s)' employer(s). AIP Publishing LLC shall own and have the right to register in its name the copyright to the proceedings issue or any other collective work in which the Work is included. Any rights granted under this License are contingent upon acceptance of the Work for publication by AIP Publishing. If for any reason and at its own discretion AIP Publishing decides not to publish the Work, this License is considered void.

Each Copyright Owner hereby grants to AIP Publishing LLC the following irrevocable rights for the full term of United States and foreign copyrights (including any extensions):

1. The exclusive right and license to publish, reproduce, distribute, transmit, display, store, translate, edit, adapt, and create derivative works from the Work (in whole or in part) throughout the world in all formats and media whether now known or later developed, and the nonexclusive right and license to do the same with the Supplementary Material.
2. The right for AIP Publishing to freely transfer and/or sublicense any or all of the exclusive rights listed in #1 above. Sublicensing includes the right to authorize requests for reuse of the Work by third parties.
3. The right for AIP Publishing to take whatever steps it considers necessary to protect and enforce, at its own expense, the exclusive rights granted herein against third parties.

Author Rights and Permitted Uses

Subject to the rights herein granted to AIP Publishing, each Copyright Owner retains ownership of copyright and all other proprietary rights such as patent rights in the Work.

Each Copyright Owner retains the following nonexclusive rights to use the Work, without obtaining permission from AIP Publishing, in keeping with professional publication ethics and provided clear credit is given to its first publication in an AIP Publishing proceeding. Any reuse must include a full credit line acknowledging AIP Publishing's publication and a link to the Version of Record (VOR) on AIP Publishing's site.

Each Copyright Owner may:

1. Reprint portions of the Work (excerpts, figures, tables) in future works created by the Author, in keeping with professional publication ethics.
2. Post the Accepted Manuscript (AM) to their personal web page or their employer's web page immediately after acceptance by AIP Publishing.
3. Deposit the AM in an institutional or funder-designated repository immediately after acceptance by AIP Publishing.
4. Use the AM for posting within scientific collaboration networks (SCNs). For a detailed description of our policy on posting to SCNs, please see our Web Posting Guidelines (https://publishing.aip.org/authors/web-posting-guidelines).
5. Reprint the Version of Record (VOR) in print collections written by the Author, or in the Author's thesis or dissertation. It is understood and agreed that the thesis or dissertation may be made available electronically on the university's site or in its repository and that copies may be offered for sale on demand.
6. Reproduce copies of the VOR for courses taught by the Author or offered at the institution where the Author is employed, provided no fee is charged for access to the Work.
7. Use the VOR for internal training and noncommercial business purposes by the Author's employer.
8. Use the VOR in oral presentations made by the Author, such as at conferences, meetings, seminars, etc., provided those receiving copies are informed that they may not further copy or distribute the Work.
9. Distribute the VOR to colleagues for noncommercial scholarly use, provided those receiving copies are informed that they may not further copy or distribute the Work.
10. Post the VOR to their personal web page or their employer's web page 12 months after publication by AIP Publishing.
11. Deposit the VOR in an institutional or funder-designated repository 12 months after publication by AIP Publishing.
12. Update a prior posting with the VOR on a noncommercial server such as arXiv, 12 months after publication by AIP Publishing.

Author Warranties

Each Author and Copyright Owner represents and warrants to AIP Publishing the following:

1. The Work is the original independent creation of each Author and does not infringe any copyright or violate any other right of any third party.
2. The Work has not been previously published and is not being considered for publication elsewhere in any form, except as a preprint on a noncommercial server such as arXiv, or in a thesis or dissertation.
3. Written permission has been obtained for any material used from other sources and copies of the permission grants have been supplied to AIP Publishing to be included in the manuscript file.
4. All third-party material for which permission has been obtained has been properly credited within the manuscript.
5. In the event that the Author is subject to university open access policies or other institutional restrictions that conflict with any of the rights or provisions of this License, such Author has obtained the necessary waiver from his or her university or institution.

This License must be signed by the Author(s) and, in the case of a Work Made for Hire, also by the Copyright Owners. One Author/Copyright Owner may sign on behalf of all the contributors/owners only if they all have authorized the signing, approval of the License, and agreed to be bound by it. The signing Author and, in the case of a Work Made for Hire, the signing Copyright Owner warrants that he/she/it has full authority to enter into this License and to make the grants this License contains.

1. The Author must please sign here (*except* if an Author is a U.S. Government employee, then please sign under #3 below):

Author(s) Signature	Print Name	Date
	U. A. Nishonov	25.11.2025

2. The Copyright Owner (if different from the Author) must please sign here:

Name of Copyright Owner	Authorized Signature and Title	Date

3. If an Author is a U.S. Government employee, such Author must please sign below.
The signing Author certifies that the Work was written as part of his/her official duties and is therefore not eligible for copyright protection in the United States.

Name of U.S. Government Institution (e.g., Naval Research Laboratory, NIST)

Author Signature	Print Name	Date

PLEASE NOTE: NATIONAL LABORATORIES THAT ARE SPONSORED BY U.S. GOVERNMENT AGENCIES BUT ARE INDEPENDENTLY RUN ARE NOT CONSIDERED GOVERNMENT INSTITUTIONS. (For example, Argonne, Brookhaven, Lawrence Livermore, Sandia, and others.) Authors at these types of institutions should sign under #1 or #2 above.

If the Work was authored under a U.S. Government contract, and the U.S. Government wishes to retain for itself and others acting on its behalf, a paid-up, nonexclusive, irrevocable, worldwide license in the Work to reproduce, prepare derivative works from, distribute copies to the public, perform publicly, and display publicly, by or on behalf of the Government, please check the box below and add the relevant Contract numbers.

☐ Contract #(s) ______________________ [1.16.1]

LICENSE TERMS DEFINED

Accepted Manuscript (AM): The final version of an author's manuscript that has been accepted for publication and incorporates all the editorial changes made to the manuscript after submission and peer review. The AM does not yet reflect any of the publisher's enhancements to the work such as copyediting, pagination, and other standard formatting.

arXiv: An electronic archive and distribution server for research article preprints in the fields of physics, mathematics, computer science, quantitative biology, quantitative finance, and statistics, which is owned and operated by Cornell University, http://arxiv.org/.

Commercial and noncommercial scholarly use: *Noncommercial* scholarly uses are those that further the research process for authors and researchers on an individual basis for their own personal purposes. They are author-to-author interactions meant for the exchange of ideas. *Commercial* uses fall outside the author-to-author exchange and include but are not limited to the copying or distribution of an article, either in hard copy form or electronically, for resale or licensing to a third party; posting of the AM or VOR of an article by a site or service where an access fee is charged or which is supported by commercial paid advertising or sponsorship; use by a for-profit entity for any type of promotional purpose. Commercial uses require the permission of AIP Publishing.

Embargo period: The period of time during which free access to the full text of an article is delayed.

Employer's web page: A web page on an employer's site that highlights the accomplishments and research interests of the company's employees, which usually includes their publications. (See also: Personal web page and Scholarly Collaboration Network).

Exclusive License to Publish: An exclusive license to publish is a written agreement in which the copyright owner gives the publisher exclusivity over certain inherent rights associated with the copyright in the work. Those rights include the right to reproduce the work, to distribute copies of the work, to perform and display the work publicly, and to authorize others to do the same. The publisher does not hold the copyright to the work, which continues to reside with the author. The terms of the AIP Publishing License to Publish encourage authors to make full use of their work and help them to comply with requirements imposed by employers, institutions, and funders.

Full Credit Line: AIP Publishing's preferred format for a credit line is as follows (you will need to insert the specific citation information in place of the capital letters): "Reproduced from [FULL CITATION], with the permission of AIP Publishing." A FULL CITATION would appear as: Journal abbreviation, volume number, article ID number *or* page number (year). For example: Appl. Phys. Lett. 107, 021102 (2015).

Institutional repository: A university or research institution's digital collection of articles that have been authored by its staff and which are usually made publicly accessible. As authors are encouraged and sometimes required to include their published articles in their institution's repository, the majority of publishers allow for deposit of the Accepted Manuscript for this purpose. AIP Publishing also allows for the VOR to be deposited 12 months after publication of the Work.

Journal editorial office: The contact point for authors concerning matters related to the publication of their manuscripts. Contact information for the journal editorial offices may be found on the journal websites under the "About" tab.

Linking to the Version of Record (VOR): To create a link to your article in an AIP Publishing journal or proceedings, you need to know the CrossRef digital object identifier (doi). You can find the doi on the article's abstract page. For instructions on linking, please refer to our Web Posting Guidelines at https://publishing.aip.org/authors/web-posting-guidelines.

National Laboratories: National laboratories are sponsored and funded by the U.S. Government but have independent nonprofit affiliations and employ private sector resources. These institutions are classified as Federally Funded Research and Development Centers (FFRDCs). Authors working at FFRCs are not considered U.S. Government employees for the purposes of copyright. The Master Government List of FFRDCs may be found at http://www.nsf.gov/statistics/ffrdclist/.

Personal web page: A web page that is hosted by the author or the author's institution and is dedicated to the author's personal research interests and publication history. An author's profile page on a social media site or scholarly collaboration network site is *not* considered a personal web page. (See also: Scholarly Collaboration Network; Employer's web page).

Peer X-Press: A web-based manuscript submission system by which authors submit their manuscripts to AIP Publishing for publication, communicate with the editorial offices, and track the status of their submissions. The Peer X-Press system provides a fully electronic means of completing the License to Publish. A hard copy of the Agreement will be supplied by the editorial office if the author is unable to complete the electronic version of the form. (Conference Proceedings authors will continue to submit their manuscripts and forms directly to the Conference Editors.)

Preprint: A version of an author's manuscript intended for publication but that has not been peer reviewed and does not reflect any editorial input or publisher enhancements.

Professional Publication Ethics: AIP Publishing provides information on what it expects from authors in its "Statement of ethics and responsibilities of authors submitting to AIP Publishing journals" (http://publishing.aip.org/authors/ethics). AIP Publishing is also a member of the Committee on Publication Ethics (COPE) (http://publicationethics.org/), which provides numerous resources and guidelines for authors, editors, and publishers with regard to ethical standards and accepted practices in scientific publishing.

Scholarly Collaboration Network (SCN): Professional networking sites that facilitate collaboration among researchers as well as the sharing of data, results, and publications. SCNs include sites such as Academia.edu, ResearchGate, and Mendeley, among others.

Supplementary Material: Related material that has been judged by peer review as being relevant to the understanding of the article but that may be too lengthy or of too limited interest for inclusion in the article itself. Supplementary Material may include data tables or sets, appendixes, movie or audio clips, or other multimedia files.

U.S. Government employees: Authors working at Government organizations who author works as part of their official duties and who are not able to license rights to the Work, since no copyright exists. Government works are in the public domain within the United States.

Version of Record (VOR): The final published version of the article as it appears in the printed journal/proceedings or on the Scitation website. It incorporates all editorial input, is formatted in the publisher's standard style, and is usually viewed in PDF form.

Waiver: A request made to a university or institution to exempt an article from its open-access policy requirements. For example, a conflict will exist with any policy that requires the author to grant a nonexclusive license to the university or institution that enables it to license the Work to others. In all such cases, the Author must obtain a waiver, which shall be included in the manuscript file.

Work: The "Work" is considered all the material that comprises the article, including but not limited to the abstract, tables, figures, images, multimedia files that are directly embedded within the text, and the text itself. The Work does not include the Supplementary Material (see Supplementary Material above).

Work Made for Hire: Under copyright law, a work prepared by an employee within the scope of employment, or a work that has been specially ordered or commissioned for which the parties have agreed in writing to consider as a Work Made for Hire. The hiring party or employer is considered the author and owner of the copyright, not the person who creates the work.