Modeling of Calculation of Deformations of Shell-type Main Pipelines Beyond Elastic Limits Under Repeated Dynamic Loading

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Abstract. The article formulates the problem of deforming a thin-walled shell type main pipeline beyond the limits of elasticity under repeated dynamic loading based on the theory of small elastoplastic deformations. The equations of motion of a cylindrical shell – main pipeline are obtained using the Hamilton–Ostrogradsky principle. When solving the obtained boundary value problem, the method of finite differences of the second order of accuracy is used. Based on the application of central difference formulas, a system of algebraic equations is obtained. Based on the developed method for calculating the shell pipeline, the calculation results for repeated static loading, taking into account internal pressure, are presented.

INTRODUCTION

As is well known, the seismodynamic theory of complex systems of underground structures created in [1,2] is based on extensive factual research on the effects of earthquakes and extensive experimental work carried out in laboratory and field conditions. The theory is based on the possibility of moving the soil surrounding the underground structure. The nature of the interaction of structures with the surrounding soil, which has elastic, elastoplastic and viscoelastic properties, has been established. The study of damage and cracks in pipelines during repeated static and dynamic loading serves to improve the theory of calculations and ensure the seismic resistance of the elements of structures.

In [3], vibrations of underground shell-type structures from seismic impacts were investigated, and the problems of earthquake resistance of underground shell structures of open and closed profiles, constant and variable thicknesses were solved. In [4], the condition of pipelines after earthquakes of varying intensity is described, reserves of strength and reliability of pipelines are identified, and engineering methods for calculating their seismic effects are proposed. An earthquake impact survey indicates a significant influence of the type of pipes and their connections on the degree of damage to pipelines during earthquakes.

The monograph [5] is devoted to the experimental and theoretical study of the interaction of structures with the ground. Based on the experimental results, local laws of interaction of extended underground structures with soils of disturbed and undisturbed structures are constructed. Numerical solutions of the problem of propagation of plane shock and continuous waves in ground media and in "structure – ground" systems are given, taking into account the elastic, viscous, plastic properties of both soil and underground structures, as well as various laws of interaction on the surface of their contact. In [6], according to the developed computational model of a thin-walled large-diameter pipeline in the form of a cylindrical shell for straight pipelines and a toroidal one for curved ones. The problems of determining the frequencies of free bending vibrations, static and dynamic stability of above-ground pressure pipelines with flowing liquid are solved. It is noted in [7] that the mathematical model of deformation of a part of the pipeline on a viscoelastic base developed by him made it possible to estimate the stress level and bearing capacity of the pipeline over time. Accounting for soil creep allowed us to analyze changes in the stress-strain state of the pipeline section. A computational model of unsteady elastoplastic deformation of a spatial pipeline with liquid flowing in it and interacting with the ground was developed in [8].

The monograph [9] describes the methods of numerical modeling of main pipeline systems. The proposed concepts and methods are the basic elements of the theoretical foundation of modern computer tools for the effective solution of a wide range of technical and technological problems of designing structures and facilities of pipeline transport. The article [10] presents experimental and numerical results of the analysis of the interaction of soil and pipe. It is noted that the interaction of soil and its structure can be used to improve the planning and design of underground pipelines and reduce the risk of damage to the material.

It is noted in [11] that significant progress has been made in recent years in identifying and quantifying seismic hazards and determining the pipeline's response under conditions of large deformations. The application of analytical methods to the assessment of the pipeline's response to constant displacement is shown. The article [12] provides an overview and analysis of pipeline integrity, and also shows that earthquake-induced soil deformation can significantly affect the stress-strain state of underground pipelines.

STATEMENT OF THE TASK

Let us formulate the problem when the main pipeline – thin-walled shell is deformed beyond the limits of elasticity under repeated dynamic loading. In this case, the stress-strain state of the pipeline is determined based on the theory of small elastoplastic deformations [13].

Following the work [14], we introduce the differences:

$$\bar{e}_{ij}^{(n)} = (-1)^n (e_{ij}^{(n-1)} - e_{ij}^{(n)}); \quad \bar{U}_i^{(n)} = (-1)^n (U_i^{(n-1)} - U_i^{(n)}); \qquad \bar{\sigma}_{ij}^{(n)} = (-1)^n (\sigma_{ij}^{(n-1)} - \sigma_{ij}^{(n)}). \tag{1}$$

To determine the components of displacements $\bar{U}_i^{(n)}$ and deformations $\bar{e}_{ij}^{(n)}$ under nth loading, we have the following relations [15]:

$$\bar{U}_\alpha^{(n)} = \bar{U}^{(n)} - \frac{\gamma}{A} \cdot \frac{\partial \bar{W}^{(n)}}{\partial \alpha}; \quad \bar{U}_\beta^{(n)} = (1 + k_2 \gamma) \bar{V}^{(n)} - \frac{\gamma}{B} \cdot \frac{\partial \bar{W}^{(n)}}{\partial \beta}; \quad \bar{U}_\gamma^{(n)} = \bar{W}^{(n)}(\alpha, \beta); \tag{2}$$

$$\bar{e}_{\alpha\alpha}^{(n)} = \frac{1}{R} \frac{\partial \bar{U}^{(n)}}{\partial \alpha} - \frac{\gamma}{R^2} \frac{\partial^2 \bar{W}^{(n)}}{\partial \alpha^2}; \quad \bar{e}_{\beta\beta}^{(n)} = \frac{\partial \bar{V}^{(n)}}{R \partial \beta} - (\gamma - k_2 \gamma^2) \frac{\partial^2 \bar{W}^{(n)}}{R^2 \partial \beta^2} + (1 - k_2 \gamma + k_2^2 \gamma^2) k_2 \bar{W}^{(n)};$$

$$\bar{e}_{\alpha\beta}^{(n)} = (1 - k_2 \gamma + k_2^2 \gamma^2) \frac{\partial \bar{U}^{(n)}}{R \partial \beta} - (\gamma - k_2 \gamma^2) \frac{\partial^2 \bar{W}^{(n)}}{R^2 \partial \alpha \partial \beta} + (1 + k_2 \gamma) \frac{\partial \bar{V}^{(n)}}{R \partial \alpha} - \frac{\gamma}{R^2} \frac{\partial^2 \bar{W}^{(n)}}{\partial \alpha \partial \beta}. \tag{3}$$

Under alternating loading, the stress and strain components are related as follows [16]:

$$\sigma_{\alpha\alpha}^{(n)} = G_1 \left\{ \left(e_{\alpha\alpha}^{(n)} + \mu e_{\beta\beta}^{(n)} \right) - \left[\omega^{(n)} \left(\bar{e}_{\alpha\alpha}^{(n)} + \mu \bar{e}_{\beta\beta}^{(n)} \right) + \sum_{m=1}^{k-1} \omega^{0(n-m)} \left(\bar{e}_{\alpha\alpha}^{0(n-m)} + \mu \bar{e}_{\beta\beta}^{0(n-m-1)} \right) \right] \right\}$$

$$\sigma_{\beta\beta}^{(n)} = G_1 \left\{ \left(e_{\beta\beta}^{(n)} + \mu e_{\alpha\alpha}^{(n)} \right) - \left[\omega^{(n)} \left(\bar{e}_{\beta\beta}^{(n)} + \mu \bar{e}_{\alpha\alpha}^{(n)} \right) + \sum_{m=1}^{k-1} \omega^{0(n-m)} \left(\bar{e}_{\beta\beta}^{0(n-m)} + \mu \bar{e}_{\alpha\alpha}^{0(n-m-1)} \right) \right] \right\},$$

$$\sigma_{\alpha\beta}^{(n)} = G_1 \left\{ e_{\alpha\beta}^{(n)} - \omega^{(n)} \bar{e}_{\alpha\beta}^{(n)} + \sum_{m=1}^{k-1} \omega^{0(n-m)} \bar{e}_{\alpha\beta}^{0(n-m-1)} \right\}. \tag{4}$$

DERIVATION OF THE EQUATION OF MOTION

Under variable loading, we use the Hamilton–Ostrogradsky principle [15] to obtain the equation of motion of the cylindrical shell of the main pipeline:

$$\int_t (\delta T^{(n)} - \delta \Pi^{(n)} + \delta A^{(n)}) dt = 0. \tag{5}$$

Here, the kinetic energy variation $\delta T^{(n)}$ is determined by the formula:

$$\int_t \delta T^{(n)}dt = \int_\beta\int_\alpha \left\{ \rho h \frac{\partial U^{(n)}}{\partial t}\delta U^{(n)} + \left[\rho\left(h + k_2^2\frac{h^3}{12}\right)\frac{\partial V^{(n)}}{\partial t} - \rho k_2 \frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t\partial\beta}\right]\delta V^{(n)} + \left[\rho h \frac{\partial W^{(n)}}{\partial t} - \rho\frac{h^3}{12R^2}\frac{\partial^3 W^{(n)}}{\partial t\partial\alpha^2} + \right.\right.$$

$$\left.\left. + \rho k_2 \frac{h^3}{12R}\frac{\partial^2 V^{(n)}}{\partial t\partial\beta} - \rho\frac{h^3}{12R^2}\frac{\partial^3 W^{(n)}}{\partial t\partial\beta^2}\right]\delta W^{(n)}\right\} R^2 d\alpha d\beta \Big|_t + \int_\beta \rho\frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t\partial\alpha}\delta W^{(n)} R d\beta \Big|_t\Big|_\alpha - \int_\alpha \left(\rho k_2\frac{h^3}{12}\frac{\partial V^{(n)}}{\partial t} - \right.$$

$$\left. - \rho\frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t^2\partial\beta}\right)\delta W^{(n)} R d\alpha \Big|_t\Big|_\beta - \int_t\int_\beta\int_\alpha \left[\rho h\frac{\partial^2 U^{(n)}}{\partial t^2}\delta U^{(n)} + \left(\rho\left(h + k_2^2\frac{h^3}{12}\right)\frac{\partial^2 V^{(n)}}{\partial t^2} - \rho k_2\frac{h^3}{12R}\frac{\partial^3 W^{(n)}}{\partial t^2\partial\beta}\right)\delta V^{(n)} - \right.$$

$$\left. + \left[\rho h\frac{\partial^2 W^{(n)}}{\partial t^2} - \rho\frac{h^3}{12R^2}\frac{\partial^4 W^{(n)}}{\partial t^2\partial\alpha^2} + \rho k_2\frac{h^3}{12R}\frac{\partial^3 V^{(n)}}{\partial t^2\partial\beta} - \rho\frac{h^3}{12R^2}\frac{\partial^4 W^{(n)}}{\partial t^2\partial\beta^2}\right]\delta W^{(n)}\right] R^2 d\alpha d\beta dt -$$

$$- \int_t\int_\beta \rho\frac{h^3}{12R}\frac{\partial^3 W^{(n)}}{\partial t^2\partial\alpha}\delta W^{(n)} R d\beta dt \Big|_\alpha + \int_t\int_\alpha \left(\rho k_2\frac{h^3}{12}\frac{\partial^2 V^{(n)}}{\partial t^2} - \rho\frac{h^3}{12R}\frac{\partial^3 W^{(n)}}{\partial t^2\partial\beta}\right)\delta W^{(n)} R d\alpha dt \Big|_\beta . \tag{6}$$

Variations of the work of external forces are also provided in the form of:

$$\int_t \delta A^{(n)}dt = \int_t\int_\beta\int_\alpha \left\{ \left[N^{(n)}\left(P_1^{(n)}\right) + N^{(n)}\left(q_1^{(n)}\right)\right]\delta U^{(n)} + \left[N^{(n)}\left(P_2^{(n)}\right) + N^{(n)}\left(q_2^{(n)}\right)\right]\delta V^{(n)} + \left[\frac{\partial}{R\partial\alpha}\left(M^{(n)}\left(P_1^{(n)}\right) + M^{(n)}\left(q_1^{(n)}\right)\right) + \right.\right.$$

$$\left.\left. + \frac{\partial}{R\partial\beta}\left(M^{(n)}\left(P_2^{(n)}\right) + M^{(n)}\left(q_2^{(n)}\right)\right) + Q^{(n)}\left(P_3^{(n)}\right) + Q^{(n)}\left(q_3^{(n)}\right)\right]\delta W^{(n)}\right\} R^2 d\alpha d\beta dt + \int_t\int_\beta \left\{ N^{(n)}\left(\varphi_1^{(n)}\right)\delta U^{(n)} + \right.$$

$$\left. + N^{(n)}\left(\varphi_2^{(n)}\right)\delta V^{(n)} + \left[q^{(n)}\left(\varphi_3^{(n)}\right) + \frac{\partial M^{(n)}\left(\varphi_2^{(n)}\right)}{R\partial\beta}\right]\delta W^{(n)} - M^{(n)}\left(\varphi_1^{(n)}\right)\delta\frac{\partial W^{(n)}}{R\partial\alpha}\right\} R d\beta dt \Big|_\alpha -$$

$$\int_t M^{(n)}\left(\varphi_2^{(n)}\right)\delta W^{(n)}dt \Big|_\alpha\Big|_\beta + \int_t\int_\alpha \left\{ N^{(n)}\left(f_1^{(n)}\right)\delta U^{(n)} + N^{(n)}\left(f_2^{(n)}\right)\delta V^{(n)} + \left[Q^{(n)}\left(f_3^{(n)}\right) + \right.\right.$$

$$\left.\left. + \frac{\partial M^{(n)}\left(f_1^{(n)}\right)}{R\partial\alpha}\right]\delta W^{(n)} - M^{(n)}\left(f_2^{(n)}\right)\delta\frac{\partial W^{(n)}}{R\partial\beta}\right\} R d\alpha dt \Big|_\beta - \int_t M^{(n)}\left(f_1^{(n)}\right)\delta W^{(n)}dt \Big|_\beta\Big|_\alpha \tag{7}$$

The variations of potential energy in this formulation are determined by the formula:

$$\int_t \delta\Pi^{(n)}dt = \int_t\int_v \left(\sigma_{\alpha\alpha}^{(n)}\delta l_{\alpha\alpha}^{(n)} + \sigma_{\beta\beta}^{(n)}\delta l_{\beta\beta}^{(n)} + \sigma_{\alpha\beta}^{(n)}\delta l_{\alpha\beta}^{(n)}\right) dV dt.. \tag{8}$$

Taking into account the relations (3) – (4), the internal effort moments are determined. Substituting them in (8) and performing integration operations in parts, after some transformations for the variation of potential energy under repeated dynamic loading, we obtain the following expressions:

$$\int_t \delta\Pi dt = \int_t\int_\beta \left(N_\alpha^{(n)}\delta U^{(n)} + N_{\alpha\beta}^{(n)}\delta V^{(n)} + \left(\frac{\partial M_\alpha^{(n)}}{R\partial\alpha} + 2\frac{\partial M_{\alpha\beta}^{(n)}}{R\partial\beta}\right)\delta W^{(n)} - M_\alpha^{(n)}\delta\frac{\partial W^{(n)}}{R\partial\alpha}\right) R d\beta dt \Big|_\alpha -$$

$$- \int_t M_{\alpha\beta}^{(n)}\delta W^{(n)}\Big|_\alpha\Big|_\beta dt + \int_t\int_\alpha \left(\left(1 + k_2^2\frac{h^2}{12}\right) N_{\alpha\beta}^{(n)}\delta U^{(n)} + N_\beta^{(n)}\delta V_\alpha^{(n)} + \left(\frac{\partial M_\beta^{(n)}}{R\partial\beta} + 2\frac{\partial M_{\alpha\beta}^{(n)}}{R\partial\alpha}\right)\delta W^{(n)} - \right.$$

$$\left. - M_\beta^{(n)}\delta\frac{\partial W^{(n)}}{R\partial\beta}\right) R d\alpha dt \Big|_\beta - \int_t M_{\alpha\beta}^{(n)}\delta W^{(n)}\Big|_\alpha\Big|_\beta dt - \int_t\int_\beta\int_\alpha \left(\frac{\partial N_\alpha^{(n)}}{R\partial\alpha} + \left(1 + k_2^2\frac{h^2}{12}\right)\frac{\partial N_{\alpha\beta}^{(n)}}{R\partial\beta}\right)\delta U^{(n)} +$$

$$+ \left(\frac{\partial N_{\alpha\beta}^{(n)}}{R\partial\alpha} + 2\frac{\partial N_\beta^{(n)}}{R\partial\beta}\right)\delta V^{(n)} + \left(\frac{\partial^2 M_\alpha^{(n)}}{R^2\partial\alpha^2} + 2\frac{\partial^2 M_{\alpha\beta}^{(n)}}{R^2\partial\alpha\partial\beta} + \frac{\partial^2 M_\alpha^{(n)}}{R^2\partial\beta^2} - \left(1 + k_2^2\frac{h^2}{12}\right) k_2 N_\beta^{(n)}\right)\delta W^{(n)}\Bigg) R d\alpha d\beta dt, \tag{9}$$

where

$$-a_{14\omega}^{0(n-m)}\frac{\partial^2}{\partial\beta^2}\left(W^{0(n-m)}-W^{0(n-m-1)}\right)-a_{13\omega}^{0(n-m)}\frac{\partial}{\partial\beta}\left(V^{0(n-m)}-V^{0(n-m-1)}\right)+a_{15\omega}^{0(n-m)}\left(W^{0(n-m)}-W^{0(n-m-1)}\right)\Big],$$

$$M_\alpha^{(n)}=G_1\Big[\left(a_{41}-a_{41\omega}^{(n)}\right)\frac{\partial^2W^{(n)}}{\partial\alpha^2}-\left(a_{42}-a_{42\omega}^{(n)}\right)\frac{\partial^2W^{(n)}}{\partial\beta^2}-\left(a_{43}-a_{43\omega}^{(n)}\right)W^{(n)}+M_\alpha^{0(n-1)}+M_\alpha^{0(n-m)}\Big],$$

$$\widetilde{M}_\alpha^{(n)}=\left(a_{41}-a_{41\omega}^{(n)}\right)\frac{\partial^2W^{(n)}}{\partial\alpha^2}-\left(a_{42}-a_{42\omega}^{(n)}\right)\frac{\partial^2W^{(n)}}{\partial\beta^2}-\left(a_{43}-a_{43\omega}^{(n)}\right)W^{(n)};$$

$$M_\alpha^{0(n-1)}=\left[-a_{41\omega}^{0(n-1)}\frac{\partial^2W^{(n0}}{\partial\alpha^2}+a_{42\omega}^{0(n-1)}\frac{\partial^2W^{(n)}}{\partial\beta^2}+a_{43\omega}^{0(n-1)}W^{(n)}\right];$$

$$M_\alpha^{0(n-m)}=\sum_{m=1}^{k-1}\left[a_{41\omega}^{0(n-1)}\frac{\partial^2}{\partial\alpha^2}\left(W^{0(n-m)}-W^{0(n-m)}\right)-a_{42\omega}^{0(n-1)}\frac{\partial^2}{\partial\beta^2}\left(W^{0(n-m)}-W^{0(n-m)}\right)-a_{43\omega}^{0(n-1)}\left(W^{0(n-m)}-W^{0(n-m)}\right)\right].$$

Relations (9) can be generalized using expressions of internal efforts and moments, according to [16,17]:

$$N_\alpha^{(n)}=\widetilde{N}_\alpha^{(n)}-N_\alpha^{0(n-1)}+N_\alpha^{0(n-m)}$$

$$M_\alpha^{(n)}=\widetilde{M}_\alpha^{(n)}-M_\alpha^{0(n-1)}+M_\alpha^{0(n-m)}.$$

Now we substitute the kinetic (6) and potential energies (9), as well as the work of external forces (7) in (5). As a result, we obtain systems of equations of pipeline motion with appropriate boundary and initial conditions:

$$\rho h\frac{\partial^2U^{(n)}}{\partial t^2}-\frac{G_1}{R}\left[\widetilde{a}_{15}\cdot\frac{\partial W^{(n)}}{\partial\alpha}+\widetilde{a}_{11}\frac{\partial^2U^{(n)}}{\partial\alpha^2}-\widetilde{a}_{12}\frac{\partial^3W^{(n)}}{\partial\alpha^3}+\widetilde{a}_{13}\frac{\partial^2V^{(n)}}{\partial\beta\partial\alpha}-\widetilde{a}_{14}\frac{\partial^3W^{(n)}}{\partial\beta^2\partial\alpha}\right]+$$
$$+\frac{G}{R}\left(1+k_2^2\frac{h^2}{12}\right)\left[\widetilde{a}_{31}\frac{\partial^2U^{(n)}}{\partial\beta^2}+\widetilde{a}_{32}\frac{\partial^3W^{(n)}}{\partial\alpha\partial\beta^2}+\widetilde{a}_{33}\frac{\partial^2V^{(n)}}{\partial\alpha\partial\beta}+\widetilde{a}_{34}\frac{\partial^3W^{(n)}}{\partial\alpha\partial\beta^2}\right]+N^{(n)}\left(P_1^{(n)}\right)+N^{(n)}\left(q_1^{(n)}\right)=0;$$

$$\rho\left(h+k_2^2\frac{h^3}{12}\right)\frac{\partial^2V^{(n)}}{\partial t^2}-\rho k_2\frac{h^3}{12R}\frac{\partial^3W^{(n)}}{\partial t^2\partial\beta}+\frac{G}{R}\left[\widetilde{a}_{31}\frac{\partial^2U^{(n)}}{\partial\beta\partial\alpha}+\widetilde{a}_{32}\frac{\partial^3W^{(n)}}{\partial\alpha^2\partial\beta}+\widetilde{a}_{33}\frac{\partial^2V^{(n)}}{\partial\alpha^2}+\widetilde{a}_{34}\frac{\partial^3W^{(n)}}{\partial\alpha^2\partial\beta}\right]+$$
$$+\frac{G_1}{R}\left[\widetilde{a}_{21}\frac{\partial^2V^{(n)}}{\partial\beta\partial\alpha}-\widetilde{a}_{22}\frac{\partial^3W}{\partial\beta^2\partial\alpha}+\widetilde{a}_{23}\frac{\partial W}{\partial\alpha}+\widetilde{a}_{24}\frac{\partial^2U}{\partial\alpha^2}-\widetilde{a}_{25}\frac{\partial^3W}{\partial\beta^2\partial\alpha}\right]+N^{(n)}\left(P_2^{(n)}\right)+N^{(n)}\left(q_2^{(n)}\right)=0;$$

$$\rho h\frac{\partial^2W^{(n)}}{\partial t^2}-\rho\frac{h^3}{12R^2}\frac{\partial^4W^{(n)}}{\partial t^2\partial\alpha^2}+\rho k_2\frac{h^3}{12R}\frac{\partial^3V^{(n)}}{\partial t^2\partial\beta}-\rho\frac{h^3}{12R^2}\frac{\partial^4W^{(n)}}{\partial t^2\partial\beta^2}+\frac{G_1}{R^2}\left[\widetilde{a}_{41}\frac{\partial^4W^{(n)}}{\partial\alpha^4}-\right.$$
$$\left.-\widetilde{a}_{42}\frac{\partial^4W^{(n)}}{\partial\beta^2\partial\alpha^2}-\widetilde{a}_{43}\frac{\partial^2W^{(n)}}{\partial\alpha^2}\right]+\frac{2G}{R^2}\left[\widetilde{a}_{61}\frac{\partial^3U^{(n)}}{\partial\beta^2\partial\alpha}+\widetilde{a}_{62}\frac{\partial^4W^{(n)}}{\partial\alpha^2\partial\beta^2}+\widetilde{a}_{63}\frac{\partial^3V^{(n)}}{\partial\alpha^2\partial\beta}\right]+$$
$$+\frac{G_1}{R^2}\left[\widetilde{a}_{41}\frac{\partial^4W^{(n)}}{\partial\alpha^2\partial\beta^2}-\widetilde{a}_{42}\frac{\partial^4W^{(n)}}{\partial\beta^4}-\widetilde{a}_{43}\frac{\partial^2W^{(n)}}{\partial\beta^2}\right]-G_1k_2\left(1+k_2^2\frac{h^2}{12}\right)\left[\widetilde{a}_{21}\frac{\partial V^{(n)}}{\partial\beta}-\widetilde{a}_{22}\frac{\partial^2W}{\partial\beta^2}+\right.\qquad(10)$$
$$\left.\widetilde{a}_{23}W+\widetilde{a}_{24}\frac{\partial U}{\partial\alpha}-\widetilde{a}_{25}\frac{\partial^2W}{\partial\beta^2}\right]+\frac{\partial}{R\partial\alpha}\left(M^{(n)}\left(P_1^{(n)}\right)+M^{(n)}\left(q_1^{(n)}\right)\right)+\frac{\partial}{R\partial\beta}\left(M^{(n)}\left(P_2^{(n)}\right)+M^{(n)}\left(q_2^{(n)}\right)\right)+$$
$$+Q^{(n)}\left(P_3^{(n)}\right)+Q^{(n)}\left(q_3^{(n)}\right)=0.$$

Boundary conditions for the parameter α:

$$\left[\widetilde{a}_{13}\frac{\partial V^{(n)}}{\partial\beta}+\widetilde{a}_{11}\frac{\partial U^{(n)}}{\partial\alpha}-\widetilde{a}_{12}\frac{\partial^2W^{(n)}}{\partial\alpha^2}-\widetilde{a}_{14}\frac{\partial^2W^{(n)}}{\partial\beta^2}+\widetilde{a}_{15}W^{(n)}+N^{(n)}\left(\varphi_1^{(n)}\right)\right]\delta U^{(n)}+$$
$$+\left[\widetilde{a}_{31}\frac{\partial U^{(n)}}{\partial\beta}+\widetilde{a}_{32}\frac{\partial^2W^{(n)}}{\partial\alpha\partial\beta}+\widetilde{a}_{33}\frac{\partial V^{(n)}}{\partial\alpha}+\widetilde{a}_{34}\frac{\partial^2W^{(n)}}{\partial\alpha\partial\beta}+N^{(n)}\left(\varphi_2^{(n)}\right)\right]\delta V^{(n)}+$$

$$+\frac{1}{R}\left[\left[\widetilde{a}_{41}\frac{\partial^3 W^{(n)}}{\partial\alpha^3}-\widetilde{a}_{42}\frac{\partial^3 W^{(n)}}{\partial\beta^2\partial\alpha}-\widetilde{a}_{43}\frac{\partial W^{(n)}}{\partial\alpha}\right]+\left[\widetilde{a}_{61}\frac{\partial^2 U^{(n)}}{\partial\beta^2}+\widetilde{a}_{62}\frac{\partial^3 W^{(n)}}{\partial\alpha\partial\beta^2}+\widetilde{a}_{63}\frac{\partial^2 V^{(n)}}{\partial\alpha\partial\beta}\right]\right]\delta W^{(n)}-.$$

$$-\left[-\widetilde{a}_{41}\frac{\partial^2 W^{(n)}}{\partial\alpha^2}-\widetilde{a}_{42}\frac{\partial^2 W^{(n)}}{\partial\beta^2}-\widetilde{a}_{43}W^{(n)}-M^{(n)}\left(f_2^{(n)}\right)\right]\delta\frac{\partial W^{(n)}}{R\partial\alpha}\Bigg|_{\alpha}=0 \tag{11}$$

The nodal effect according to the parameters α and β:

$$\left[\widetilde{a}_{61}\frac{\partial U^{(n)}}{\partial\beta}+\widetilde{a}_{62}\frac{\partial^2 W^{(n)}}{\partial\alpha\partial\beta}+\widetilde{a}_{63}\frac{\partial V^{(n)}}{\partial\alpha}+M^{(n)}\left(\varphi_2^{(n)}\right)\right]\delta W^{(n)}\Big|_{\beta}\Big|_{\alpha}=0. \tag{12}$$

Initial conditions for the t parameter:

$$\rho h\frac{\partial U^{(n)}}{\partial t}\delta U^{(n)}\Bigg|_{e}=0;\qquad \left[\rho\left(h+k_2^2\frac{h^3}{12}\right)\frac{\partial V^{(n)}}{\partial t}-\rho k_2\frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t\partial\beta}\right]\delta V^{(n)}\Bigg|_{t}=0;$$

$$\left[\rho h\frac{\partial W^{(n)}}{\partial t}-\rho\frac{h^3}{12R^2}\frac{\partial^3 W^{(n)}}{\partial t\partial\alpha^2}+\rho k_2\frac{h^3}{12R}\frac{\partial^2 V^{(n)}}{\partial t\partial\beta}-\rho\frac{h^3}{12R^2}\frac{\partial^3 W^{(n)}}{\partial t\partial\beta^2}\right]\delta W^{(n)}\Bigg|\Bigg|_{e}=0. \tag{13}$$

Boundary and initial effects on parameters t, α and β:

$$\int_{\beta}\rho\frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t\partial\alpha}\delta W^{(n)}Rd\beta\Bigg|_{t}\Bigg|_{\alpha}=0;\qquad \int_{\alpha}\left(\rho k_2\frac{h^3}{12}\frac{\partial V^{(n)}}{\partial t}-\rho\frac{h^3}{12R}\frac{\partial^2 W^{(n)}}{\partial t^2\partial\beta}\right)\delta W^{(n)}Rd\alpha\Bigg|_{t}\Bigg|_{\beta}=0. \tag{14}$$

CALCULATION METHOD

Let's consider the solutions of the boundary value problem and the calculation algorithm for the elastic case (first approximation). The movement of the pipeline, according to the Bubnov−Galerkin method, is represented as

$$U=\sum_n U_n(\alpha,t)\cos\frac{n\pi\beta}{\beta_1},\quad V=\sum_n V_n(\alpha,t)\sin\frac{n\pi\beta}{\beta_1},\quad W=\sum_n W_n(\alpha,t)\cos\frac{n\pi\beta}{\beta_1}.. \tag{15}$$

For this case, we will rewrite the system of differential equations of the main pipeline – cylindrical shell in the following form:

$$\begin{aligned}&-\alpha_1^{(1)}\frac{\partial^2 W_n}{\partial t^2}+\alpha_2^{(1)}\frac{\partial^4 W_n}{\partial t^2\partial\alpha^2}-\alpha_3^{(1)}\frac{\partial^2 V_n}{\partial t^2}-\alpha_4^{(1)}\frac{\partial^4 W_n}{\partial\alpha^4}+\alpha_5^{(1)}\frac{\partial^2 W_n}{\partial\alpha^2}-\alpha_7^{(1)}W_n-\alpha_8^{(1)}V_n-\alpha_6^{(1)}\frac{\partial U_n}{\partial\alpha}+Z_n=0\\&-a_2^{(2)}\frac{\partial^2 U_n}{\partial t^2}+a_2^{(2)}\frac{\partial^2 U_n}{\partial\alpha^2}+a_4^{(2)}\frac{\partial V_n}{\partial\alpha}+a_3^{(2)}\frac{\partial^2 W_n}{\partial\alpha}-a_5^{(2)}U_n+X_n=0;\\&-\alpha_2^{(3)}\frac{\partial^2 V_n}{\partial t^2}+\alpha_1^{(3)}\frac{\partial W_n}{\partial t^2}-\alpha_4^{(3)}\frac{\partial U_n}{\partial t}+\alpha_3^{(3)}\frac{\partial^2 V_n}{\partial\alpha^2}+\alpha_5^{(3)}W_n-\alpha_6^{(3)}V_n+Y_n=0\end{aligned} \tag{16}$$

Here

$$a_1^{(1)}=\left[1+k^2\cdot n^2\frac{h^2}{12}\right];\quad a_2^{(1)}=\frac{h^2}{12}\frac{1}{R^2};\quad a_3^{(1)}=\frac{k}{R}\frac{h^2}{12};\quad a_4^{(1)}=-\frac{h^2}{12R^2};$$

$$a_5^{(1)}=\frac{h^2}{12}(2n^2\frac{1}{R^2}-k^2);\quad a_6^{(1)}=\frac{\lambda}{\lambda+2\mu}\left(1+k^2\frac{h^2}{R^2}\right);$$

$$a_7^{(1)}=1+\frac{h^2}{12R^2}\left(2k^2+\frac{n^2}{R}-k^2n^2\right);\quad a_8^{(1)}=n\cdot\left[1+k^2\cdot\frac{h^2}{12}\right];$$

$$Z_n=\frac{R^2}{\pi(\lambda+2\mu)h^2}\int_0^{2\pi}\left[Q(P_3)+Q(q_3)+\frac{\partial}{\partial\alpha}\left(M(P_1)+M(q_1)\right)+\frac{\partial}{\partial\beta}\left(M(P_2)+M(q_2)\right)\right]\cos\frac{n\pi\beta}{\beta_1}d\beta;$$

$$a_1^{(2)}=1;\ a_2^{(2)}=1;\ \ \alpha_3^{(2)}=\frac{1}{\lambda+2\mu}\left[\lambda\left(1+k^2\frac{h^2}{12}\right)kR-(\lambda+\mu)\frac{k}{R}\frac{h^2}{12}\right];$$

$$\alpha_4^{(2)}=\frac{1}{\lambda+2\mu}\left(\lambda+\mu\left(1+k^2\frac{h^2}{12}\right)\right)n;\ \ \alpha_5^{(2)}=\frac{\mu}{\lambda+2\mu}\left(1+2k^2\frac{h^2}{12}\right)n^2;$$

$$X_n=\frac{R^2}{\pi(\lambda+2\mu)h^2}\int_0^{2\pi}\left(N(p_1)+N(q_1)\right)\cos\frac{n\pi\beta}{\beta_1}d\beta \tag{17}$$

$$a_1^{(3)}=\frac{h^2}{12}\frac{k}{R}n;\ \ a_2^{(3)}=1+k^2\frac{h^2}{12};\ \ a_3^{(3)}=\frac{\lambda}{\lambda+2\mu};\ \ a_4^{(3)}=\frac{\left(\lambda+\mu\left(1+k^2\left(h^2/12\right)\right)\right)n}{\lambda+2\mu};$$

$$\alpha_5^{(3)}=\left(\frac{k}{R}\frac{h^2}{12}n^2-\left(1+k^2\frac{h^2}{12}\right)Rkn\right);\ \ \alpha_6^{(3)}=n^2;\ \ Y_n=\frac{R^2}{\pi(\lambda+2\mu)h^2}\int_0^{2\pi}\left(N(p_2)+N(q_2)\right)\sin\frac{n\pi\beta}{\beta_1}d\beta.$$

Boundary conditions of the shell pipeline according to the parameter α:

$$\left.\left[b_1^{(1)}\frac{\partial^3W_n}{\partial\alpha^3}-b_2^{(1)}\frac{\partial W_n}{\partial\alpha}-b_3^{(1)}\frac{\partial V_n}{\partial\alpha}-b_4^{(1)}U_n+\bar{Z}_n\right]h\delta W_n\right|_\alpha=0;$$

$$\left.\left[-b_1^{(2)}\frac{\partial U_n}{\partial\alpha}+b_2^{(2)}W_n-b_3^{(2)}V_n+\bar{X}_n\right]h\delta U_n\right|_\alpha=0; \tag{18}$$

$$\left.\left[b_1^{(3)}\frac{\partial W_n}{\partial\alpha}-b_2^{(3)}\frac{\partial V_n}{\partial\alpha}+b_3^{(3)}U_n+\bar{Y}_n\right]h\delta V_n\right|_\alpha=0$$

$$\left.\left[-b_1^{(4)}\frac{\partial^2W_n}{\partial\alpha^2}+b_2^{(4)}W_n\bar{M}_n\right]h\delta\frac{\partial W_n}{\partial\alpha}\right|_\alpha=0$$

Initial conditions:

$$\left.\left[m_1^{(1)}\frac{\partial^2W_n}{\partial t^2}-m_2^{(1)}\frac{\partial^3W_n}{\partial t\partial\alpha^2}+m_3^{(1)}\frac{\partial^2V_n}{\partial t\partial\alpha}\right]\cdot t_0h\delta W_n\right|_t=0;$$

$$\left.m_1^{(2)}\frac{\partial U_n}{\partial t}t_0h\delta U_n\right|_t=0;\ \ \left.\left[m_1^{(3)}\frac{\partial W_n}{\partial t}+m_2^{(3)}\frac{\partial V_n}{\partial t}\right]\cdot t_0h\delta V_n\right|_t=0. \tag{19}$$

The system of differential equations (16) is represented in vector form by introducing the following vectors:

$$U_n=(W_nU_nV_n)^T,\quad F_n=(Z_nX_nY_n)^T.. \tag{20}$$

Taking into account (20), the system of differential equations (16) is written as follows:

$$A_1\ddot{U}_n+A_2\ddot{U}_n^{II}+A_3U_n^{IV}+A_4U_n^{II}+A_5U_n^{I}+A_6U_n+F_n=0.. \quad (21$$

where the matrices are of the third order

$$A_1=\begin{pmatrix}-a_1^{(1)}&-a_6^{(1)}&-a_3^{(1)}\\0&-a_1^{(2)}&0\\a_1^{(3)}&0&-a_2^{(3)}\end{pmatrix};\ A_2=\begin{pmatrix}a_2^{(1)}&0&0\\0&0&0\\0&0&0\end{pmatrix};\ A_3=\begin{pmatrix}-a_4^{(1)}&0&0\\0&0&0\\0&0&0\end{pmatrix};$$

$$A_4=\begin{pmatrix}a_5^{(1)}&0&0\\0&a_2^{(2)}&0\\0&0&a_3^{(3)}\end{pmatrix};\ A_5=\begin{pmatrix}0&-a_6^{(1)}&0\\a_3^{(2)}&0&a_4^{(2)}\\0&-a_4^{(3)}&0\end{pmatrix};\ A_6=\begin{pmatrix}-a_7^{(1)}&0&-a_8^{(1)}\\0&-a_5^{(2)}&0\\a_5^{(3)}&0&-a_6^{(3)}\end{pmatrix} \tag{22}$$

The matrix elements are shown in relations (17).

When solving boundary value problems (16), (18) and (19), the method of finite differences of the second order of accuracy is used. Based on the application of central difference formulas, the following system of algebraic equations is obtained:

$$B_n U_{n,i-1}^{k+1} + C_n U_{n,i}^{k+1} + B_n U_{n,i+1}^{k+1} + \overline{A}_n U_{n,i+1}^{k+1} + \overline{B}_n U_{n,i-1}^{k} + \overline{C}_n U_{n,i}^{k} + \overline{D}_n U_{n,i+1}^{k} + \overline{A}_n U_{n,i+2}^{k} + B_n U_{n,i-1}^{k-1} + \\ + C_n U_{n,i}^{k-1} + B_n U_{n,i+1}^{k-1} + \tau^2 F_{n,i}^{k} = 0 \tag{23}$$

The initial condition (19), after approximation, will take the following form:

$$\left[\overline{M}_1 U_{n,i-1}^{k+1} + \overline{M}_2 U_{n,i}^{k+1} + \overline{M}_3 U_{n,i+1}^{k+1} - \overline{M}_1 U_{n,i+1}^{k+1} - \overline{M}_1 U_{n,i-1}^{k-1} - \overline{M}_2 U_{n,i}^{k-1} - \overline{M}_3 U_{n,i+1}^{k-1}\right] \cdot t_0 h \delta U_{n,i+1}^{k-1} = 0 \tag{24}$$

The difference boundary value problem is solved using the run-through method [16]. It is assumed that the displacements and their velocities are set at the initial moment of time, and also, the pipeline is pinched at *α=0* and *α=1*. In vector form, the boundary conditions are expressed as follows:

$$U_{n,0}^{j} = 0; \quad A' U_{n,-1}^{j} = A' U_{n,1}^{j}; \; U_{n,N}^{j} = 0; \quad A' U_{n,N+1}^{j} = A' U_{n,N-1}^{j} \quad \text{by} \quad i = 1,2,..., N-1 \tag{25}$$

taking into account the boundary conditions (18), the system of equations (23) is written as

$$B_n U_{n,i-1}^{k+1} + C_n U_{n,i}^{k+1} + B_n U_{n,i+1}^{k+1} = b_{n,j} \tag{26}$$

where

$$b_{n,i} = \tau^2 F_{n,i}^{k} - (\overline{A}_n U_{n,i-2}^{k} + \overline{B}_n U_{n,i-1}^{k} + \overline{C}_n U_{n,i}^{k} + \overline{D}_n U_{n,i+1}^{k} + \overline{A}_n U_{n,i+2}^{k} + B_n U_{n,i-1}^{k-1} + C_n U_{n,i}^{k-1} + B_n U_{n,i+1}^{k-1})$$

From equation (26), we can deduce the solution for the *i*–th equation [18-20]:

$$U_{n,i}^{K+1} = f_i - H_i U_{n,i+1}^{K+i} \tag{27}$$

where

$$f_i = (C_n - B_n H_{i-1})^{-1}(b_{n,i} - B_n f_{i+1}), \quad H_i = (C_n - B_n H_{i-1})^{-1} B_n .$$

CALCULATION RESULTS

As an illustration, based on the developed method for calculating the shell pipeline, Table 1 shows the calculation results for repeated static loading, taking into account internal pressure. Geometric and mechanical characteristics of the cylinder: *h*=0.01sm; *μ*=0.3; *R*=150 sm; *L*=1120 sm; *β*=2π; *q*=1; $E = 2 \cdot 10^5$ MPa. The dimensionless results obtained are presented in the form of graphs and tables.

Table 1. The nature of the change in the calculated values along the length of the shell

α	$W \cdot 10^3$	$U \cdot 10^3$	M_1	M_2	N_1	N_2
0	0,0	0,0	-9,71608	-4,17511	-2,16607	-0,99421
0.1	-0,05403	-0,87381	-3,56012	-1,80509	-1,07511	-2,58409
0.2	-0,16721	0,06788	0,07746	-0,30733	-0,35142	-6,17013
0.3	-0,26960	2, 62508	2,15907	0,63612	-0,63503	-10,11210
0.4	-0,34052	5,70209	3,28106	1,24514	-1,78410	-13,10121
0.5	-0,36473	8,34110	3,53810	1,58013	-3,17805	-14,11012

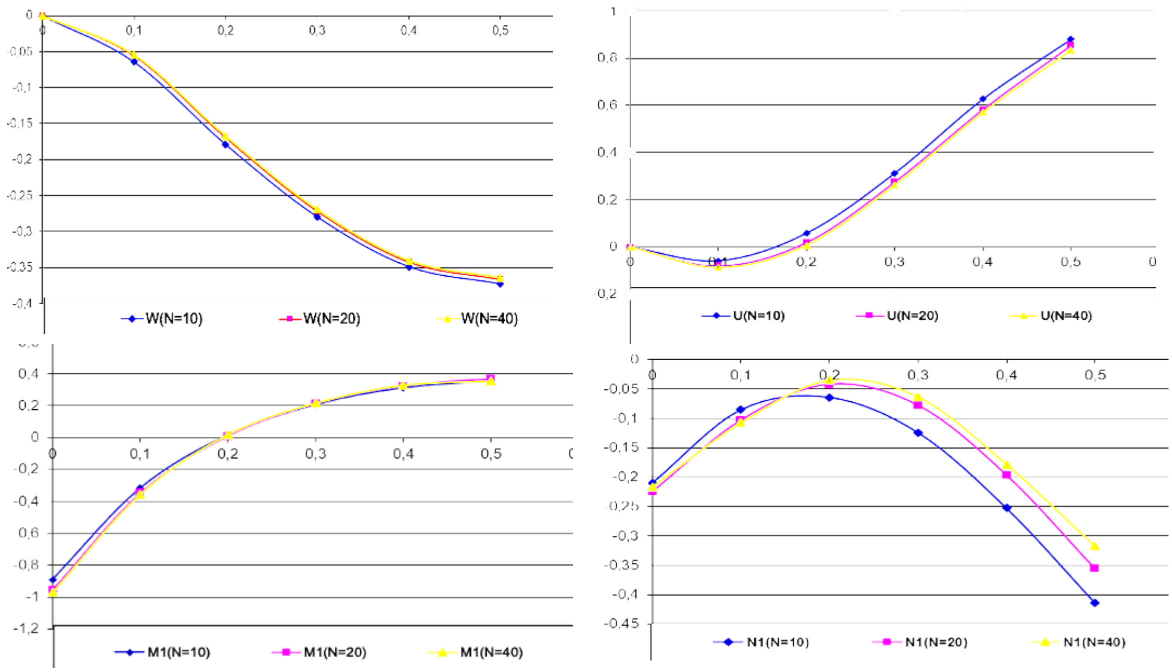

FIGURE 1. The nature of the convergence of the calculated values for different values of the grid pitch h

CONCLUSION

The results of the study of the stressed deformed state of the shell under loading show that plastic deformations occurring in the pipeline are localized in relatively narrow zones. Therefore, the destruction of the shell during loading is mainly determined by the accumulation of damage.

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