Forced Vibrations of a Rod Mounted on an Elastic Base

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Abstract. In this work, the problem of investigating the forced vibration of a rod with its lower end rigidly fixed and a load applied to its upper end was addressed. The equations of forced vibration were derived using classical theories for the load applied to the end of the rod, and their solutions were obtained. Using the obtained solutions, the oscillatory motion trajectory of the load was graphically illustrated with the help of the Maple software. During the laboratory process, the forced vibration of a rod mounted on a similar elastic base is studied. In this case, the lower end of the rod is rigidly fixed. A body with mass is placed on the other end. A load-sensitive sensor is installed on the upper end of the rod. The sensor is connected to a monitor, and the obtained results are compared with those obtained analytically, after which appropriate conclusions are drawn.

Keywords. Beam, rod, deflection, graph, elastic force.

INTRODUCTION

Nowadays, in the field of engineering, studying the dynamic state of devices, determining their vibration characteristics, and assessing in advance the resonance conditions that occur under the influence of external forces have great practical importance. The stable operation of elements encountered in mechanical engineering, construction, and various micromechanical systems depends on their resistance to vibrations. Therefore, an in-depth study of the forced vibrations of structural elements, including rods, is considered one of the urgent scientific problems today [1–3]. A vertically positioned rod system, with one end clamped and a load applied to the other end, is a widely used dynamic system. Such rods are found in various types of structures. In these kinds of systems, external forces cause forced vibrations. If the vibration frequency of these external forces coincides with the natural frequency of the rod, a resonance phenomenon occurs. As a result, excessive deformation and mechanical failures appear in the structure [4]. To prevent such phenomena, that is, to reduce the vibration amplitude of the system, numerous research studies have been conducted [5–7]. Currently, the forced vibrations of rods mounted on elastic foundations are analyzed using various numerical and experimental methods. Initially, S. Timoshenko and J. Goodier [8] analyzed the bending vibrations of rods based on classical theory, while later Meirovitch [9] generalized the vibration theory of mechanical systems. Today, finite element and analytical-asymptotic methods are widely applied [10–12].

In this study, the forced vibrations of a vertically positioned rod, with its lower end rigidly clamped and a load with mass attached to the upper end, are investigated. The work examines the interaction of the rod with an elastic foundation, as well as the effects of the load mass and the frequency of external forces on the vibration amplitude. Analytical and experimental results are compared, and recommendations are provided to ensure the stability of the structure under resonance conditions [13-17].

FORMULATION OF THE PROBLEM

When an earthquake occurs, tall buildings undergo forced oscillatory motion in the horizontal direction. Schematically, such a building can be modeled as an elastic rod of length l. The lower end of the rod is clamped, while a load of mass m is attached to its upper end. A horizontal force of magnitude $F(t)$ acts on the load (see Fig. 1). We study the problem of determining the forced oscillatory motion of the load under the action of this horizontal force.

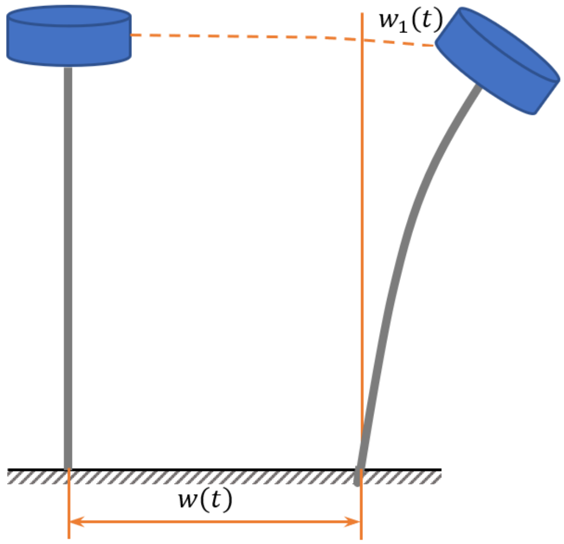

FIGURE 1. Research object

RESEARCH RESULTS

The load is subjected to a disturbing force $F_0(t)$, which initiates motion, and a restoring force $F_1 = -cw$, which tends to return it to the equilibrium position. In addition, the load is also affected by the gravitational force $P = mg$. In this case, the fundamental differential equation of motion for the load can be written in the following form.

$$m\frac{d^2w}{dt^2} + cw = F_0(t); \tag{1}$$

The differential equation of forced oscillatory motion written for the rod in equation (1) can be expressed in the following form:

$$\frac{d^2w}{dt^2} + \omega^2 w = F(t); \tag{2}$$

here,

$$\omega = \sqrt{\frac{c}{m}}\ ;\ F(t) = \frac{F_0(t)}{m}.$$

The general solution of the differential equation of forced oscillatory motion (2) consists of the sum of the general solution of its homogeneous part and a particular solution obtained by taking into account the right-hand side of the equation. The general solution of the homogeneous part of equation (2) can be written in the following form:

$$w = A\sin(\omega t + \alpha); \tag{3}$$

Here, A is the vibration amplitude, and α is the initial phase. They are determined from the initial conditions.

The particular solution of the forced oscillatory motion equation caused by the action of the force $F_0(t)$ is given as follows:

$$w(t) = \frac{1}{m\omega}\int_0^t F_0(t)\sin\omega(t-\tau)\,d\tau; \tag{4}$$

Thus, the motion of the load placed on the elastic rod under the action of the exciting force $F_0(t)$ is expressed as follows:

$$w(t) = A\sin(\omega t + \alpha)\frac{1}{m\omega}\int_0^t F_0(t)\sin\omega(t-\tau)\,d\tau\,; \qquad (5)$$

The first term of this equation determines the free vibration of the point, while the second term represents the forced vibration caused by the action of the force $F_0(t)$.

Practical issue. For the purpose of solving a practical problem, we take the value of the exciting force $F_0(t)$ in the form $5 \cdot e^{-10t}$. The mass of the load is assumed to be $1.84\ kg$. The vibration frequency is taken as 0.5 s^{-1}. Then, the oscillatory motion of the load over a 20-second interval is plotted in a graph (Fig. 2). To verify the accuracy of the obtained graph, an experiment is conducted using the “Alisher 2025” laboratory setup (Fig. 3).

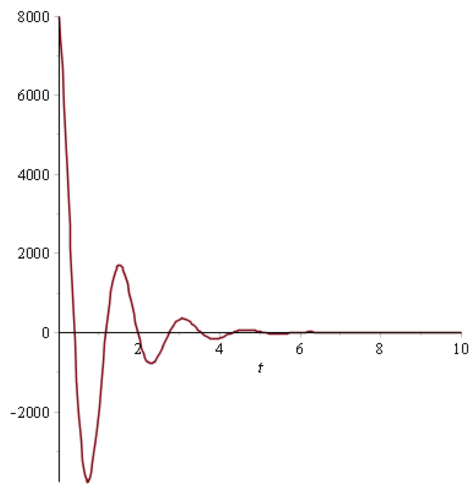

FIGURE 2. Analytical solution

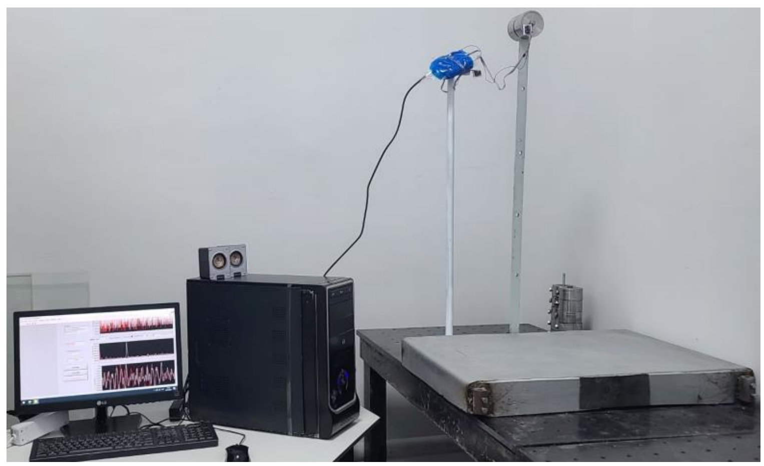

FIGURE 3. Alisher 2025 laboratory equipment

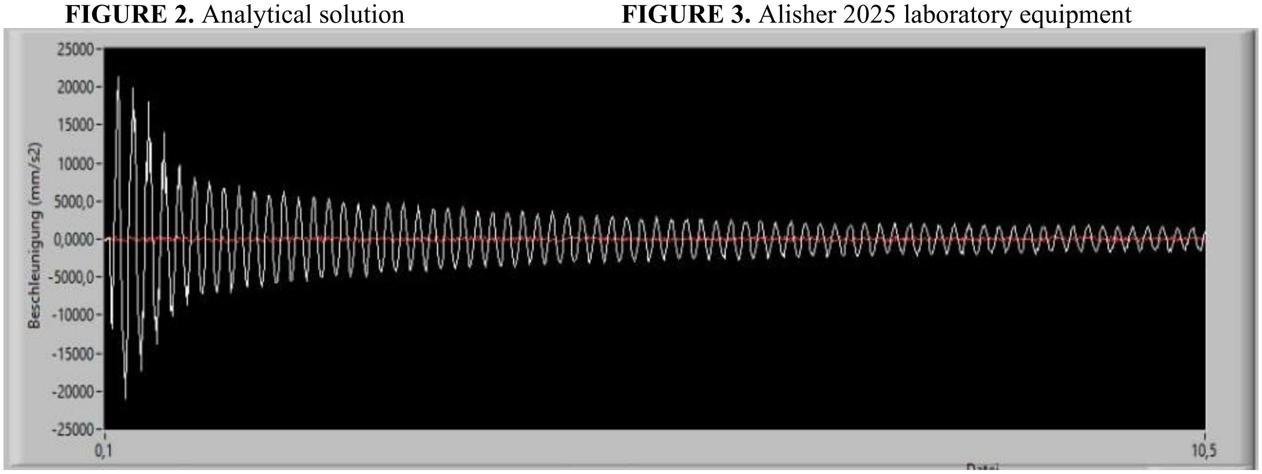

FIGURE 4. Vibrational motion of the load obtained on the Alisher 2025 laboratory equipment

CONCLUSION

The “Alisher 2025” laboratory setup has the following structure: a rigid base, a rod firmly fixed to the base, and a mass stone attached to the end of the rod. Initially, a sensor is installed on the load shown in Fig. 3. Then, a horizontal impulse is applied to the load. Using the sensor, the oscillation graph is transmitted to the monitor. Figure 4 shows the vibration graph of the load obtained from the monitor. As can be seen from Fig. 4, the vibration graph of the load replicates the graph obtained theoretically in Fig. 2. In conclusion, it can be stated that the solution obtained theoretically is a reliable solution.

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